Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/30-1.1.4.2-c-x-^m-a-x^j+b-x^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [454]. This is test number [30].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (454)	0.00 (0)
Mathematica	100.00 (454)	0.00 (0)
Maple	84.80 (385)	15.20 (69)
Fricas	70.26 (319)	29.74 (135)
Giac	57.49 (261)	42.51 (193)
Mupad	42.51 (193)	57.49 (261)
Maxima	33.70 (153)	66.30 (301)
Sympy	25.33 (115)	74.67 (339)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

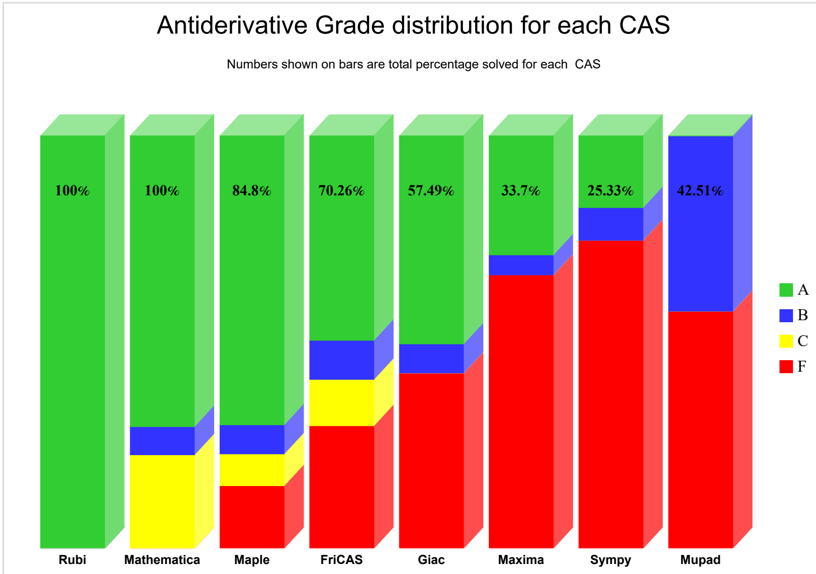
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

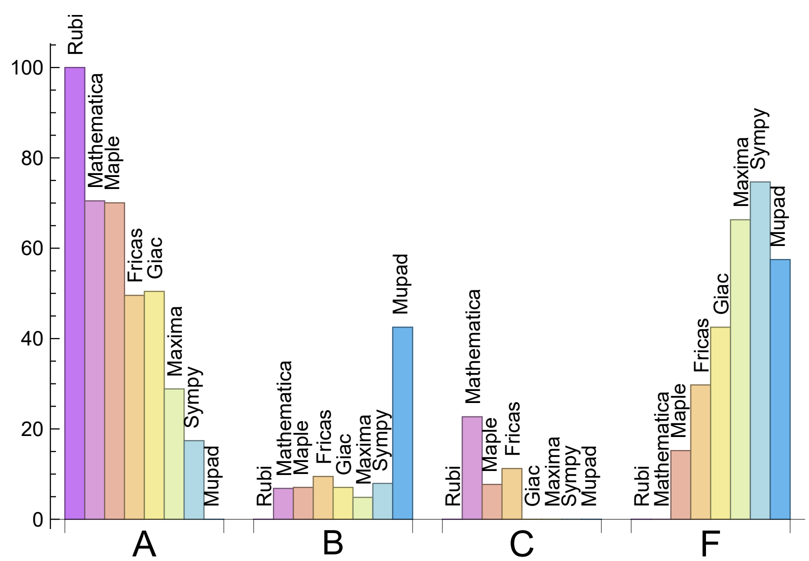
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.48	6.83	22.69	0.00
Maple	70.04	7.05	7.71	15.20
Giac	50.44	7.05	0.00	42.51
Fricas	49.56	9.47	11.23	29.74
Maxima	28.85	4.85	0.00	66.30
Sympy	17.40	7.93	0.00	74.67
Mupad	N/A	42.51	0.00	57.49

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	69	100.00 %	0.00 %	0.00 %
Fricas	135	37.78 %	28.89 %	33.33 %
Giac	193	97.41 %	0.00 %	2.59 %
Maxima	301	100.00 %	0.00 %	0.00 %
Sympy	339	89.97 %	5.01 %	5.01 %
Mupad	261	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

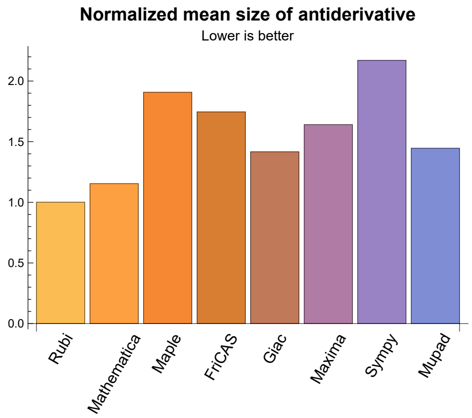
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

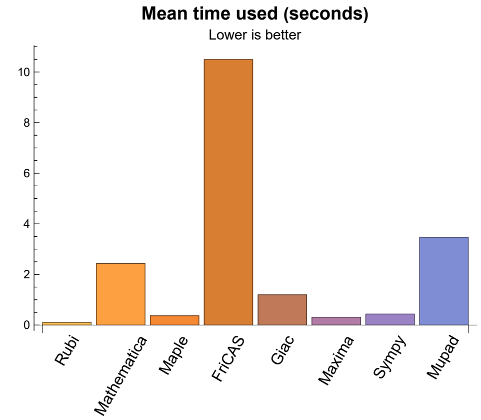
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	119.28	1.00	74.00	1.00
Mathematica	2.43	67.54	1.15	59.00	0.92
Maple	0.37	191.97	1.91	79.00	0.94
Maxima	0.31	42.06	1.64	27.00	0.89
Fricas	10.49	114.06	1.75	54.00	1.00
Sympy	0.44	46.94	2.17	26.00	0.88
Giac	1.20	80.54	1.42	53.00	0.96
Mupad	3.47	46.89	1.45	37.00	0.87

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

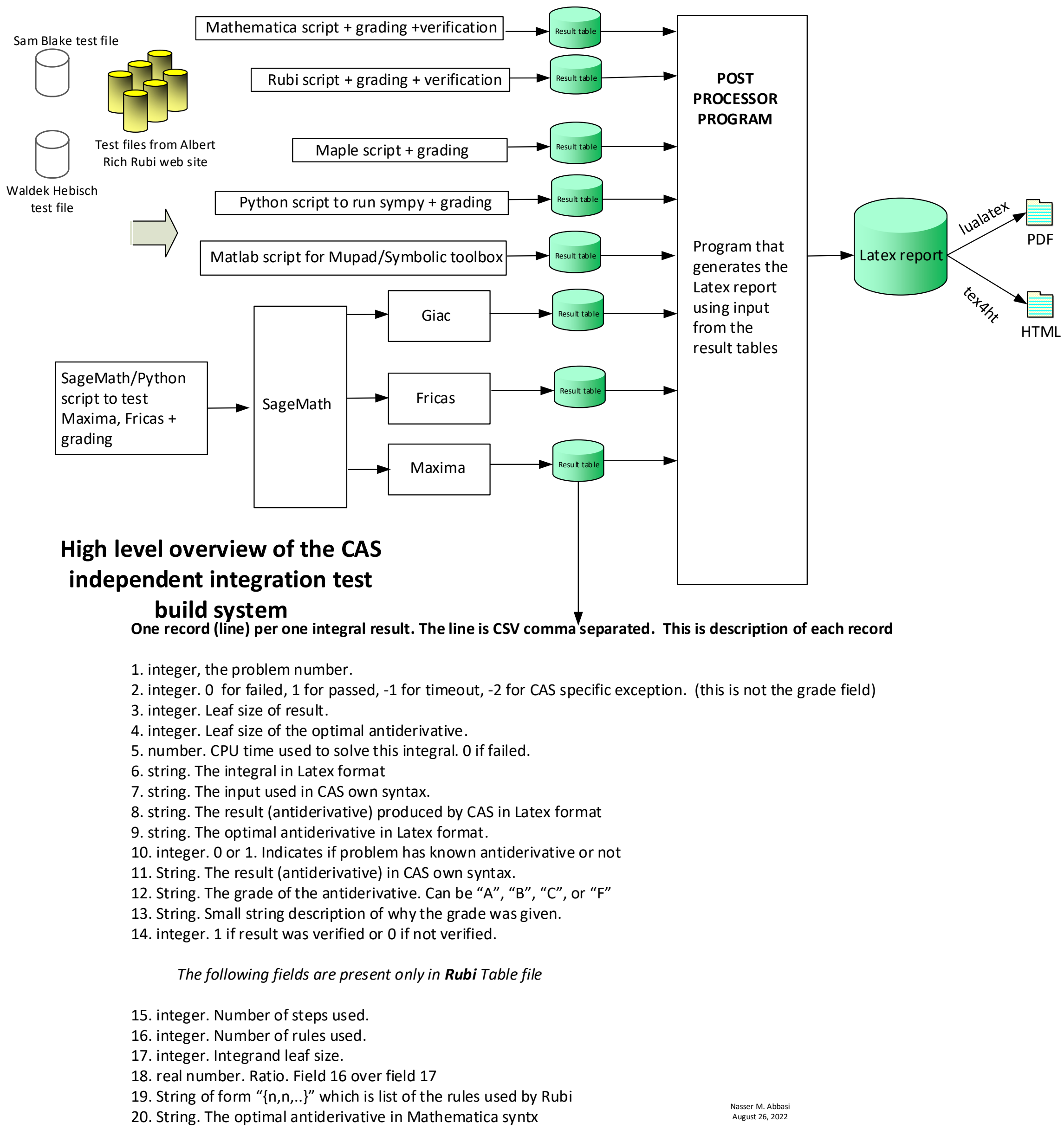
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 411, 412, 413, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 26, 28, 30, 32, 34, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 349, 350, 351, 352, 353, 354, 392, 410, 414, 415, 416, 417, 418, 419, 420, 421 }

C grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 181, 182, 183, 184, 200, 201, 202, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 437, 438 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 94, 95, 96, 103, 105, 117, 118, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 303, 306, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 352, 354, 355, 356, 358, 360, 362, 363, 369, 377, 382, 383, 384, 385, 386, 388, 389, 394, 402, 408, 412, 431, 432, 433, 434, 435, 436, 448, 449, 450 }

B grade: { 28, 32, 86, 104, 110, 111, 112, 119, 120, 124, 125, 126, 127, 272, 313, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 350, 351, 353, 357, 359, 361, 387 }

C grade: { 92, 93, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 407, 411 }

F grade: { 277, 278, 279, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 34, 35, 36, 37, 94, 95, 96, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 260, 261, 262, 263, 284, 285, 286, 303, 306, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 338, 339, 340, 344, 345, 346, 347, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 383, 387, 394, 402, 433, 434, 435, 436, 448, 449, 450 }

B grade: { 28, 30, 32, 283, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 348, 350, 351, 353 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 338, 339, 340, 344, 345, 346, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 387, 388, 389, 394, 402, 407, 408, 409, 410, 411, 412, 413, 414, 416, 419, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 28, 30, 32, 34, 79, 84, 127, 167, 178, 179, 185, 186, 187, 188, 189, 194, 195, 196, 197, 283, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354 }

C grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 98, 99, 102, 289, 290, 291, 292, 293, 294, 301, 302, 304, 305, 307 }

F grade: { 97, 100, 101, 103, 104, 105, 110, 111, 112, 117, 118, 119, 120, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 190, 191, 192, 193, 198, 199, 200, 201, 202, 277, 278, 279, 295, 296, 298, 299, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 415, 417, 418, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 308, 309, 319, 320, 321, 322, 324, 338, 339, 340, 344, 345, 346, 347, 358, 359, 360, 362, 363, 369, 377, 394, 433 }

B grade: { 9, 13, 15, 17, 19, 21, 26, 28, 30, 32, 283, 323, 325, 326, 327, 328, 329, 330, 332, 333, 334, 336, 341, 342, 343, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 402 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 331, 335, 337, 348, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 352, 354, 360, 361, 382, 384, 386, 387, 388, 408, 412, 432, 433, 434, 435, 436 }

B grade: { 26, 28, 30, 32, 34, 176, 177, 178, 179, 235, 240, 241, 242, 243, 244, 245, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 350, 351, 353, 362, 383, 431 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 99, 100, 101, 102, 114, 115, 116, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 325, 326, 327, 337, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 48, 61, 72, 94, 95, 96, 98, 105, 113, 134, 142, 153, 162, 170, 178, 189, 198, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 258, 260, 261, 262, 263, 266, 268, 280, 281, 282, 283, 284, 285, 286, 291, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 371, 382, 383, 384, 386, 387, 388, 392, 415, 416, 417, 418, 419, 420, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 444 }

C grade: { }

F grade: { 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 235, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 259, 264, 265, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 364, 365, 366, 367, 368, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 421, 422, 423, 424, 425, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.002	0.084	0.306	1.945	0.006	1.499	0.022

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.001	0.049	0.280	1.132	0.009	1.412	0.022

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	13	13	12	13	13
N.S.	1	1.00	1.00	0.88	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.002	0.000	0.046	0.282	2.011	0.010	1.360	0.019

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.015	0.289	1.743	0.005	1.466	0.017

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	14	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.08	0.85
time (sec)	N/A	0.003	0.001	0.018	0.281	1.699	0.021	1.576	0.024

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.010	0.001	0.343	0.290	1.416	0.011	1.161	0.041

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.013	0.001	0.373	0.287	2.168	0.008	1.616	0.033

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.007	0.002	0.365	0.295	1.307	0.007	1.732	0.030

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	24	24	24	24	24
N.S.	1	1.00	1.00	0.94	1.50	1.50	1.50	1.50	1.50
time (sec)	N/A	0.005	0.001	0.348	0.281	1.822	0.009	1.005	0.031

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.008	0.001	0.331	0.276	1.288	0.008	1.316	0.030

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	36	42	36	36
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.91	0.78	0.78
time (sec)	N/A	0.010	0.002	0.376	0.292	2.267	0.011	1.234	0.039

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.015	0.004	0.351	0.285	1.368	0.050	1.256	0.040

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.011	0.006	0.355	0.511	2.239	0.058	1.169	4.913

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.005	0.002	0.408	0.281	1.356	0.036	1.147	4.916

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.005	0.003	0.368	0.502	3.543	0.049	1.036	0.045

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	18	15	24	18
N.S.	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.008	0.003	0.337	0.285	2.147	0.084	2.205	0.063

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.010	0.009	0.353	0.509	1.337	0.069	2.572	4.956

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	33	31	43	31
N.S.	1	1.00	1.00	0.91	0.89	0.94	0.89	1.23	0.89
time (sec)	N/A	0.018	0.005	0.346	0.277	1.667	0.121	1.363	0.060

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
N.S.	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.015	0.014	0.368	0.513	1.292	0.100	0.918	4.937

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	44	45	42	57	46
N.S.	1	1.00	1.00	0.90	0.90	0.92	0.86	1.16	0.94
time (sec)	N/A	0.023	0.005	0.345	0.298	1.453	0.138	0.886	0.062

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.010	0.018	0.374	0.512	3.026	0.114	1.432	4.955

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	42	34	47	34	47	34
N.S.	1	1.00	0.87	1.11	0.89	1.24	0.89	1.24	0.89
time (sec)	N/A	0.021	0.012	0.359	0.289	2.394	0.134	1.462	0.049

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	45	49	136	92	47	44
N.S.	1	1.00	0.95	0.79	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.012	0.024	0.354	0.509	1.823	0.145	1.174	4.975

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	55	50	73	51	51	51
N.S.	1	1.00	0.84	1.12	1.02	1.49	1.04	1.04	1.04
time (sec)	N/A	0.028	0.024	0.359	0.287	1.349	0.176	1.371	0.053

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	55	64	172	114	59	58
N.S.	1	1.00	0.99	0.81	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.020	0.028	0.358	0.508	1.764	0.159	1.783	5.026

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	29	22	21	21	19	23	11
N.S.	1	1.00	2.23	1.69	1.62	1.62	1.46	1.77	0.85
time (sec)	N/A	0.008	0.004	0.350	0.295	1.599	0.030	1.081	4.977

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	18	14	14	15	14
N.S.	1	1.00	0.90	0.95	0.90	0.70	0.70	0.75	0.70
time (sec)	N/A	0.010	0.003	0.345	0.283	2.103	0.019	1.019	0.036

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	22	17	16	16	14	18	6
N.S.	1	1.00	3.67	2.83	2.67	2.67	2.33	3.00	1.00
time (sec)	N/A	0.005	0.002	0.352	0.298	1.162	0.026	1.717	0.058

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	13	8	8	15	8
N.S.	1	1.00	1.00	1.17	1.08	0.67	0.67	1.25	0.67
time (sec)	N/A	0.004	0.002	0.374	0.286	1.710	0.021	1.256	0.029

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.003	0.002	0.362	0.278	1.352	0.027	1.151	0.033

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	11	10	16	11
N.S.	1	1.00	1.00	1.07	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.006	0.002	0.360	0.279	3.929	0.029	1.262	4.960

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	24	19	18	20	15	20	8
N.S.	1	1.00	3.00	2.38	2.25	2.50	1.88	2.50	1.00
time (sec)	N/A	0.005	0.002	0.351	0.289	2.496	0.035	1.316	0.033

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	24	17	26	16
N.S.	1	1.00	1.00	0.95	0.91	1.09	0.77	1.18	0.73
time (sec)	N/A	0.010	0.003	0.372	0.279	2.805	0.033	0.995	0.033

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	31	24	25	30	24	27	13
N.S.	1	1.00	2.07	1.60	1.67	2.00	1.60	1.80	0.87
time (sec)	N/A	0.007	0.003	0.355	0.279	2.565	0.039	1.038	4.924

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	27	30	22	33	23
N.S.	1	1.00	1.00	0.90	0.93	1.03	0.76	1.14	0.79
time (sec)	N/A	0.012	0.003	0.361	0.291	2.703	0.040	1.552	0.033

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	18	14
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.93
time (sec)	N/A	0.007	0.003	0.348	0.267	2.405	0.055	1.797	4.950

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	15	15	12	18	16
N.S.	1	1.00	1.00	0.89	0.83	0.83	0.67	1.00	0.89
time (sec)	N/A	0.007	0.003	0.346	0.274	1.609	0.052	1.089	0.052

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	95	168	0	59	0	0	-1
N.S.	1	1.00	0.58	1.03	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.125	10.044	0.347	0.000	0.492	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	80	197	0	57	0	0	-1
N.S.	1	1.00	0.28	0.70	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.187	10.028	0.358	0.000	0.410	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	79	146	0	49	0	0	-1
N.S.	1	1.00	0.58	1.07	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.084	9.562	0.346	0.000	0.369	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	51	175	0	43	0	0	40
N.S.	1	1.00	0.20	0.69	0.00	0.17	0.00	0.00	0.16
time (sec)	N/A	0.125	7.781	0.351	0.000	0.847	0.000	0.000	5.051

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	48	124	0	34	0	0	-1
N.S.	1	1.00	0.42	1.10	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.060	6.860	0.347	0.000	0.591	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	51	177	0	40	0	0	-1
N.S.	1	1.00	0.21	0.71	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.146	10.013	0.353	0.000	0.421	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	53	123	0	36	0	0	-1
N.S.	1	1.00	0.46	1.06	0.00	0.31	0.00	0.00	-0.01
time (sec)	N/A	0.063	10.019	0.388	0.000	0.470	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	53	201	0	54	0	0	-1
N.S.	1	1.00	0.19	0.71	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.183	10.014	0.363	0.000	0.452	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	94	188	0	70	0	0	-1
N.S.	1	1.00	0.51	1.01	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.163	10.038	0.365	0.000	0.661	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	84	217	0	68	0	0	-1
N.S.	1	1.00	0.28	0.71	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.217	10.041	0.364	0.000	0.358	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	83	166	0	60	0	0	40
N.S.	1	1.00	0.53	1.05	0.00	0.38	0.00	0.00	0.25
time (sec)	N/A	0.093	10.033	0.368	0.000	0.542	0.000	0.000	5.005

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	52	195	0	58	0	0	-1
N.S.	1	1.00	0.19	0.71	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.318	10.015	0.349	0.000	0.574	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	49	144	0	47	0	0	-1
N.S.	1	1.00	0.37	1.07	0.00	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.186	9.205	0.346	0.000	0.441	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	52	194	0	52	0	0	-1
N.S.	1	1.00	0.19	0.71	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.267	10.012	0.347	0.000	0.386	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	54	139	0	45	0	0	-1
N.S.	1	1.00	0.40	1.04	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.089	10.015	0.348	0.000	0.422	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	54	196	0	51	0	0	-1
N.S.	1	1.00	0.19	0.71	0.00	0.18	0.00	0.00	-0.00
time (sec)	N/A	0.178	10.017	0.350	0.000	0.388	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	54	142	0	44	0	0	-1
N.S.	1	1.00	0.39	1.04	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.015	0.356	0.000	0.429	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	54	223	0	67	0	0	-1
N.S.	1	1.00	0.18	0.73	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.215	10.022	0.371	0.000	0.833	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	54	169	0	59	0	0	-1
N.S.	1	1.00	0.33	1.04	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.124	10.013	0.352	0.000	0.508	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	80	149	0	48	0	0	-1
N.S.	1	1.00	0.57	1.06	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.101	10.028	0.352	0.000	0.388	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	66	178	0	43	0	0	-1
N.S.	1	1.00	0.26	0.69	0.00	0.17	0.00	0.00	-0.00
time (sec)	N/A	0.151	10.034	0.343	0.000	0.539	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	127	0	34	0	0	-1
N.S.	1	1.00	0.55	1.09	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.065	10.030	0.391	0.000	0.316	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	53	158	0	22	0	0	-1
N.S.	1	1.00	0.23	0.69	0.00	0.10	0.00	0.00	-0.00
time (sec)	N/A	0.111	10.023	0.381	0.000	0.485	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	49	108	0	14	0	0	40
N.S.	1	1.00	0.53	1.17	0.00	0.15	0.00	0.00	0.43
time (sec)	N/A	0.035	10.011	0.351	0.000	0.293	0.000	0.000	5.035

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	48	182	0	42	0	0	-1
N.S.	1	1.00	0.19	0.72	0.00	0.17	0.00	0.00	-0.00
time (sec)	N/A	0.145	10.012	0.352	0.000	0.390	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	53	129	0	36	0	0	-1
N.S.	1	1.00	0.45	1.08	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.063	10.023	0.359	0.000	0.381	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	53	204	0	56	0	0	-1
N.S.	1	1.00	0.19	0.71	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.180	10.013	0.368	0.000	0.452	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	80	172	0	80	0	0	-1
N.S.	1	1.00	0.50	1.07	0.00	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.129	10.036	0.410	0.000	0.297	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	68	200	0	76	0	0	-1
N.S.	1	1.00	0.24	0.72	0.00	0.27	0.00	0.00	-0.00
time (sec)	N/A	0.191	10.019	0.397	0.000	0.507	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	67	147	0	68	0	0	-1
N.S.	1	1.00	0.49	1.07	0.00	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.095	10.021	0.385	0.000	0.334	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	57	182	0	61	0	0	-1
N.S.	1	1.00	0.23	0.72	0.00	0.24	0.00	0.00	-0.00
time (sec)	N/A	0.152	10.036	0.354	0.000	0.506	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	54	130	0	51	0	0	-1
N.S.	1	1.00	0.47	1.13	0.00	0.44	0.00	0.00	-0.01
time (sec)	N/A	0.068	10.020	0.345	0.000	0.409	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	56	184	0	60	0	0	-1
N.S.	1	1.00	0.22	0.72	0.00	0.24	0.00	0.00	-0.00
time (sec)	N/A	0.152	10.018	0.345	0.000	0.398	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	54	132	0	51	0	0	-1
N.S.	1	1.00	0.47	1.16	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.058	9.989	0.342	0.000	0.648	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	51	206	0	72	0	0	40
N.S.	1	1.00	0.19	0.75	0.00	0.26	0.00	0.00	0.15
time (sec)	N/A	0.162	10.024	0.497	0.000	0.506	0.000	0.000	5.168

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	56	150	0	68	0	0	-1
N.S.	1	1.00	0.40	1.08	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.097	10.018	0.408	0.000	0.413	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	56	228	0	90	0	0	-1
N.S.	1	1.00	0.18	0.75	0.00	0.29	0.00	0.00	-0.00
time (sec)	N/A	0.221	10.017	0.406	0.000	0.306	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	122	212	0	376	0	100	-1
N.S.	1	1.00	0.77	1.33	0.00	2.36	0.00	0.63	-0.01
time (sec)	N/A	0.163	0.203	0.421	0.000	2.843	0.000	1.513	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	68	72	0	108	0	80	-1
N.S.	1	1.00	0.54	0.57	0.00	0.86	0.00	0.63	-0.01
time (sec)	N/A	0.134	0.038	0.394	0.000	2.137	0.000	1.487	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	110	198	0	348	0	86	-1
N.S.	1	1.00	0.85	1.52	0.00	2.68	0.00	0.66	-0.01
time (sec)	N/A	0.136	0.164	0.346	0.000	3.429	0.000	1.455	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	61	0	97	0	64	-1
N.S.	1	1.00	0.63	0.60	0.00	0.96	0.00	0.63	-0.01
time (sec)	N/A	0.108	0.038	0.346	0.000	1.846	0.000	1.294	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	61	0	17	-1
N.S.	1	1.00	1.00	1.16	0.00	2.44	0.00	0.68	-0.04
time (sec)	N/A	0.027	0.082	0.352	0.000	1.256	0.000	1.391	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	46	50	0	86	0	50	-1
N.S.	1	1.00	0.61	0.66	0.00	1.13	0.00	0.66	-0.01
time (sec)	N/A	0.080	0.033	0.373	0.000	2.228	0.000	1.528	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	38	39	0	76	0	29	-1
N.S.	1	1.00	0.75	0.76	0.00	1.49	0.00	0.57	-0.02
time (sec)	N/A	0.051	0.080	0.364	0.000	1.267	0.000	1.365	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	39	0	75	0	33	-1
N.S.	1	1.00	0.69	0.76	0.00	1.47	0.00	0.65	-0.02
time (sec)	N/A	0.051	0.029	0.358	0.000	1.750	0.000	1.619	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	56	50	0	87	0	43	-1
N.S.	1	1.00	0.74	0.66	0.00	1.14	0.00	0.57	-0.01
time (sec)	N/A	0.075	0.077	0.363	0.000	1.024	0.000	0.990	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	63	0	23	-1
N.S.	1	1.00	1.00	1.16	0.00	2.52	0.00	0.92	-0.04
time (sec)	N/A	0.025	0.010	0.369	0.000	1.624	0.000	0.956	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	58	61	0	95	0	55	-1
N.S.	1	1.00	0.57	0.60	0.00	0.94	0.00	0.54	-0.01
time (sec)	N/A	0.106	0.072	0.363	0.000	1.363	0.000	0.972	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	106	217	0	360	0	114	-1
N.S.	1	1.00	0.82	1.67	0.00	2.77	0.00	0.88	-0.01
time (sec)	N/A	0.135	0.092	0.498	0.000	1.437	0.000	0.798	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	68	72	0	110	0	90	-1
N.S.	1	1.00	0.54	0.57	0.00	0.87	0.00	0.71	-0.01
time (sec)	N/A	0.134	0.093	0.456	0.000	1.571	0.000	0.675	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	122	234	0	396	0	104	-1
N.S.	1	1.00	0.77	1.47	0.00	2.49	0.00	0.65	-0.01
time (sec)	N/A	0.166	0.153	0.439	0.000	1.551	0.000	1.189	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	79	83	0	121	0	147	-1
N.S.	1	1.00	0.52	0.55	0.00	0.80	0.00	0.97	-0.01
time (sec)	N/A	0.159	0.101	0.389	0.000	1.758	0.000	0.846	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	134	247	0	422	0	138	-1
N.S.	1	1.00	0.71	1.31	0.00	2.23	0.00	0.73	-0.01
time (sec)	N/A	0.199	0.162	0.447	0.000	1.510	0.000	0.769	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	90	94	0	132	0	202	-1
N.S.	1	1.00	0.50	0.52	0.00	0.73	0.00	1.12	-0.01
time (sec)	N/A	0.195	0.118	0.404	0.000	1.970	0.000	0.866	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	81	997	0	133	0	45	-1
N.S.	1	1.00	1.47	18.13	0.00	2.42	0.00	0.82	-0.02
time (sec)	N/A	0.050	0.170	0.370	0.000	1.522	0.000	0.820	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	61	979	0	94	0	23	-1
N.S.	1	1.00	1.91	30.59	0.00	2.94	0.00	0.72	-0.03
time (sec)	N/A	0.021	0.125	0.346	0.000	2.075	0.000	0.692	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	19	0	14	19
N.S.	1	1.00	1.00	0.87	1.13	0.83	0.00	0.61	0.83
time (sec)	N/A	0.022	0.142	0.356	0.294	1.635	0.000	0.862	5.134

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	31	41	38	29	0	30	27
N.S.	1	1.00	0.65	0.85	0.79	0.60	0.00	0.62	0.56
time (sec)	N/A	0.045	0.179	0.357	0.292	1.418	0.000	0.886	5.129

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	63	50	40	0	47	40
N.S.	1	1.00	0.59	0.85	0.68	0.54	0.00	0.64	0.54
time (sec)	N/A	0.072	1.216	0.359	0.300	1.805	0.000	0.813	5.266

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	64	688	0	0	0	0	-1
N.S.	1	1.00	0.29	3.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	10.036	0.365	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	49	671	0	16	0	39	40
N.S.	1	1.00	0.25	3.41	0.00	0.08	0.00	0.20	0.20
time (sec)	N/A	0.102	10.015	0.342	0.000	0.287	0.000	0.483	5.157

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	53	696	0	43	0	0	-1
N.S.	1	1.00	0.24	3.09	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.140	10.015	0.357	0.000	0.552	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	66	1079	0	0	0	0	-1
N.S.	1	1.00	0.13	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	10.035	0.356	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	53	1054	0	0	0	0	-1
N.S.	1	1.00	0.11	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	10.030	0.388	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	48	1083	0	24	0	0	-1
N.S.	1	1.00	0.10	2.18	0.00	0.05	0.00	0.00	-0.00
time (sec)	N/A	0.362	10.013	0.396	0.000	0.453	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	113	223	0	0	0	111	-1
N.S.	1	1.00	0.65	1.28	0.00	0.00	0.00	0.64	-0.01
time (sec)	N/A	0.109	0.220	0.392	0.000	0.000	0.000	0.821	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	95	181	0	0	0	83	-1
N.S.	1	1.00	0.82	1.56	0.00	0.00	0.00	0.72	-0.01
time (sec)	N/A	0.067	0.169	0.379	0.000	0.000	0.000	1.146	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	65	83	0	0	0	54	72
N.S.	1	1.00	1.16	1.48	0.00	0.00	0.00	0.96	1.29
time (sec)	N/A	0.041	0.119	0.383	0.000	0.000	0.000	1.196	5.243

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	160	0	19	0	25	-1
N.S.	1	1.00	1.00	6.40	0.00	0.76	0.00	1.00	-0.04
time (sec)	N/A	0.026	0.071	0.390	0.000	6.338	0.000	1.309	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	48	218	0	42	0	84	-1
N.S.	1	1.00	0.57	2.60	0.00	0.50	0.00	1.00	-0.01
time (sec)	N/A	0.080	0.095	0.375	0.000	4.042	0.000	2.366	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	72	262	0	64	0	146	-1
N.S.	1	1.00	0.51	1.85	0.00	0.45	0.00	1.03	-0.01
time (sec)	N/A	0.138	0.121	0.381	0.000	3.519	0.000	2.852	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	96	306	0	86	0	208	-1
N.S.	1	1.00	0.48	1.53	0.00	0.43	0.00	1.04	-0.00
time (sec)	N/A	0.201	0.131	0.390	0.000	3.845	0.000	1.149	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	137	549	0	0	0	150	-1
N.S.	1	1.00	0.70	2.79	0.00	0.00	0.00	0.76	-0.01
time (sec)	N/A	0.127	0.317	0.391	0.000	0.000	0.000	0.800	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	119	503	0	0	0	122	-1
N.S.	1	1.00	0.86	3.62	0.00	0.00	0.00	0.88	-0.01
time (sec)	N/A	0.091	0.331	0.391	0.000	0.000	0.000	0.705	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	90	237	0	0	0	94	-1
N.S.	1	1.00	1.17	3.08	0.00	0.00	0.00	1.22	-0.01
time (sec)	N/A	0.052	0.235	0.380	0.000	0.000	0.000	0.625	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	404	0	36	0	34	40
N.S.	1	1.00	1.24	16.16	0.00	1.44	0.00	1.36	1.60
time (sec)	N/A	0.004	0.121	0.385	0.000	6.192	0.000	0.594	5.426

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	55	524	0	63	0	0	-1
N.S.	1	1.00	0.70	6.63	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.152	0.456	0.000	2.942	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	81	570	0	87	0	0	-1
N.S.	1	1.00	0.59	4.16	0.00	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.167	0.382	0.000	2.409	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	105	614	0	109	0	0	-1
N.S.	1	1.00	0.54	3.15	0.00	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.187	0.395	0.000	4.430	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	126	245	0	0	0	125	-1
N.S.	1	1.00	0.62	1.20	0.00	0.00	0.00	0.61	-0.00
time (sec)	N/A	0.121	0.244	0.390	0.000	0.000	0.000	0.552	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	108	203	0	0	0	97	-1
N.S.	1	1.00	0.74	1.39	0.00	0.00	0.00	0.66	-0.01
time (sec)	N/A	0.088	0.204	0.392	0.000	0.000	0.000	0.685	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	160	0	0	0	69	-1
N.S.	1	1.00	0.99	1.84	0.00	0.00	0.00	0.79	-0.01
time (sec)	N/A	0.056	0.174	0.380	0.000	0.000	0.000	0.586	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	133	0	0	0	37	-1
N.S.	1	1.00	1.18	3.91	0.00	0.00	0.00	1.09	-0.03
time (sec)	N/A	0.034	0.073	0.388	0.000	0.000	0.000	0.630	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	35	194	0	29	0	53	-1
N.S.	1	1.00	0.65	3.59	0.00	0.54	0.00	0.98	-0.02
time (sec)	N/A	0.051	0.086	0.519	0.000	3.736	0.000	0.595	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	59	240	0	50	0	115	-1
N.S.	1	1.00	0.53	2.14	0.00	0.45	0.00	1.03	-0.01
time (sec)	N/A	0.103	0.101	0.391	0.000	3.590	0.000	0.538	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	83	284	0	72	0	177	-1
N.S.	1	1.00	0.49	1.67	0.00	0.42	0.00	1.04	-0.01
time (sec)	N/A	0.164	0.115	0.407	0.000	2.974	0.000	0.537	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	124	527	0	0	0	136	-1
N.S.	1	1.00	0.73	3.08	0.00	0.00	0.00	0.80	-0.01
time (sec)	N/A	0.103	0.288	0.391	0.000	0.000	0.000	0.640	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	440	0	0	0	108	-1
N.S.	1	1.00	0.94	3.89	0.00	0.00	0.00	0.96	-0.01
time (sec)	N/A	0.071	0.291	0.386	0.000	0.000	0.000	0.728	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	76	240	0	0	0	0	-1
N.S.	1	1.00	1.27	4.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.168	0.431	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	111	0	54	0	26	-1
N.S.	1	1.00	1.53	3.70	0.00	1.80	0.00	0.87	-0.03
time (sec)	N/A	0.035	0.143	0.416	0.000	3.331	0.000	0.553	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	70	548	0	79	0	0	-1
N.S.	1	1.00	0.65	5.12	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.171	0.388	0.000	2.800	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	96	592	0	101	0	0	-1
N.S.	1	1.00	0.58	3.59	0.00	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.187	0.395	0.000	2.398	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	120	636	0	123	0	0	-1
N.S.	1	1.00	0.54	2.85	0.00	0.55	0.00	0.00	-0.00
time (sec)	N/A	0.230	0.206	0.384	0.000	3.529	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	155	264	0	0	0	0	-1
N.S.	1	1.00	0.51	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	10.132	0.351	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	136	273	0	0	0	0	-1
N.S.	1	1.00	0.33	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	10.108	0.370	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	118	198	0	0	0	0	-1
N.S.	1	1.00	0.55	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	10.081	0.343	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	94	207	0	0	0	0	40
N.S.	1	1.00	0.29	0.64	0.00	0.00	0.00	0.00	0.12
time (sec)	N/A	0.230	10.042	0.349	0.000	0.000	0.000	0.000	5.180

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	54	132	0	0	0	0	-1
N.S.	1	1.00	0.44	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	10.029	0.357	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	59	213	0	0	0	0	-1
N.S.	1	1.00	0.18	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	10.036	0.343	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	59	179	0	0	0	0	-1
N.S.	1	1.00	0.31	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	10.031	0.356	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	59	281	0	0	0	0	-1
N.S.	1	1.00	0.14	0.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	10.033	0.352	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	59	245	0	0	0	0	-1
N.S.	1	1.00	0.21	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	10.040	0.397	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	142	196	0	0	0	0	-1
N.S.	1	1.00	0.48	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	10.125	0.379	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	123	261	0	0	0	0	-1
N.S.	1	1.00	0.30	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.772	10.071	0.365	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	106	163	0	0	0	0	40
N.S.	1	1.00	0.51	0.78	0.00	0.00	0.00	0.00	0.19
time (sec)	N/A	0.211	10.067	0.350	0.000	0.000	0.000	0.000	5.193

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	60	228	0	0	0	0	-1
N.S.	1	1.00	0.19	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	10.027	0.354	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	60	130	0	0	0	0	-1
N.S.	1	1.00	0.42	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	10.046	0.347	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	62	339	0	0	0	0	-1
N.S.	1	1.00	0.18	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	10.042	0.342	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	62	168	0	0	0	0	-1
N.S.	1	1.00	0.29	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	10.041	0.338	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	62	411	0	0	0	0	-1
N.S.	1	1.00	0.14	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	10.034	0.346	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	62	201	0	0	0	0	-1
N.S.	1	1.00	0.21	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	10.039	0.344	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	161	196	0	0	0	0	-1
N.S.	1	1.00	0.53	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	10.079	0.350	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	143	261	0	0	0	0	-1
N.S.	1	1.00	0.35	0.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	10.064	0.346	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	124	163	0	0	0	0	-1
N.S.	1	1.00	0.57	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	10.062	0.345	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	106	228	0	0	0	0	-1
N.S.	1	1.00	0.33	0.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	10.052	0.349	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	80	127	0	0	0	0	40
N.S.	1	1.00	0.63	1.01	0.00	0.00	0.00	0.00	0.32
time (sec)	N/A	0.083	10.046	0.380	0.000	0.000	0.000	0.000	5.268

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	54	253	0	0	0	0	-1
N.S.	1	1.00	0.18	0.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	10.042	0.381	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	59	142	0	0	0	0	-1
N.S.	1	1.00	0.36	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	10.041	0.351	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	59	363	0	0	0	0	-1
N.S.	1	1.00	0.15	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	10.044	0.346	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	59	179	0	0	0	0	-1
N.S.	1	1.00	0.24	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	10.038	0.355	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	131	384	0	0	0	0	-1
N.S.	1	1.00	0.30	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	10.079	0.347	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	124	260	0	0	0	0	-1
N.S.	1	1.00	0.52	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	10.084	0.349	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	94	312	0	0	0	0	-1
N.S.	1	1.00	0.27	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	10.062	0.347	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	82	184	0	0	0	0	-1
N.S.	1	1.00	0.55	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	10.049	0.360	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	62	242	0	0	0	0	40
N.S.	1	1.00	0.21	0.82	0.00	0.00	0.00	0.00	0.14
time (sec)	N/A	0.181	10.028	0.354	0.000	0.000	0.000	0.000	5.351

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	62	180	0	0	0	0	-1
N.S.	1	1.00	0.39	1.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	10.049	0.355	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	64	339	0	0	0	0	-1
N.S.	1	1.00	0.17	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	10.040	0.357	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	64	261	0	0	0	0	-1
N.S.	1	1.00	0.26	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	10.038	0.350	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	64	411	0	0	0	0	-1
N.S.	1	1.00	0.14	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	10.040	0.352	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	185	156	0	1293	0	396	-1
N.S.	1	1.00	0.50	0.42	0.00	3.49	0.00	1.07	-0.00
time (sec)	N/A	0.422	0.110	0.383	0.000	291.525	0.000	1.826	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	133	123	0	0	0	312	-1
N.S.	1	1.00	0.47	0.43	0.00	0.00	0.00	1.10	-0.00
time (sec)	N/A	0.299	0.093	0.366	0.000	0.000	0.000	1.524	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	96	90	0	0	0	228	-1
N.S.	1	1.00	0.49	0.46	0.00	0.00	0.00	1.17	-0.01
time (sec)	N/A	0.184	0.064	0.354	0.000	0.000	0.000	1.200	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	74	57	0	0	0	143	40
N.S.	1	1.00	0.68	0.52	0.00	0.00	0.00	1.31	0.37
time (sec)	N/A	0.093	0.047	0.346	0.000	0.000	0.000	1.605	5.192

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	0	0	0	23	-1
N.S.	1	1.00	1.00	1.17	0.00	0.00	0.00	1.00	-0.04
time (sec)	N/A	0.031	0.038	0.359	0.000	0.000	0.000	1.513	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	79	0	0	0	72	-1
N.S.	1	1.00	0.84	0.88	0.00	0.00	0.00	0.80	-0.01
time (sec)	N/A	0.097	0.151	0.353	0.000	0.000	0.000	1.749	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	112	125	0	0	0	126	-1
N.S.	1	1.00	0.63	0.70	0.00	0.00	0.00	0.71	-0.01
time (sec)	N/A	0.198	0.220	0.352	0.000	0.000	0.000	1.531	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	149	167	0	0	0	177	-1
N.S.	1	1.00	0.56	0.63	0.00	0.00	0.00	0.67	-0.00
time (sec)	N/A	0.321	0.261	0.350	0.000	0.000	0.000	1.638	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	186	209	0	0	0	228	-1
N.S.	1	1.00	0.53	0.59	0.00	0.00	0.00	0.64	-0.00
time (sec)	N/A	0.436	0.325	0.360	0.000	0.000	0.000	1.640	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	168	145	0	0	0	770	-1
N.S.	1	1.00	0.49	0.42	0.00	0.00	0.00	2.24	-0.00
time (sec)	N/A	0.413	4.815	0.359	0.000	0.000	0.000	1.151	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	131	112	0	0	0	602	-1
N.S.	1	1.00	0.51	0.44	0.00	0.00	0.00	2.36	-0.00
time (sec)	N/A	0.283	4.860	0.360	0.000	0.000	0.000	1.675	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	94	79	0	768	0	434	40
N.S.	1	1.00	0.56	0.47	0.00	4.54	0.00	2.57	0.24
time (sec)	N/A	0.172	4.760	0.357	0.000	287.483	0.000	2.260	5.141

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	59	48	0	501	0	265	-1
N.S.	1	1.00	0.70	0.57	0.00	5.96	0.00	3.15	-0.01
time (sec)	N/A	0.097	4.750	0.459	0.000	234.821	0.000	1.839	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	88	69	0	0	0	83	-1
N.S.	1	1.00	1.13	0.88	0.00	0.00	0.00	1.06	-0.01
time (sec)	N/A	0.095	10.083	0.395	0.000	0.000	0.000	1.541	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	61	93	0	0	0	92	-1
N.S.	1	1.00	0.54	0.82	0.00	0.00	0.00	0.81	-0.01
time (sec)	N/A	0.127	10.060	0.389	0.000	0.000	0.000	2.584	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	61	139	0	0	0	143	-1
N.S.	1	1.00	0.30	0.68	0.00	0.00	0.00	0.70	-0.00
time (sec)	N/A	0.236	10.051	0.358	0.000	0.000	0.000	1.494	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	61	181	0	0	0	194	-1
N.S.	1	1.00	0.21	0.62	0.00	0.00	0.00	0.67	-0.00
time (sec)	N/A	0.346	10.056	0.343	0.000	0.000	0.000	1.715	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	61	223	0	0	0	245	-1
N.S.	1	1.00	0.16	0.59	0.00	0.00	0.00	0.65	-0.00
time (sec)	N/A	0.477	10.061	0.361	0.000	0.000	0.000	1.703	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	185	167	0	1294	0	206	-1
N.S.	1	1.00	0.46	0.42	0.00	3.23	0.00	0.51	-0.00
time (sec)	N/A	0.490	0.112	0.355	0.000	270.067	0.000	1.354	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	148	134	0	1031	0	164	-1
N.S.	1	1.00	0.47	0.43	0.00	3.29	0.00	0.52	-0.00
time (sec)	N/A	0.357	0.087	0.360	0.000	242.118	0.000	1.134	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	111	101	0	768	0	122	-1
N.S.	1	1.00	0.49	0.45	0.00	3.41	0.00	0.54	-0.00
time (sec)	N/A	0.226	0.077	0.352	0.000	176.966	0.000	1.161	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	74	68	0	502	0	80	-1
N.S.	1	1.00	0.54	0.50	0.00	3.66	0.00	0.58	-0.01
time (sec)	N/A	0.121	0.056	0.344	0.000	271.299	0.000	1.347	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	238	0	36	40
N.S.	1	1.00	0.77	0.77	0.00	5.06	0.00	0.77	0.85
time (sec)	N/A	0.035	0.035	0.343	0.000	211.263	0.000	1.221	5.219

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	61	0	0	0	51	-1
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.84	-0.02
time (sec)	N/A	0.065	0.112	0.337	0.000	0.000	0.000	1.275	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	101	126	0	0	0	109	-1
N.S.	1	1.00	0.66	0.82	0.00	0.00	0.00	0.71	-0.01
time (sec)	N/A	0.161	0.171	0.336	0.000	0.000	0.000	1.042	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	138	188	0	0	0	160	-1
N.S.	1	1.00	0.57	0.78	0.00	0.00	0.00	0.66	-0.00
time (sec)	N/A	0.277	0.186	0.338	0.000	0.000	0.000	1.962	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	175	248	0	0	0	211	-1
N.S.	1	1.00	0.53	0.75	0.00	0.00	0.00	0.64	-0.00
time (sec)	N/A	0.392	0.240	0.342	0.000	0.000	0.000	1.387	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	161	143	0	2566	0	214	-1
N.S.	1	1.00	0.48	0.43	0.00	7.64	0.00	0.64	-0.00
time (sec)	N/A	0.394	4.179	0.389	0.000	197.195	0.000	2.088	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	122	110	0	2083	0	163	-1
N.S.	1	1.00	0.49	0.44	0.00	8.40	0.00	0.66	-0.00
time (sec)	N/A	0.268	4.124	0.375	0.000	224.791	0.000	1.901	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	85	77	0	1598	0	112	-1
N.S.	1	1.00	0.53	0.48	0.00	9.99	0.00	0.70	-0.01
time (sec)	N/A	0.155	4.366	0.359	0.000	192.097	0.000	1.940	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	45	45	0	1107	0	60	-1
N.S.	1	1.00	0.66	0.66	0.00	16.28	0.00	0.88	-0.01
time (sec)	N/A	0.059	4.127	0.350	0.000	214.171	0.000	1.626	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	71	56	0	0	0	71	40
N.S.	1	1.00	1.18	0.93	0.00	0.00	0.00	1.18	0.67
time (sec)	N/A	0.040	1.553	0.343	0.000	0.000	0.000	1.843	5.358

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	110	88	0	0	0	105	-1
N.S.	1	1.00	0.75	0.60	0.00	0.00	0.00	0.72	-0.01
time (sec)	N/A	0.167	4.209	0.349	0.000	0.000	0.000	1.264	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	48	126	0	0	0	156	-1
N.S.	1	1.00	0.20	0.53	0.00	0.00	0.00	0.66	-0.00
time (sec)	N/A	0.275	10.078	0.347	0.000	0.000	0.000	1.453	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	48	159	0	0	0	207	-1
N.S.	1	1.00	0.15	0.49	0.00	0.00	0.00	0.64	-0.00
time (sec)	N/A	0.387	10.068	0.334	0.000	0.000	0.000	1.781	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	48	192	0	0	0	258	-1
N.S.	1	1.00	0.12	0.47	0.00	0.00	0.00	0.63	-0.00
time (sec)	N/A	0.539	10.064	0.342	0.000	0.000	0.000	2.407	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.007	0.001	0.036	0.288	2.515	0.010	1.638	0.022

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.001	0.019	0.277	2.973	0.007	1.281	0.022

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.002	0.000	0.020	0.274	3.556	0.007	1.460	0.022

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.004	0.001	0.019	0.284	1.138	0.006	1.118	0.020

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.003	0.001	0.018	0.271	2.593	0.006	1.471	0.017

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.023	0.002	0.430	0.283	1.282	0.009	1.439	0.040

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.013	0.002	0.345	0.277	1.135	0.009	1.440	0.033

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.011	0.001	0.334	0.286	1.316	0.008	1.419	0.031

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.012	0.001	0.339	0.290	1.214	0.008	1.380	0.034

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.012	0.001	0.330	0.267	1.147	0.009	0.983	0.032

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	51
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89
time (sec)	N/A	0.026	0.003	0.389	0.298	1.164	0.053	0.917	5.093

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	40
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91
time (sec)	N/A	0.018	0.003	0.354	0.291	1.430	0.043	1.743	0.042

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.015	0.003	0.347	0.270	1.365	0.038	1.371	0.041

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.011	0.002	0.354	0.290	1.348	0.031	1.032	0.036

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.005	0.001	0.337	0.285	1.583	0.010	2.611	0.022

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	16	10	20	15
N.S.	1	1.00	1.00	1.06	1.00	0.89	0.56	1.11	0.83
time (sec)	N/A	0.005	0.003	0.342	0.294	1.495	0.062	1.902	5.120

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.011	0.003	0.360	0.269	2.324	0.084	1.261	0.052

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.015	0.004	0.343	0.269	1.895	0.085	1.617	0.059

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	48
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86
time (sec)	N/A	0.020	0.004	0.366	0.268	1.326	0.126	1.085	0.061

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	62	62
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.07
time (sec)	N/A	0.028	0.015	0.368	0.279	1.386	0.081	1.117	0.038

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	46	47	62	44	48	50
N.S.	1	1.00	0.93	1.00	1.02	1.35	0.96	1.04	1.09
time (sec)	N/A	0.022	0.010	0.350	0.286	1.428	0.073	1.624	0.045

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	34	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.09
time (sec)	N/A	0.016	0.010	0.358	0.267	1.470	0.066	1.670	0.040

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.00
time (sec)	N/A	0.013	0.005	0.362	0.280	1.390	0.048	1.705	0.037

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00
time (sec)	N/A	0.005	0.002	0.406	0.282	1.430	0.045	1.672	5.169

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	0.90
time (sec)	N/A	0.014	0.010	0.358	0.285	1.081	0.086	1.277	0.045

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	41
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	0.98
time (sec)	N/A	0.017	0.027	0.356	0.270	1.767	0.116	1.112	5.343

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	64	57
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.10	0.98
time (sec)	N/A	0.022	0.035	0.369	0.290	1.236	0.133	1.770	5.315

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	73	69
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.00
time (sec)	N/A	0.026	0.038	0.372	0.273	1.579	0.146	1.093	0.071

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	86	79
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.02	0.94
time (sec)	N/A	0.034	0.030	0.381	0.279	1.681	0.187	1.219	0.077

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	53	57	53	62	0	131	62
N.S.	1	1.00	0.50	0.54	0.50	0.59	0.00	1.25	0.59
time (sec)	N/A	0.079	0.025	0.362	0.303	1.200	0.000	1.412	5.487

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	46	42	51	0	108	51
N.S.	1	1.00	0.52	0.58	0.52	0.64	0.00	1.35	0.64
time (sec)	N/A	0.049	0.022	0.366	0.276	1.406	0.000	1.252	5.475

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	35	30	39	0	81	39
N.S.	1	1.00	0.79	0.67	0.58	0.75	0.00	1.56	0.75
time (sec)	N/A	0.028	0.016	0.376	0.289	1.558	0.000	1.665	5.302

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	12	26	0	50	-1
N.S.	1	1.00	0.92	1.08	0.48	1.04	0.00	2.00	-0.04
time (sec)	N/A	0.025	0.010	0.362	0.270	1.354	0.000	1.525	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	51	0	111	0	67	73
N.S.	1	1.00	1.04	1.00	0.00	2.18	0.00	1.31	1.43
time (sec)	N/A	0.033	0.035	0.371	0.000	1.796	0.000	1.264	5.357

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	64	56	0	127	0	45	-1
N.S.	1	1.00	1.23	1.08	0.00	2.44	0.00	0.87	-0.02
time (sec)	N/A	0.033	0.054	0.508	0.000	1.425	0.000	0.959	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	73	0	149	0	72	-1
N.S.	1	1.00	0.96	0.87	0.00	1.77	0.00	0.86	-0.01
time (sec)	N/A	0.065	0.089	0.424	0.000	1.737	0.000	1.369	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	93	89	0	175	0	92	-1
N.S.	1	1.00	0.83	0.79	0.00	1.56	0.00	0.82	-0.01
time (sec)	N/A	0.092	0.109	0.392	0.000	1.328	0.000	1.495	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	80	79	86	95	0	282	80
N.S.	1	1.00	0.50	0.49	0.53	0.59	0.00	1.75	0.50
time (sec)	N/A	0.156	0.033	0.369	0.296	1.768	0.000	1.281	5.238

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	69	68	75	84	0	246	69
N.S.	1	1.00	0.51	0.50	0.55	0.62	0.00	1.81	0.51
time (sec)	N/A	0.114	0.030	0.373	0.295	1.284	0.000	1.491	5.237

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	58	57	64	73	0	210	58
N.S.	1	1.00	0.54	0.53	0.59	0.68	0.00	1.94	0.54
time (sec)	N/A	0.093	0.027	0.360	0.284	1.461	0.000	1.388	5.187

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	47	46	53	62	0	173	47
N.S.	1	1.00	0.59	0.58	0.66	0.78	0.00	2.16	0.59
time (sec)	N/A	0.089	0.025	0.365	0.295	1.242	0.000	1.873	5.175

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	41	50	0	136	36
N.S.	1	1.00	0.60	0.67	0.79	0.96	0.00	2.62	0.69
time (sec)	N/A	0.059	0.022	0.363	0.290	1.100	0.000	1.336	5.174

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	28	37	0	89	28
N.S.	1	1.00	0.92	1.08	1.12	1.48	0.00	3.56	1.12
time (sec)	N/A	0.029	0.012	0.359	0.281	2.046	0.000	1.561	5.618

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	61	0	130	0	85	-1
N.S.	1	1.00	0.92	0.82	0.00	1.76	0.00	1.15	-0.01
time (sec)	N/A	0.066	0.049	0.371	0.000	2.751	0.000	1.282	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	72	0	136	0	62	-1
N.S.	1	1.00	0.90	0.99	0.00	1.86	0.00	0.85	-0.01
time (sec)	N/A	0.063	0.072	0.375	0.000	2.039	0.000	1.669	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	82	74	0	154	0	70	-1
N.S.	1	1.00	1.01	0.91	0.00	1.90	0.00	0.86	-0.01
time (sec)	N/A	0.063	0.103	0.375	0.000	1.098	0.000	1.030	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	87	0	175	0	92	-1
N.S.	1	1.00	0.86	0.80	0.00	1.61	0.00	0.84	-0.01
time (sec)	N/A	0.090	0.123	0.371	0.000	1.814	0.000	1.561	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	104	101	0	197	0	109	-1
N.S.	1	1.00	0.76	0.74	0.00	1.44	0.00	0.80	-0.01
time (sec)	N/A	0.121	0.141	0.508	0.000	0.940	0.000	2.269	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	116	113	0	219	0	126	-1
N.S.	1	1.00	0.70	0.68	0.00	1.33	0.00	0.76	-0.01
time (sec)	N/A	0.157	0.153	0.434	0.000	1.939	0.000	1.584	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	53	55	53	51	0	64	51
N.S.	1	1.00	0.51	0.53	0.51	0.50	0.00	0.62	0.50
time (sec)	N/A	0.101	0.025	0.366	0.286	1.229	0.000	1.752	5.188

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	42	44	42	40	0	52	40
N.S.	1	1.00	0.56	0.59	0.56	0.53	0.00	0.69	0.53
time (sec)	N/A	0.066	0.022	0.356	0.287	2.168	0.000	1.781	5.201

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	33	30	28	0	38	31
N.S.	1	1.00	0.61	0.67	0.61	0.57	0.00	0.78	0.63
time (sec)	N/A	0.037	0.016	0.349	0.282	2.877	0.000	1.145	5.164

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	12	21	0	27	17
N.S.	1	1.00	0.91	1.09	0.52	0.91	0.00	1.17	0.74
time (sec)	N/A	0.007	0.009	0.358	0.283	1.354	0.000	1.371	5.142

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	39	0	74	0	45	-1
N.S.	1	1.00	1.53	1.30	0.00	2.47	0.00	1.50	-0.03
time (sec)	N/A	0.007	0.020	0.364	0.000	1.359	0.000	1.250	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	55	0	127	0	51	-1
N.S.	1	1.00	1.11	1.02	0.00	2.35	0.00	0.94	-0.02
time (sec)	N/A	0.035	0.053	0.372	0.000	1.993	0.000	1.309	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	77	0	153	0	73	44
N.S.	1	1.00	0.95	0.89	0.00	1.76	0.00	0.84	0.51
time (sec)	N/A	0.061	0.076	0.382	0.000	1.286	0.000	1.114	5.408

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	96	95	0	175	0	88	-1
N.S.	1	1.00	0.83	0.83	0.00	1.52	0.00	0.77	-0.01
time (sec)	N/A	0.092	0.072	0.390	0.000	2.033	0.000	0.956	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	50	56	41	60	0	79	57
N.S.	1	1.00	0.51	0.57	0.42	0.61	0.00	0.81	0.58
time (sec)	N/A	0.096	0.025	0.372	0.286	1.101	0.000	1.654	5.282

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	39	46	30	49	0	64	47
N.S.	1	1.00	0.54	0.64	0.42	0.68	0.00	0.89	0.65
time (sec)	N/A	0.068	0.023	0.367	0.305	1.865	0.000	1.135	5.223

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	26	34	19	38	0	48	35
N.S.	1	1.00	0.55	0.72	0.40	0.81	0.00	1.02	0.74
time (sec)	N/A	0.041	0.016	0.371	0.292	1.451	0.000	0.884	5.175

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	12	29	0	27	28
N.S.	1	1.00	0.90	1.29	0.57	1.38	0.00	1.29	1.33
time (sec)	N/A	0.012	0.010	0.362	0.314	1.667	0.000	1.321	5.074

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	54	54	0	156	0	77	-1
N.S.	1	1.00	1.04	1.04	0.00	3.00	0.00	1.48	-0.02
time (sec)	N/A	0.041	0.030	0.370	0.000	1.473	0.000	1.314	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	62	62	0	189	0	72	-1
N.S.	1	1.00	0.83	0.83	0.00	2.52	0.00	0.96	-0.01
time (sec)	N/A	0.057	0.063	0.404	0.000	1.603	0.000	1.184	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	84	76	0	219	0	92	42
N.S.	1	1.00	0.76	0.69	0.00	1.99	0.00	0.84	0.38
time (sec)	N/A	0.074	0.095	0.395	0.000	1.518	0.000	1.437	5.432

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	96	86	0	241	0	107	-1
N.S.	1	1.00	0.70	0.62	0.00	1.75	0.00	0.78	-0.01
time (sec)	N/A	0.154	0.112	0.456	0.000	1.926	0.000	1.432	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	106	100	0	263	0	122	44
N.S.	1	1.00	0.64	0.60	0.00	1.58	0.00	0.73	0.27
time (sec)	N/A	0.284	0.148	0.459	0.000	1.599	0.000	1.126	5.683

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	102	103	0	180	0	83	-1
N.S.	1	1.00	0.82	0.82	0.00	1.44	0.00	0.66	-0.01
time (sec)	N/A	0.260	0.085	0.392	0.000	1.528	0.000	1.817	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	92	0	159	0	71	-1
N.S.	1	1.00	0.96	0.97	0.00	1.67	0.00	0.75	-0.01
time (sec)	N/A	0.200	0.073	0.380	0.000	1.332	0.000	1.956	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	71	79	0	131	0	55	-1
N.S.	1	1.00	1.18	1.32	0.00	2.18	0.00	0.92	-0.02
time (sec)	N/A	0.106	0.056	0.386	0.000	2.896	0.000	1.625	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	53	58	0	77	0	37	-1
N.S.	1	1.00	1.56	1.71	0.00	2.26	0.00	1.09	-0.03
time (sec)	N/A	0.030	0.036	0.372	0.000	2.841	0.000	2.386	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	0	21	0	34	-1
N.S.	1	1.00	0.92	1.08	0.00	0.84	0.00	1.36	-0.04
time (sec)	N/A	0.024	0.020	0.365	0.000	2.860	0.000	1.810	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	31	33	0	29	0	54	-1
N.S.	1	1.00	0.55	0.59	0.00	0.52	0.00	0.96	-0.02
time (sec)	N/A	0.056	0.061	0.372	0.000	1.725	0.000	1.537	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	44	46	0	40	0	81	-1
N.S.	1	1.00	0.51	0.53	0.00	0.47	0.00	0.94	-0.01
time (sec)	N/A	0.079	0.069	0.375	0.000	2.140	0.000	1.541	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	55	57	0	51	0	107	-1
N.S.	1	1.00	0.47	0.49	0.00	0.44	0.00	0.92	-0.01
time (sec)	N/A	0.111	0.078	0.364	0.000	1.840	0.000	2.174	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.031	0.070	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	61	59	0	0	0	0	0	-1
N.S.	1	1.27	1.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.025	0.074	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.024	0.085	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	0	38	0	0	54
N.S.	1	1.00	0.94	1.12	0.00	1.19	0.00	0.00	1.69
time (sec)	N/A	0.018	0.028	0.405	0.000	1.592	0.000	0.000	5.281

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	50	0	70	0	0	98
N.S.	1	1.00	0.63	0.71	0.00	1.00	0.00	0.00	1.40
time (sec)	N/A	0.039	0.045	0.396	0.000	2.060	0.000	0.000	5.275

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	72	84	0	111	0	0	157
N.S.	1	1.00	0.62	0.72	0.00	0.96	0.00	0.00	1.35
time (sec)	N/A	0.066	0.038	0.401	0.000	2.294	0.000	0.000	5.365

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.007	0.006	0.372	0.282	1.978	0.157	0.936	5.110

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	48	46	42	0	65	42
N.S.	1	1.00	0.58	0.60	0.58	0.52	0.00	0.81	0.52
time (sec)	N/A	0.079	0.029	0.372	0.298	1.226	0.000	1.863	5.184

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	34	37	34	30	0	48	33
N.S.	1	1.00	0.65	0.71	0.65	0.58	0.00	0.92	0.63
time (sec)	N/A	0.045	0.023	0.372	0.303	3.074	0.000	1.948	5.327

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	14	21	0	29	21
N.S.	1	1.00	1.00	1.08	0.56	0.84	0.00	1.16	0.84
time (sec)	N/A	0.013	0.015	0.388	0.288	1.525	0.000	1.774	5.190

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	43	0	75	0	47	-1
N.S.	1	1.00	1.69	1.34	0.00	2.34	0.00	1.47	-0.03
time (sec)	N/A	0.009	0.022	0.386	0.000	1.326	0.000	1.730	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	66	0	127	0	55	-1
N.S.	1	1.00	1.29	1.12	0.00	2.15	0.00	0.93	-0.02
time (sec)	N/A	0.037	0.059	0.429	0.000	1.781	0.000	2.046	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	68	248	0	37	0	0	-1
N.S.	1	1.00	0.29	1.04	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.092	10.026	0.431	0.000	0.439	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	52	231	0	14	0	0	-1
N.S.	1	1.00	0.25	1.09	0.00	0.07	0.00	0.00	-0.00
time (sec)	N/A	0.049	10.017	0.370	0.000	0.383	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	55	248	0	38	0	0	44
N.S.	1	1.00	0.23	1.02	0.00	0.16	0.00	0.00	0.18
time (sec)	N/A	0.087	10.021	0.387	0.000	0.348	0.000	0.000	5.712

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	68	676	0	45	0	0	-1
N.S.	1	1.00	0.13	1.32	0.00	0.09	0.00	0.00	-0.00
time (sec)	N/A	0.216	10.030	0.405	0.000	0.506	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	55	394	0	22	0	0	-1
N.S.	1	1.00	0.11	0.81	0.00	0.05	0.00	0.00	-0.00
time (sec)	N/A	0.132	10.016	0.455	0.000	0.675	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	50	673	0	46	0	0	-1
N.S.	1	1.00	0.10	1.32	0.00	0.09	0.00	0.00	-0.00
time (sec)	N/A	0.238	10.017	0.375	0.000	0.271	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	86	2017	0	0	0	0	-1
N.S.	1	1.00	0.32	7.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	10.036	0.476	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	70	2586	0	0	0	0	-1
N.S.	1	1.00	0.13	4.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	10.033	0.469	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	81	3347	0	148	0	52	-1
N.S.	1	1.00	1.25	51.49	0.00	2.28	0.00	0.80	-0.02
time (sec)	N/A	0.064	0.022	0.580	0.000	2.026	0.000	0.681	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	70	1793	0	0	0	0	-1
N.S.	1	1.00	0.30	7.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	10.039	0.591	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	57	2374	0	0	0	0	-1
N.S.	1	1.00	0.12	4.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	10.024	0.508	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	59	480	0	101	0	40	-1
N.S.	1	1.00	1.64	13.33	0.00	2.81	0.00	1.11	-0.03
time (sec)	N/A	0.032	0.008	0.524	0.000	2.765	0.000	0.532	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	55	437	0	16	0	0	-1
N.S.	1	1.00	0.27	2.15	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	0.120	10.015	0.518	0.000	0.432	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	55	2860	0	24	0	0	-1
N.S.	1	1.00	0.11	5.51	0.00	0.05	0.00	0.00	-0.00
time (sec)	N/A	0.368	10.018	0.531	0.000	0.568	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	26	21	0	28	-1
N.S.	1	1.00	1.00	1.07	0.96	0.78	0.00	1.04	-0.04
time (sec)	N/A	0.027	0.007	0.453	0.292	2.712	0.000	0.502	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	57	1795	0	48	0	0	-1
N.S.	1	1.00	0.24	7.64	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.158	10.016	0.515	0.000	0.384	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	57	3048	0	55	0	0	-1
N.S.	1	1.00	0.10	5.49	0.00	0.10	0.00	0.00	-0.00
time (sec)	N/A	0.427	10.023	0.612	0.000	0.504	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	37	38	31	0	43	-1
N.S.	1	1.00	0.62	0.66	0.68	0.55	0.00	0.77	-0.02
time (sec)	N/A	0.057	0.009	0.433	0.297	2.298	0.000	0.536	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	57	2009	0	62	0	0	-1
N.S.	1	1.00	0.22	7.58	0.00	0.23	0.00	0.00	-0.00
time (sec)	N/A	0.207	10.016	0.496	0.000	0.356	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.013	0.004	0.346	0.287	1.567	0.068	0.495	5.209

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.014	0.004	0.356	0.288	2.154	0.079	0.473	5.724

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	104	120	0	171	0	106	-1
N.S.	1	1.00	0.93	1.07	0.00	1.53	0.00	0.95	-0.01
time (sec)	N/A	0.124	0.022	0.428	0.000	1.870	0.000	0.756	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	93	98	0	150	0	90	-1
N.S.	1	1.00	1.08	1.14	0.00	1.74	0.00	1.05	-0.01
time (sec)	N/A	0.086	0.018	0.377	0.000	1.867	0.000	0.666	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	73	78	0	122	0	71	-1
N.S.	1	1.00	1.30	1.39	0.00	2.18	0.00	1.27	-0.02
time (sec)	N/A	0.057	0.013	0.368	0.000	1.525	0.000	0.610	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	57	56	0	74	0	49	-1
N.S.	1	1.00	1.78	1.75	0.00	2.31	0.00	1.53	-0.03
time (sec)	N/A	0.024	0.007	0.354	0.000	2.427	0.000	0.724	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	39	0	21	0	27	21
N.S.	1	1.00	0.91	1.70	0.00	0.91	0.00	1.17	0.91
time (sec)	N/A	0.004	0.005	0.356	0.000	1.979	0.000	0.630	5.140

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	48	0	29	0	53	42
N.S.	1	1.00	0.60	0.92	0.00	0.56	0.00	1.02	0.81
time (sec)	N/A	0.032	0.007	0.345	0.000	1.656	0.000	0.587	5.062

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	61	0	40	0	82	40
N.S.	1	1.00	0.52	0.76	0.00	0.50	0.00	1.02	0.50
time (sec)	N/A	0.064	0.008	0.362	0.000	1.968	0.000	0.896	5.140

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	53	72	0	51	0	111	92
N.S.	1	1.00	0.49	0.67	0.00	0.47	0.00	1.03	0.85
time (sec)	N/A	0.089	0.012	0.366	0.000	1.941	0.000	0.568	5.137

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	64	83	0	62	0	140	116
N.S.	1	1.00	0.47	0.61	0.00	0.46	0.00	1.03	0.85
time (sec)	N/A	0.118	0.086	0.486	0.000	3.319	0.000	0.613	5.145

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	28	22	32	22
N.S.	1	1.00	1.00	0.88	0.85	1.08	0.85	1.23	0.85
time (sec)	N/A	0.013	0.004	0.377	0.273	1.706	0.086	0.603	0.050

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	22	28	22	32	22
N.S.	1	1.00	1.00	0.85	0.81	1.04	0.81	1.19	0.81
time (sec)	N/A	0.014	0.004	0.376	0.286	1.906	0.090	0.577	5.209

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	14	8	5	15	8
N.S.	1	1.00	1.75	1.12	1.75	1.00	0.62	1.88	1.00
time (sec)	N/A	0.002	0.002	0.016	0.336	2.786	0.012	0.532	5.274

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	18	15	16	10
N.S.	1	1.00	1.00	1.10	1.60	1.80	1.50	1.60	1.00
time (sec)	N/A	0.002	0.002	0.014	0.287	2.226	0.015	0.531	0.032

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	16	26	27	16	26
N.S.	1	1.00	1.00	0.92	1.33	2.17	2.25	1.33	2.17
time (sec)	N/A	0.003	0.003	0.020	0.274	2.326	0.016	0.543	0.037

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00
time (sec)	N/A	0.001	0.001	0.375	0.298	1.547	0.010	0.473	0.027

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	21	22	32	0	19
N.S.	1	1.00	1.00	0.95	1.05	1.10	1.60	0.00	0.95
time (sec)	N/A	0.004	0.003	0.021	0.276	1.330	0.261	0.000	5.178

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	40	36	82	0	21
N.S.	1	1.00	1.00	1.05	2.00	1.80	4.10	0.00	1.05
time (sec)	N/A	0.006	0.002	0.025	0.280	2.180	0.384	0.000	5.135

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	53	52	119	0	21
N.S.	1	1.00	1.00	1.05	2.65	2.60	5.95	0.00	1.05
time (sec)	N/A	0.007	0.002	0.025	0.297	3.242	0.465	0.000	5.122

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.003	0.004	0.359	0.281	2.571	0.033	0.477	5.157

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.008	0.005	0.565	0.272	1.679	0.036	0.468	5.156

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.008	0.006	1.053	0.276	2.253	0.039	0.467	5.153

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	193	339	275	231	0	285	287
N.S.	1	1.00	7.15	12.56	10.19	8.56	0.00	10.56	10.63
time (sec)	N/A	0.011	0.069	0.398	0.295	1.691	0.000	0.537	5.875

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.003	0.003	0.343	0.280	2.033	0.032	0.478	0.002

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.003	0.004	0.537	0.283	1.446	0.034	0.496	5.188

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.003	0.005	1.076	0.292	1.593	0.036	0.461	5.158

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	193	287	275	205	0	205	285
N.S.	1	1.00	7.15	10.63	10.19	7.59	0.00	7.59	10.56
time (sec)	N/A	0.006	0.045	0.359	0.291	1.923	0.000	0.483	5.950

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	88	126	121	93	178	93	124
N.S.	1	1.00	3.26	4.67	4.48	3.44	6.59	3.44	4.59
time (sec)	N/A	0.007	0.046	0.403	0.274	1.678	35.483	0.502	5.437

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	39	66	82	0	0	38
N.S.	1	1.00	1.00	1.44	2.44	3.04	0.00	0.00	1.41
time (sec)	N/A	0.010	0.051	0.438	0.296	1.885	0.000	0.000	5.211

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.003	0.003	0.016	0.294	1.820	0.041	0.497	0.046

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.003	0.019	0.283	1.723	0.051	0.490	5.124

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.004	0.023	0.278	1.530	0.061	0.828	0.047

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.007	0.036	0.280	1.947	0.132	0.613	0.042

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.008	0.046	0.286	2.502	0.217	0.589	0.062

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.009	0.065	0.290	1.391	0.315	0.656	5.184

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.008	0.001	0.026	0.282	1.450	0.025	0.658	0.042

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	38	37	46	34
N.S.	1	1.00	1.00	0.88	0.85	0.95	0.92	1.15	0.85
time (sec)	N/A	0.018	0.005	0.075	0.278	1.557	0.049	0.689	0.037

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	44	48	49	45	47
N.S.	1	1.00	1.00	0.90	0.88	0.96	0.98	0.90	0.94
time (sec)	N/A	0.016	0.005	0.036	0.292	1.198	0.060	0.565	0.048

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	144	165	124	637	36	112	197
N.S.	1	1.00	0.78	0.89	0.67	3.44	0.19	0.61	1.06
time (sec)	N/A	0.247	0.086	0.069	0.514	4.333	0.827	0.555	5.914

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	232	363	325	297	0	269	363
N.S.	1	1.00	8.00	12.52	11.21	10.24	0.00	9.28	12.52
time (sec)	N/A	0.013	0.108	0.635	0.292	2.241	0.000	1.059	6.777

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.002	0.004	0.400	0.276	1.202	0.028	0.577	5.325

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.007	0.005	0.545	0.278	1.466	0.036	0.588	0.002

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.009	0.005	1.132	0.284	1.866	0.039	0.559	5.222

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.002	0.003	0.532	0.293	1.623	0.030	0.577	5.202

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.006	0.004	1.102	0.279	1.645	0.039	0.500	0.002

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.002	0.004	1.072	0.285	1.641	0.031	0.540	5.177

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	35	37	27	48	0	26
N.S.	1	1.00	1.00	1.52	1.61	1.17	2.09	0.00	1.13
time (sec)	N/A	0.008	0.014	0.416	0.274	1.887	0.303	0.000	5.260

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	36	27	28	41	0	31
N.S.	1	1.00	1.09	1.57	1.17	1.22	1.78	0.00	1.35
time (sec)	N/A	0.009	0.026	0.411	0.280	2.110	0.630	0.000	5.227

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	37	19	28	39	0	34
N.S.	1	1.00	1.20	2.47	1.27	1.87	2.60	0.00	2.27
time (sec)	N/A	0.006	0.018	0.402	0.295	2.076	0.671	0.000	5.225

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	27	16	26	20	0	26
N.S.	1	1.00	1.09	1.23	0.73	1.18	0.91	0.00	1.18
time (sec)	N/A	0.008	0.022	0.539	0.282	2.281	0.507	0.000	5.231

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	30	11	26	20	0	28
N.S.	1	1.00	1.20	2.00	0.73	1.73	1.33	0.00	1.87
time (sec)	N/A	0.005	0.015	0.401	0.284	2.408	0.547	0.000	5.203

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	12	8	8	8	9	8
N.S.	1	1.00	0.83	1.00	0.67	0.67	0.67	0.75	0.67
time (sec)	N/A	0.003	0.008	0.358	0.273	1.399	0.049	0.555	0.096

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	25	116	17	8	22	18	8
N.S.	1	1.00	1.79	8.29	1.21	0.57	1.57	1.29	0.57
time (sec)	N/A	0.003	0.012	0.689	0.271	1.186	0.099	0.565	5.296

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	32	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	0.67
time (sec)	N/A	0.003	0.011	0.349	0.504	2.001	0.069	0.606	0.094

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	28	31	24	31	0	26
N.S.	1	1.00	1.00	1.17	1.29	1.00	1.29	0.00	1.08
time (sec)	N/A	0.012	0.024	0.441	0.496	0.995	0.183	0.000	5.249

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	104	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.179	0.111	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	109	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.050	0.027	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	103	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.388	0.070	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	99	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.126	0.008	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.356	0.070	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	39	58	97	78	0	-1
N.S.	1	1.00	0.88	0.76	1.14	1.90	1.53	0.00	-0.02
time (sec)	N/A	0.020	0.035	0.653	0.496	1.742	0.771	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.078	0.083	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	77	0	0	0	0	0	97
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	1.59
time (sec)	N/A	0.061	0.069	0.089	0.000	0.000	0.000	0.000	5.169

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.080	0.082	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	131	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.191	0.026	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	126	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.554	0.097	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	117	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.141	0.008	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	120	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.387	0.075	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	58	51	73	120	88	0	-1
N.S.	1	1.00	0.79	0.70	1.00	1.64	1.21	0.00	-0.01
time (sec)	N/A	0.027	0.053	0.649	0.490	2.806	1.571	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.087	0.069	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.087	0.073	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.415	0.100	0.077	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.338	0.092	0.091	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	58	47	0	93	0	67	67
N.S.	1	1.00	1.14	0.92	0.00	1.82	0.00	1.31	1.31
time (sec)	N/A	0.071	0.018	0.085	0.000	2.585	0.000	0.473	5.184

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	63	53	108	0	69	55
N.S.	1	1.00	1.48	1.50	1.26	2.57	0.00	1.64	1.31
time (sec)	N/A	0.017	0.037	0.049	0.499	1.241	0.000	0.562	5.574

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	66	55	0	104	0	71	63
N.S.	1	1.00	1.29	1.08	0.00	2.04	0.00	1.39	1.24
time (sec)	N/A	0.045	0.040	0.057	0.000	1.569	0.000	0.490	5.337

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	69	74	0	112	0	0	-1
N.S.	1	1.00	1.13	1.21	0.00	1.84	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.013	0.879	0.000	1.680	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	55	0	98	0	61	67
N.S.	1	1.00	1.25	1.04	0.00	1.85	0.00	1.15	1.26
time (sec)	N/A	0.059	0.036	0.100	0.000	1.333	0.000	0.482	5.182

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	68	81	34	118	0	63	54
N.S.	1	1.00	1.58	1.88	0.79	2.74	0.00	1.47	1.26
time (sec)	N/A	0.018	0.039	0.055	0.498	2.214	0.000	0.442	5.384

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	73	73	0	109	0	65	63
N.S.	1	1.00	1.38	1.38	0.00	2.06	0.00	1.23	1.19
time (sec)	N/A	0.051	0.045	0.095	0.000	1.830	0.000	0.454	5.354

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	77	105	0	118	0	0	-1
N.S.	1	1.00	1.22	1.67	0.00	1.87	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.048	0.740	0.000	2.322	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	98	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.134	0.118	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	-1
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.304	0.079	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.010	0.064	0.076	0.000	0.000	0.000	0.000	5.387

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	87	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.191	0.076	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	42	76	27	0	-1
N.S.	1	1.00	1.00	0.84	1.35	2.45	0.87	0.00	-0.03
time (sec)	N/A	0.014	0.031	0.690	0.520	1.441	0.564	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.059	0.082	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	66	0	0	0	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.060	0.079	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.065	0.101	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	117	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.157	0.109	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	109	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	1.640	0.078	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	91	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.138	0.079	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.327	0.080	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	42	61	148	185	0	-1
N.S.	1	1.00	0.96	0.78	1.13	2.74	3.43	0.00	-0.02
time (sec)	N/A	0.023	0.054	0.421	0.501	1.616	1.144	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.078	0.071	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	74	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.079	0.074	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.087	0.077	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	74	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.080	0.080	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	63	477	0	102	0	0	-1
N.S.	1	1.00	1.97	14.91	0.00	3.19	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.015	0.413	0.000	1.706	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	59	49	0	80	0	40	-1
N.S.	1	1.00	1.84	1.53	0.00	2.50	0.00	1.25	-0.03
time (sec)	N/A	0.012	0.102	0.059	0.000	1.677	0.000	0.543	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	63	0	0	102	0	0	-1
N.S.	1	1.00	1.97	0.00	0.00	3.19	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.186	0.030	0.000	3.921	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	76	0	0	102	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	2.76	0.00	0.00	-0.03
time (sec)	N/A	0.017	0.054	0.087	0.000	2.163	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	466	0	111	0	0	-1
N.S.	1	1.00	2.00	14.12	0.00	3.36	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.141	1.052	0.000	3.105	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	62	51	0	88	0	47	-1
N.S.	1	1.00	1.88	1.55	0.00	2.67	0.00	1.42	-0.03
time (sec)	N/A	0.011	0.107	0.067	0.000	2.897	0.000	0.554	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	0	0	111	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	3.36	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.197	0.028	0.000	3.746	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	78	0	0	106	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	2.79	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.062	0.085	0.000	1.923	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.016	0.068	0.083	0.000	0.000	0.000	0.000	5.209

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	109	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	2.95	0.00	0.00	1.81
time (sec)	N/A	0.014	0.017	0.082	0.000	1.388	0.000	0.000	5.323

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.013	0.016	0.081	0.000	0.000	0.000	0.000	5.127

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.014	0.072	0.086	0.000	0.000	0.000	0.000	5.166

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	109	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	2.87	0.00	0.00	1.74
time (sec)	N/A	0.013	0.018	0.089	0.000	2.157	0.000	0.000	5.102

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.013	0.017	0.090	0.000	0.000	0.000	0.000	5.117

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	218	0	0	0	0	0	-1
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.266	0.032	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	156	0	0	0	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.157	0.030	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	106	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.088	0.106	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	116	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.123	0.114	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	166	0	0	0	0	0	-1
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.364	0.108	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	177	0	0	0	0	0	82
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.039	0.156	0.007	0.000	0.000	0.000	0.000	5.245

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	134	0	0	0	0	0	82
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.038	0.120	0.008	0.000	0.000	0.000	0.000	5.225

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	0	0	0	0	83
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.037	0.057	0.092	0.000	0.000	0.000	0.000	5.273

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0	83
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.042	0.077	0.096	0.000	0.000	0.000	0.000	5.484

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	185	0	0	0	0	0	83
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.042	0.210	0.089	0.000	0.000	0.000	0.000	5.604

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	26	0	16	0	50	15
N.S.	1	1.00	1.06	1.44	0.00	0.89	0.00	2.78	0.83
time (sec)	N/A	0.005	0.016	0.057	0.000	1.348	0.000	1.314	5.262

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	19	0	11	27
N.S.	1	1.00	1.00	0.90	0.00	0.95	0.00	0.55	1.35
time (sec)	N/A	0.002	0.032	0.368	0.000	1.566	0.000	1.506	5.322

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.003	0.009	0.435	0.506	1.668	0.068	1.124	5.244

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.88
time (sec)	N/A	0.007	0.018	0.376	0.291	1.398	0.000	1.049	5.231

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16
time (sec)	N/A	0.007	0.002	0.384	0.300	1.610	0.000	0.924	5.258

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	14	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.56	0.88
time (sec)	N/A	0.006	0.008	0.383	0.302	1.517	0.000	2.260	5.211

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	988	988	61	0	0	0	0	0	42
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	1.558	10.019	0.050	0.000	0.000	0.000	0.000	5.410

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	61	0	0	0	0	0	42
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.712	10.018	0.052	0.000	0.000	0.000	0.000	5.251

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	92	92	0	0	0	0	0	-1
N.S.	1	1.03	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.096	0.196	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.108	0.198	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	74	74	0	0	0	0	0	-1
N.S.	1	1.12	1.12	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.091	0.207	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.142	0.202	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.124	0.208	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	64	0	0	76
N.S.	1	1.00	0.98	0.00	0.00	1.45	0.00	0.00	1.73
time (sec)	N/A	0.012	0.056	0.110	0.000	2.298	0.000	0.000	5.303

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	61	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.003	0.128	0.000	2.214	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	76	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	1.65	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.064	0.203	0.000	2.083	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	79	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	1.72	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.005	0.213	0.000	2.446	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	-1
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.048	0.438	0.298	2.380	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	-1
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.044	0.431	0.291	1.982	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	-1
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.047	0.429	0.303	3.007	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	47	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.82	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.045	0.105	0.000	2.853	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	0	0	54	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.043	0.104	0.000	2.403	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	0	0	76	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	1.95	0.00	0.00	-0.03
time (sec)	N/A	0.038	0.092	0.194	0.000	2.383	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	38	0	0	64	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	1.60	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.074	0.184	0.000	3.471	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [454] had the largest ratio of [28]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	13	0.077
2	A	2	1	1.00	11	0.091
3	A	1	0	1.00	9	0.000
4	A	2	1	1.00	13	0.077
5	A	2	1	1.00	13	0.077
6	A	3	2	1.00	15	0.133
7	A	4	3	1.00	13	0.231
8	A	3	2	1.00	11	0.182
9	A	2	2	1.00	15	0.133
10	A	3	2	1.00	15	0.133
11	A	3	2	1.00	11	0.182
12	A	4	3	1.00	15	0.200
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	15	0.133
15	A	2	2	1.00	13	0.154
16	A	5	5	1.00	11	0.454
17	A	3	3	1.00	15	0.200
18	A	4	3	1.00	15	0.200
19	A	4	3	1.00	15	0.200
20	A	4	3	1.00	15	0.200
21	A	3	3	1.00	15	0.200
22	A	4	3	1.00	13	0.231
23	A	4	4	1.00	11	0.364
24	A	4	3	1.00	15	0.200
25	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	13	0.231
27	A	4	3	1.00	13	0.231
28	A	3	3	1.00	13	0.231
29	A	2	2	1.00	13	0.154
30	A	2	2	1.00	11	0.182
31	A	5	5	1.00	9	0.556
32	A	3	3	1.00	13	0.231
33	A	4	3	1.00	13	0.231
34	A	4	3	1.00	13	0.231
35	A	4	3	1.00	13	0.231
36	A	5	5	1.00	9	0.556
37	A	5	5	1.00	11	0.454
38	A	6	5	1.00	17	0.294
39	A	7	7	1.00	17	0.412
40	A	5	5	1.00	15	0.333
41	A	6	6	1.00	13	0.462
42	A	4	4	1.00	17	0.235
43	A	6	6	1.00	17	0.353
44	A	4	4	1.00	17	0.235
45	A	7	7	1.00	17	0.412
46	A	7	5	1.00	17	0.294
47	A	8	7	1.00	15	0.467
48	A	6	6	1.00	13	0.462
49	A	7	7	1.00	17	0.412
50	A	5	4	1.00	17	0.235
51	A	7	7	1.00	17	0.412
52	A	5	5	1.00	17	0.294
53	A	7	6	1.00	17	0.353
54	A	5	4	1.00	17	0.235
55	A	8	7	1.00	17	0.412
56	A	6	5	1.00	17	0.294
57	A	5	4	1.00	17	0.235
58	A	6	6	1.00	17	0.353
59	A	4	4	1.00	17	0.235
60	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	13	0.231
62	A	6	6	1.00	17	0.353
63	A	4	4	1.00	17	0.235
64	A	7	6	1.00	17	0.353
65	A	6	5	1.00	17	0.294
66	A	7	7	1.00	17	0.412
67	A	5	5	1.00	17	0.294
68	A	6	6	1.00	17	0.353
69	A	4	4	1.00	17	0.235
70	A	6	6	1.00	17	0.353
71	A	4	4	1.00	15	0.267
72	A	7	7	1.00	13	0.538
73	A	5	5	1.00	17	0.294
74	A	8	7	1.00	17	0.412
75	A	7	4	1.00	19	0.210
76	A	5	2	1.00	19	0.105
77	A	6	3	1.00	19	0.158
78	A	4	2	1.00	19	0.105
79	A	1	1	1.00	19	0.053
80	A	3	2	1.00	19	0.105
81	A	2	2	1.00	19	0.105
82	A	2	2	1.00	19	0.105
83	A	3	2	1.00	19	0.105
84	A	1	1	1.00	19	0.053
85	A	4	2	1.00	19	0.105
86	A	6	3	1.00	19	0.158
87	A	5	2	1.00	19	0.105
88	A	7	4	1.00	19	0.210
89	A	6	3	1.00	19	0.158
90	A	8	4	1.00	19	0.210
91	A	7	3	1.00	19	0.158
92	A	3	3	1.00	17	0.176
93	A	2	2	1.00	15	0.133
94	A	1	1	1.00	17	0.059
95	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
96	A	3	2	1.00	17	0.118
97	A	4	4	1.00	17	0.235
98	A	3	3	1.00	13	0.231
99	A	4	4	1.00	17	0.235
100	A	6	6	1.00	17	0.353
101	A	5	5	1.00	17	0.294
102	A	6	6	1.00	17	0.353
103	A	8	5	1.00	19	0.263
104	A	6	5	1.00	17	0.294
105	A	4	4	1.00	15	0.267
106	A	1	1	1.00	19	0.053
107	A	3	2	1.00	19	0.105
108	A	5	2	1.00	19	0.105
109	A	7	2	1.00	19	0.105
110	A	9	6	1.00	19	0.316
111	A	7	6	1.00	19	0.316
112	A	5	5	1.00	17	0.294
113	A	1	1	1.00	15	0.067
114	A	3	3	1.00	19	0.158
115	A	5	3	1.00	19	0.158
116	A	7	3	1.00	19	0.158
117	A	9	5	1.00	21	0.238
118	A	7	5	1.00	21	0.238
119	A	5	5	1.00	21	0.238
120	A	3	3	1.00	21	0.143
121	A	2	2	1.00	21	0.095
122	A	4	2	1.00	21	0.095
123	A	6	2	1.00	21	0.095
124	A	8	6	1.00	21	0.286
125	A	6	6	1.00	21	0.286
126	A	4	4	1.00	21	0.190
127	A	2	2	1.00	21	0.095
128	A	4	3	1.00	21	0.143
129	A	6	3	1.00	21	0.143
130	A	8	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	11	6	1.00	19	0.316
132	A	11	8	1.00	19	0.421
133	A	8	6	1.00	17	0.353
134	A	8	8	1.00	15	0.533
135	A	5	5	1.00	19	0.263
136	A	8	8	1.00	19	0.421
137	A	7	6	1.00	19	0.316
138	A	11	8	1.00	19	0.421
139	A	10	6	1.00	19	0.316
140	A	11	6	1.00	19	0.316
141	A	11	8	1.00	17	0.471
142	A	8	7	1.00	15	0.467
143	A	8	8	1.00	19	0.421
144	A	6	6	1.00	19	0.316
145	A	9	8	1.00	19	0.421
146	A	8	6	1.00	19	0.316
147	A	12	8	1.00	19	0.421
148	A	11	6	1.00	19	0.316
149	A	11	5	1.00	19	0.263
150	A	11	7	1.00	19	0.368
151	A	8	5	1.00	19	0.263
152	A	8	7	1.00	17	0.412
153	A	5	5	1.00	15	0.333
154	A	7	7	1.00	19	0.368
155	A	6	5	1.00	19	0.263
156	A	10	7	1.00	19	0.368
157	A	9	5	1.00	19	0.263
158	A	12	8	1.00	19	0.421
159	A	9	6	1.00	19	0.316
160	A	9	8	1.00	19	0.421
161	A	6	6	1.00	17	0.353
162	A	7	7	1.00	15	0.467
163	A	6	6	1.00	19	0.316
164	A	10	8	1.00	19	0.421
165	A	9	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	13	8	1.00	19	0.421
167	A	13	3	1.00	19	0.158
168	A	10	3	1.00	19	0.158
169	A	7	3	1.00	17	0.176
170	A	4	3	1.00	15	0.200
171	A	1	1	1.00	19	0.053
172	A	4	4	1.00	19	0.210
173	A	7	4	1.00	19	0.210
174	A	10	4	1.00	19	0.210
175	A	13	4	1.00	19	0.210
176	A	12	3	1.00	19	0.158
177	A	9	3	1.00	17	0.176
178	A	6	3	1.00	15	0.200
179	A	3	2	1.00	19	0.105
180	A	4	3	1.00	19	0.158
181	A	5	4	1.00	19	0.210
182	A	8	4	1.00	19	0.210
183	A	11	4	1.00	19	0.210
184	A	14	4	1.00	19	0.210
185	A	14	3	1.00	19	0.158
186	A	11	3	1.00	19	0.158
187	A	8	3	1.00	19	0.158
188	A	5	3	1.00	17	0.176
189	A	2	2	1.00	15	0.133
190	A	3	3	1.00	19	0.158
191	A	6	3	1.00	19	0.158
192	A	9	3	1.00	19	0.158
193	A	12	3	1.00	19	0.158
194	A	12	4	1.00	19	0.210
195	A	9	4	1.00	19	0.210
196	A	6	4	1.00	19	0.210
197	A	3	3	1.00	17	0.176
198	A	3	3	1.00	15	0.200
199	A	6	4	1.00	19	0.210
200	A	9	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	12	4	1.00	19	0.210
202	A	15	4	1.00	19	0.210
203	A	2	1	1.00	15	0.067
204	A	2	1	1.00	13	0.077
205	A	1	0	1.00	11	0.000
206	A	2	1	1.00	15	0.067
207	A	2	1	1.00	15	0.067
208	A	3	2	1.00	17	0.118
209	A	3	2	1.00	15	0.133
210	A	3	2	1.00	13	0.154
211	A	3	2	1.00	17	0.118
212	A	3	2	1.00	17	0.118
213	A	3	2	1.00	17	0.118
214	A	3	2	1.00	17	0.118
215	A	3	2	1.00	17	0.118
216	A	3	2	1.00	17	0.118
217	A	2	2	1.00	17	0.118
218	A	4	4	1.00	15	0.267
219	A	3	2	1.00	13	0.154
220	A	3	2	1.00	17	0.118
221	A	3	2	1.00	17	0.118
222	A	3	2	1.00	17	0.118
223	A	3	2	1.00	17	0.118
224	A	3	2	1.00	17	0.118
225	A	3	2	1.00	17	0.118
226	A	2	2	1.00	17	0.118
227	A	3	2	1.00	17	0.118
228	A	3	2	1.00	17	0.118
229	A	3	2	1.00	15	0.133
230	A	3	2	1.00	13	0.154
231	A	3	2	1.00	17	0.118
232	A	4	3	1.00	19	0.158
233	A	3	3	1.00	17	0.176
234	A	2	2	1.00	15	0.133
235	A	1	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	19	0.158
237	A	3	3	1.00	19	0.158
238	A	4	4	1.00	19	0.210
239	A	5	4	1.00	19	0.210
240	A	6	3	1.00	19	0.158
241	A	5	3	1.00	17	0.176
242	A	4	3	1.00	15	0.200
243	A	3	2	1.00	19	0.105
244	A	2	2	1.00	19	0.105
245	A	1	1	1.00	19	0.053
246	A	4	3	1.00	19	0.158
247	A	4	4	1.00	19	0.210
248	A	4	3	1.00	19	0.158
249	A	5	4	1.00	19	0.210
250	A	6	4	1.00	19	0.210
251	A	7	4	1.00	19	0.210
252	A	4	2	1.00	19	0.105
253	A	3	2	1.00	19	0.105
254	A	2	2	1.00	19	0.105
255	A	1	1	1.00	17	0.059
256	A	2	2	1.00	15	0.133
257	A	3	3	1.00	19	0.158
258	A	4	3	1.00	19	0.158
259	A	5	3	1.00	19	0.158
260	A	4	3	1.00	19	0.158
261	A	3	3	1.00	19	0.158
262	A	2	2	1.00	19	0.105
263	A	1	1	1.00	19	0.053
264	A	3	3	1.00	19	0.158
265	A	4	4	1.00	17	0.235
266	A	5	4	1.00	15	0.267
267	A	6	4	1.00	19	0.210
268	A	7	4	1.00	19	0.210
269	A	5	3	1.00	21	0.143
270	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	3	1.00	21	0.143
272	A	2	2	1.00	21	0.095
273	A	1	1	1.00	21	0.048
274	A	2	2	1.00	21	0.095
275	A	3	2	1.00	21	0.095
276	A	4	2	1.00	21	0.095
277	A	3	3	1.00	21	0.143
278	A	3	3	1.27	19	0.158
279	A	3	3	1.00	21	0.143
280	A	1	1	1.00	21	0.048
281	A	2	2	1.00	21	0.095
282	A	3	2	1.00	21	0.095
283	A	2	2	1.00	17	0.118
284	A	3	2	1.00	19	0.105
285	A	2	2	1.00	19	0.105
286	A	1	1	1.00	19	0.053
287	A	2	2	1.00	15	0.133
288	A	3	3	1.00	19	0.158
289	A	3	3	1.00	19	0.158
290	A	2	2	1.00	17	0.118
291	A	3	3	1.00	19	0.158
292	A	5	5	1.00	19	0.263
293	A	4	4	1.00	19	0.210
294	A	5	5	1.00	19	0.263
295	A	5	4	1.00	21	0.190
296	A	6	6	1.00	21	0.286
297	A	3	3	1.00	21	0.143
298	A	4	4	1.00	21	0.190
299	A	5	5	1.00	21	0.238
300	A	2	2	1.00	21	0.095
301	A	3	3	1.00	21	0.143
302	A	6	6	1.00	21	0.286
303	A	1	1	1.00	21	0.048
304	A	4	4	1.00	21	0.190
305	A	7	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	2	2	1.00	21	0.095
307	A	5	4	1.00	21	0.190
308	A	3	2	1.00	15	0.133
309	A	3	2	1.00	13	0.154
310	A	5	3	1.00	19	0.158
311	A	4	3	1.00	19	0.158
312	A	3	3	1.00	19	0.158
313	A	2	2	1.00	17	0.118
314	A	1	1	1.00	15	0.067
315	A	2	2	1.00	19	0.105
316	A	3	2	1.00	19	0.105
317	A	4	2	1.00	19	0.105
318	A	5	2	1.00	19	0.105
319	A	4	3	1.00	11	0.273
320	A	4	3	1.00	13	0.231
321	A	3	3	1.00	9	0.333
322	A	3	3	1.00	9	0.333
323	A	3	3	1.00	9	0.333
324	A	3	3	1.00	13	0.231
325	A	3	3	1.00	13	0.231
326	A	3	3	1.00	13	0.231
327	A	3	3	1.00	13	0.231
328	A	2	2	1.00	11	0.182
329	A	2	2	1.00	15	0.133
330	A	2	2	1.00	15	0.133
331	A	2	2	1.00	23	0.087
332	A	2	2	1.00	11	0.182
333	A	2	2	1.00	13	0.154
334	A	2	2	1.00	13	0.154
335	A	2	2	1.00	17	0.118
336	A	2	2	1.00	17	0.118
337	A	2	2	1.00	17	0.118
338	A	2	2	1.00	11	0.182
339	A	2	2	1.00	11	0.182
340	A	2	2	1.00	11	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	2	2	1.00	11	0.182
342	A	2	2	1.00	13	0.154
343	A	2	2	1.00	13	0.154
344	A	3	2	1.00	11	0.182
345	A	4	3	1.00	11	0.273
346	A	3	2	1.00	11	0.182
347	A	7	7	1.00	9	0.778
348	A	2	2	1.00	22	0.091
349	A	1	1	1.00	13	0.077
350	A	2	2	1.00	15	0.133
351	A	2	2	1.00	17	0.118
352	A	1	1	1.00	13	0.077
353	A	2	2	1.00	15	0.133
354	A	1	1	1.00	13	0.077
355	A	2	2	1.00	11	0.182
356	A	5	5	1.00	13	0.385
357	A	2	2	1.00	15	0.133
358	A	5	5	1.00	13	0.385
359	A	2	2	1.00	15	0.133
360	A	2	2	1.00	11	0.182
361	A	2	2	1.00	11	0.182
362	A	2	2	1.00	9	0.222
363	A	5	5	1.00	11	0.454
364	A	3	3	1.00	25	0.120
365	A	4	4	1.00	27	0.148
366	A	4	4	1.00	23	0.174
367	A	4	4	1.00	22	0.182
368	A	4	4	1.00	21	0.190
369	A	5	5	1.00	18	0.278
370	A	4	4	1.00	23	0.174
371	A	3	3	1.00	15	0.200
372	A	4	4	1.00	23	0.174
373	A	5	4	1.00	27	0.148
374	A	5	4	1.00	23	0.174
375	A	5	4	1.00	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	5	4	1.00	21	0.190
377	A	6	5	1.00	18	0.278
378	A	5	4	1.00	23	0.174
379	A	5	5	1.00	22	0.227
380	A	5	4	1.00	23	0.174
381	A	5	4	1.00	22	0.182
382	A	5	5	1.00	13	0.385
383	A	5	5	1.00	15	0.333
384	A	4	4	1.00	15	0.267
385	A	4	4	1.00	15	0.267
386	A	5	5	1.00	15	0.333
387	A	5	5	1.00	17	0.294
388	A	4	4	1.00	17	0.235
389	A	4	4	1.00	17	0.235
390	A	3	3	1.00	27	0.111
391	A	3	3	1.00	23	0.130
392	A	2	2	1.00	15	0.133
393	A	3	3	1.00	21	0.143
394	A	4	4	1.00	18	0.222
395	A	3	3	1.00	23	0.130
396	A	3	3	1.00	22	0.136
397	A	3	3	1.00	23	0.130
398	A	4	4	1.00	27	0.148
399	A	4	4	1.00	23	0.174
400	A	4	4	1.00	22	0.182
401	A	4	4	1.00	21	0.190
402	A	5	5	1.00	18	0.278
403	A	4	4	1.00	23	0.174
404	A	4	4	1.00	22	0.182
405	A	4	4	1.00	23	0.174
406	A	4	4	1.00	22	0.182
407	A	3	3	1.00	15	0.200
408	A	3	3	1.00	15	0.200
409	A	3	3	1.00	15	0.200
410	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
411	A	3	3	1.00	16	0.188
412	A	3	3	1.00	16	0.188
413	A	3	3	1.00	16	0.188
414	A	3	3	1.00	20	0.150
415	A	3	3	1.00	19	0.158
416	A	3	3	1.00	17	0.176
417	A	3	3	1.00	17	0.176
418	A	3	3	1.00	20	0.150
419	A	3	3	1.00	19	0.158
420	A	3	3	1.00	18	0.167
421	A	3	3	1.00	21	0.143
422	A	3	3	1.00	21	0.143
423	A	3	3	1.00	21	0.143
424	A	3	3	1.00	21	0.143
425	A	3	3	1.00	21	0.143
426	A	3	3	1.00	15	0.200
427	A	3	3	1.00	15	0.200
428	A	3	3	1.00	15	0.200
429	A	3	3	1.00	15	0.200
430	A	3	3	1.00	15	0.200
431	A	2	2	1.00	11	0.182
432	A	1	1	1.00	11	0.091
433	A	3	3	1.00	13	0.231
434	A	1	1	1.00	17	0.059
435	A	1	1	1.00	17	0.059
436	A	2	2	1.00	15	0.133
437	A	11	10	1.00	19	0.526
438	A	9	8	1.00	19	0.421
439	A	3	3	1.03	17	0.176
440	A	3	3	1.00	22	0.136
441	A	3	3	1.12	22	0.136
442	A	3	3	1.00	27	0.111
443	A	3	3	1.00	27	0.111
444	A	1	1	1.00	18	0.056
445	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	2	2	1.00	22	0.091
447	A	1	1	1.00	23	0.043
448	A	2	2	1.00	19	0.105
449	A	2	2	1.00	19	0.105
450	A	2	2	1.00	19	0.105
451	A	2	2	1.00	19	0.105
452	A	2	2	1.00	19	0.105
453	A	1	1	1.00	25	0.040
454	A	2	2	1.00	28	0.071

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2(ax + bx^3)\, dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2\left(ax+bx^3\right)\, dx &= \int \left(ax^3+bx^5\right)\, dx \\ &= \frac{ax^4}{4}+\frac{bx^6}{6} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.82

method	result	size
default	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
gosper	$\frac{x^4(2b\,x^2+3a)}{12}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Maxima [A]
time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Fricas [A]
time = 1.95, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

Giac [**A**]
time = 1.50, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Mupad [**B**]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^6}{6}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

3.2 $\int x(ax+bx^3)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{14\}$

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3\right)\,dx &= \int \left(ax^2+bx^4\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.82

method	result	size
default	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}$	14
gosper	$\frac{x^{3}\left(3b\,x^{2}+5a\right)}{15}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A]
time = 1.13, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^{3}}{3}+\frac{bx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

Giac [A]
time = 1.41, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^5}{5} + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

3.3 $\int (ax + bx^3) \, dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a*x + b*x^3,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rubi steps

$$\int (ax + bx^3) \, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x + b*x^3,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Maple [A]
time = 0.05, size = 15, normalized size = 0.88

method	result	size
norman	$\frac{1}{2}a\,x^2 + \frac{1}{4}b\,x^4$	14
risch	$\frac{1}{2}a\,x^2 + \frac{1}{4}b\,x^4$	14

gosper	$\frac{x^2(bx^2+2a)}{4}$	15
default	$\frac{(bx^2+a)^2}{4b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^2/b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Fricas [A]
time = 2.01, size = 13, normalized size = 0.76

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

Giac [A]
time = 1.36, size = 13, normalized size = 0.76

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x + b*x^3,x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

3.4 $\int \frac{ax+bx^3}{x} \, dx$

Optimal. Leaf size=12

$$ax + \frac{bx^3}{3}$$

[Out] a*x+1/3*b*x^3

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)/x,x]

[Out] a*x + (b*x^3)/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3}{x} \, dx &= \int \left(a+bx^2\right) \, dx \\ &= ax + \frac{bx^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)/x,x]

[Out] a*x + (b*x^3)/3

Maple [A]
time = 0.02, size = 11, normalized size = 0.92

method	result	size
default	$ax+\frac{1}{3}b\,x^{3}$	11
norman	$ax+\frac{1}{3}b\,x^{3}$	11
risch	$ax+\frac{1}{3}b\,x^{3}$	11
gosper	$\frac{x\left(b\,x^{2}+3a\right)}{3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/3*b*x^3
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

Fricas [A]
time = 1.74, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$ax+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3
```

Giac [A]
time = 1.47, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + a*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)/x,x)
```

```
[Out] a*x + (b*x^3)/3
```

3.5 $\int \frac{ax+bx^3}{x^2} \, dx$

Optimal. Leaf size=13

$$\frac{bx^2}{2} + a\log(x)$$

[Out] 1/2*b*x^2+a*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$a\log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)/x^2,x]

[Out] (b*x^2)/2 + a*Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3}{x^2} \, dx &= \int \left(\frac{a}{x} + bx\right) \, dx \\ &= \frac{bx^2}{2} + a\log(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{bx^2}{2} + a\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)/x^2,x]

[Out] (b*x^2)/2 + a*Log[x]

Maple [A]
time = 0.02, size = 12, normalized size = 0.92

method	result	size
default	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b*x^2+a*ln(x)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^{2}+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*log(x)
```

Fricas [A]
time = 1.70, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^{2}+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*log(x)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.77

$$a\log\left(x\right)+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x**2,x)
[Out] a*log(x) + b*x**2/2
```

Giac [A]
time = 1.58, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^2}{2}+a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)/x^2,x)
```

```
[Out] (b*x^2)/2 + a*log(x)
```

3.6 $\int x^2(ax+bx^3)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 276}

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax+bx^3\right)^2\,dx &= \int x^4\left(a+bx^2\right)^2\,dx\\ &= \int \left(a^2x^4+2abx^6+b^2x^8\right)\,dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Maple [A]
time = 0.34, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{5}a^{2}x^{5}+\frac{2}{7}ab\,x^{7}+\frac{1}{9}b^{2}x^{9}$	25
norman	$\frac{1}{5}a^{2}x^{5}+\frac{2}{7}ab\,x^{7}+\frac{1}{9}b^{2}x^{9}$	25
risch	$\frac{1}{5}a^{2}x^{5}+\frac{2}{7}ab\,x^{7}+\frac{1}{9}b^{2}x^{9}$	25
gosper	$\frac{x^{5}\left(35b^{2}x^{4}+90ab\,x^{2}+63a^{2}\right)}{315}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^{2}x^{9}+\frac{2}{7}\,abx^{7}+\frac{1}{5}\,a^{2}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Fricas [A]
time = 1.42, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^{2}x^{9}+\frac{2}{7}\,abx^{7}+\frac{1}{5}\,a^{2}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^{2}x^{5}}{5}+\frac{2abx^{7}}{7}+\frac{b^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

Giac [A]
time = 1.16, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^{2}x^{9}+\frac{2}{7}\,abx^{7}+\frac{1}{5}\,a^{2}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

3.7 $\int x(ax+bx^3)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {1598, 272, 45}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3\right)^2\,dx&=\int x^3\left(a+bx^2\right)^2\,dx\\&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^2\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^2x+2abx^2+b^2x^3\right)\,dx,x,x^2\right)\\&=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Maple [A]
time = 0.37, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
gosper	$\frac{x^4\left(3b^2x^4+8ab\,x^2+6a^2\right)}{24}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Fricas [A]
time = 2.17, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Giac [A]
time = 1.62, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

3.8 $\int (ax+bx^3)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 276}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^3\right)^2\,dx&=\int x^2\left(a+bx^2\right)^2\,dx\\&=\int \left(a^2x^2+2abx^4+b^2x^6\right)\,dx\\&=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Maple [A]
time = 0.36, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
gosper	$\frac{x^3\left(15b^2x^4+42ab\,x^2+35a^2\right)}{105}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Fricas [A]
time = 1.31, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Giac [A]
time = 1.73, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

3.9 $\int \frac{(ax+bx^3)^2}{x}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^3}{6b}$$

```
[Out] 1/6*(b*x^2+a)^3/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 267}

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^3)^2}{x}\,dx &= \int x\left(a+bx^2\right)^2\,dx\\ &= \frac{(a+bx^2)^3}{6b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Maple [A]
time = 0.35, size = 15, normalized size = 0.94

method	result	size
default	$\frac{\left(b\,x^{2}+a\right)^{3}}{6b}$	15
norman	$\frac{1}{6}b^{2}x^{6}+\frac{1}{2}ab\,x^{4}+\frac{1}{2}a^{2}x^{2}$	25
gosper	$\frac{x^{2}\left(b^{2}x^{4}+3ab\,x^{2}+3a^{2}\right)}{6}$	26
risch	$\frac{b^{2}x^{6}}{6}+\frac{ab\,x^{4}}{2}+\frac{a^{2}x^{2}}{2}+\frac{a^{3}}{6b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x^2+a)^3/b
```

Maxima [A]
time = 0.28, size = 24, normalized size = 1.50

$$\frac{1}{6}\,b^{2}x^{6}+\frac{1}{2}\,abx^{4}+\frac{1}{2}\,a^{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Fricas [A]
time = 1.82, size = 24, normalized size = 1.50

$$\frac{1}{6}\,b^{2}x^{6}+\frac{1}{2}\,abx^{4}+\frac{1}{2}\,a^{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 0.01, size = 24, normalized size = 1.50

$$\frac{a^{2}x^{2}}{2}+\frac{abx^{4}}{2}+\frac{b^{2}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Giac [A]
time = 1.00, size = 24, normalized size = 1.50

$$\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Mupad [B]
time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^2}{2}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2/x,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

3.10 $\int \frac{(ax+bx^3)^2}{x^2}\,dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 200}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax+bx^3\right)^2}{x^2}\,dx &= \int \left(a+bx^2\right)^2\,dx\\
&= \int \left(a^2+2abx^2+b^2x^4\right)\,dx\\
&= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A]
time = 0.33, size = 22, normalized size = 0.88

method	result	size
default	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
gosper	$\frac{x\left(3b^2x^4+10ab\,x^2+15a^2\right)}{15}$	25
norman	$\frac{a^2x^2+\frac{1}{5}b^2x^6+\frac{2}{3}ab\,x^4}{x}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A]
time = 1.29, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.88

$$a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Giac [A]
time = 1.32, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2/x^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.11 $\int \left(-4x+3x^3\right)^6 \, dx$

Optimal. Leaf size=46

$$\frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}$$

```
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 276}

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \left(-4x+3x^3\right)^6 \, dx &= \int x^6\left(-4+3x^2\right)^6 \, dx\\
&= \int \left(4096x^6-18432x^8+34560x^{10}-34560x^{12}+19440x^{14}-5832x^{16}+729x^{18}\right) dx\\
&= \frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 46, normalized size = 1.00

$$\frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*x + 3*x^3)^6,x]
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

Maple [A]
time = 0.38, size = 37, normalized size = 0.80

method	result	size
default	$\frac{4096}{7}x^7-2048x^9+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
norman	$\frac{4096}{7}x^7-2048x^9+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
risch	$\frac{4096}{7}x^7-2048x^9+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
gosper	$\frac{x^7(12405393x^{12}-110918808x^{10}+419026608x^8-859541760x^6+1015822080x^4-662165504x^2+189190144)}{323323}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x)^6,x,method=_RETURNVERBOSE)
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

Maxima [A]
time = 0.29, size = 36, normalized size = 0.78

$$\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^9+\frac{4096}{7}\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="maxima")
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

Fricas [A]
time = 2.27, size = 36, normalized size = 0.78

$$\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^9+\frac{4096}{7}\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="fricas")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

Sympy [A]
time = 0.01, size = 42, normalized size = 0.91

$$\frac{729x^{19}}{19} - \frac{5832x^{17}}{17} + 1296x^{15} - \frac{34560x^{13}}{13} + \frac{34560x^{11}}{11} - 2048x^9 + \frac{4096x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x)**6,x)
```

```
[Out] 729*x**19/19 - 5832*x**17/17 + 1296*x**15 - 34560*x**13/13 + 34560*x**11/11
 - 2048*x**9 + 4096*x**7/7
```

Giac [A]
time = 1.23, size = 36, normalized size = 0.78

$$\frac{729}{19}\,x^{19} - \frac{5832}{17}\,x^{17} + 1296\,x^{15} - \frac{34560}{13}\,x^{13} + \frac{34560}{11}\,x^{11} - 2048\,x^9 + \frac{4096}{7}\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="giac")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.78

$$\frac{729\,x^{19}}{19} - \frac{5832\,x^{17}}{17} + 1296\,x^{15} - \frac{34560\,x^{13}}{13} + \frac{34560\,x^{11}}{11} - 2048\,x^9 + \frac{4096\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^3)^6,x)
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

3.12 $\int \frac{x^4}{ax+bx^3} \, dx$

Optimal. Leaf size=27

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 272, 45}

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax+bx^3}\,dx &= \int \frac{x^3}{a+bx^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx, x, x^2\right)\\ &= \frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3),x]
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.35, size = 24, normalized size = 0.89

method	result	size
default	$\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)}{2b^2}$	24
norman	$\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)}{2b^2}$	24
risch	$\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)}{2b^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Maxima [A]
time = 0.29, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,b}-\frac{a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 1.37, size = 22, normalized size = 0.81

$$\frac{bx^2 - a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a + bx^2\right)}{2b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x),x)
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Giac [A]
time = 1.26, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,b} - \frac{a\log\left(|bx^2 + a|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^2 + a\right) - b\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3),x)
[Out] -(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

3.13 $\int \frac{x^3}{ax+bx^3} \, dx$

Optimal. Leaf size=31

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] x/b-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 327, 211}

$$\frac{x}{b} - \frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax+bx^3}\,dx &= \int \frac{x^2}{a+bx^2}\,dx\\ &= \frac{x}{b} - \frac{a\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{x}{b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Maple [A]
time = 0.36, size = 27, normalized size = 0.87

method	result	size
default	$\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$\frac{x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^2}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2b^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Fricas [A]
time = 2.24, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 0.06, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

Giac [A]
time = 1.17, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Mupad [B]
time = 4.91, size = 23, normalized size = 0.74

$$\frac{x}{b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3),x)
[Out] x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

3.14 $\int \frac{x^2}{ax+bx^3} \, dx$

Optimal. Leaf size=15

$$\frac{\log(a+bx^2)}{2b}$$

```
[Out] 1/2*ln(b*x^2+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 266}

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{ax+bx^3} \, dx &= \int \frac{x}{a+bx^2} \, dx \\ &= \frac{\log(a+bx^2)}{2b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Maple [A]
time = 0.41, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\ln\left(b\,x^{2}+a\right)}{2b}$	14
norman	$\frac{\ln\left(b\,x^{2}+a\right)}{2b}$	14
risch	$\frac{\ln\left(b\,x^{2}+a\right)}{2b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(b*x^2+a)/b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Fricas [A]
time = 1.36, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x),x)
```

```
[Out] log(a + b*x**2)/(2*b)
```

Giac [A]
time = 1.15, size = 14, normalized size = 0.93

$$\frac{\log\left(|bx^2 + a|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b
```

Mupad [B]
time = 4.92, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3),x)
```

```
[Out] log(a + b*x^2)/(2*b)
```

3.15 $\int \frac{x}{ax+bx^3}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1598, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x}{ax+bx^3}\,dx = \int \frac{1}{a+bx^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.37, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 3.54, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.
time = 0.05, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [A]
time = 1.04, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 0.04, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.16 $\int \frac{1}{ax+bx^3}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1607, 272, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx^3}\,dx &= \int \frac{1}{x\,(a+bx^2)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-1),x]
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Maple [A]
time = 0.34, size = 21, normalized size = 0.95

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\ln(b\,x^2+a)}{2a}$	21
norman	$\frac{\ln(x)}{a} - \frac{\ln(b\,x^2+a)}{2a}$	21
risch	$\frac{\ln(x)}{a} - \frac{\ln(b\,x^2+a)}{2a}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.91

$$-\frac{\log\left(bx^2+a\right)}{2\,a} + \frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + a)/a + log(x)/a
```

Fricas [**A**]
time = 2.15, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^2+a\right)-2\,\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [**A**]
time = 0.08, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

Giac [**A**]
time = 2.21, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a}-\frac{\log\left(|bx^2+a|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

Mupad [**B**]
time = 0.06, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3),x)
```

```
[Out] -(log(a + b*x^2) - 2*log(x))/(2*a)
```

3.17 $\int \frac{1}{x(ax+bx^3)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -1/a/x-arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 331, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx = \int \frac{1}{x^2\left(a+bx^2\right)}\,dx$$
$$= -\frac{1}{ax} - \frac{b\int \frac{1}{a+bx^2}\,dx}{a}$$
$$= -\frac{1}{ax} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$-\frac{1}{ax} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)),x]
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.35, size = 30, normalized size = 0.88

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}} - \frac{1}{ax}$	30
risch	$-\frac{1}{ax} + \frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a^2} - \frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a^2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] -b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x
```

Maxima [A]
time = 0.51, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Fricas [A]
time = 1.34, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 0.07, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

Giac [A]
time = 2.57, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Mupad [B]
time = 4.96, size = 26, normalized size = 0.76

$$-\frac{1}{a\,x}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)),x)
[Out] - 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.18 $\int \frac{1}{x^2(ax+bx^3)}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 272, 46}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^2(ax+bx^3)}\,dx = \int \frac{1}{x^3(a+bx^2)}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2)}{2a^2}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$-\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)),x]
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.35, size = 32, normalized size = 0.91

method	result	size
default	$-\frac{1}{2a\,x^2} - \frac{b\ln(x)}{a^2} + \frac{b\ln(b\,x^2+a)}{2a^2}$	32
norman	$-\frac{1}{2a\,x^2} - \frac{b\ln(x)}{a^2} + \frac{b\ln(b\,x^2+a)}{2a^2}$	32
risch	$-\frac{1}{2a\,x^2} - \frac{b\ln(x)}{a^2} + \frac{b\ln(-b\,x^2-a)}{2a^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.89

$$\frac{b\log(bx^2+a)}{2\,a^2} - \frac{b\log(x)}{a^2} - \frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)
```

Fricas [A]
time = 1.67, size = 33, normalized size = 0.94

$$\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log\left(x\right)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A]
time = 0.12, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Giac [A]
time = 1.36, size = 43, normalized size = 1.23

$$-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}-\frac{b\,\ln\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

3.19 $\int \frac{1}{x^3(ax+bx^3)}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 331, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*x + b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^3(ax+bx^3)}\,dx = \int \frac{1}{x^4(a+bx^2)}\,dx$$
$$= -\frac{1}{3ax^3} - \frac{b\int \frac{1}{x^2(a+bx^2)}\,dx}{a}$$
$$= -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$-\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*x + b*x^3)),x]
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A]
time = 0.37, size = 39, normalized size = 0.91

method	result	size
default	$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2\sqrt{ab}} - \frac{1}{3a\,x^3} + \frac{b}{a^2x}$	39
risch	$\frac{\frac{b\,x^2}{a^2} - \frac{1}{3a}}{x^3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2+b^3\right)} \_R\ln\left(\left(3a^5\_R^2+2b^3\right)x-a^3b\_R\right)\right)}{2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/3/a/x^3+b/a^2/x
```

Maxima [A]
time = 0.51, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} + \frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Fricas [A]
time = 1.29, size = 106, normalized size = 2.47

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\ \frac{3\,bx^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(37) = 74$.
time = 0.10, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\,\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\,\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Giac [A]
time = 0.92, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Mupad [B]
time = 4.94, size = 37, normalized size = 0.86

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{3\,a}-\frac{b\,x^2}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^3)),x)
[Out] (b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

3.20 $\int \frac{1}{x^4(ax+bx^3)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}$$

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 272, 46}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(ax+bx^3)}\,dx &= \int \frac{1}{x^5(a+bx^2)}\,dx\\ &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

Maple [A]
time = 0.34, size = 44, normalized size = 0.90

method	result	size
default	$-\frac{1}{4a\,x^4}+\frac{b}{2a^2x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(b\,x^2+a)}{2a^3}$	44
norman	$\frac{-\frac{1}{4a}+\frac{b\,x^2}{2a^2}}{x^4}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(b\,x^2+a)}{2a^3}$	46
risch	$\frac{-\frac{1}{4a}+\frac{b\,x^2}{2a^2}}{x^4}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(b\,x^2+a)}{2a^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Maxima [A]
time = 0.30, size = 44, normalized size = 0.90

$$-\frac{b^2\log(bx^2+a)}{2\,a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx^2-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)
```

Fricas [A]
time = 1.45, size = 45, normalized size = 0.92

$$-\frac{2\,b^2x^4\log\left(bx^2+a\right)-4\,b^2x^4\log\left(x\right)-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

Sympy [A]
time = 0.14, size = 42, normalized size = 0.86

$$\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log\left(x\right)}{a^3}-\frac{b^2\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a*x),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

Giac [A]
time = 0.89, size = 57, normalized size = 1.16

$$\frac{b^2\log\left(x^2\right)}{2\,a^3}-\frac{b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^3}-\frac{3\,b^2x^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

Mupad [B]
time = 0.06, size = 46, normalized size = 0.94

$$\frac{b^2\,\ln\left(x\right)}{a^3}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3}-\frac{\frac{1}{4\,a}-\frac{b\,x^2}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^3)),x)
```

```
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^2
))/x^4
```

3.21 $\int \frac{x^2}{(ax+bx^3)^2} dx$

Optimal. Leaf size=45

$$\frac{x}{2a(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

```
[Out] 1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(ax+bx^3)^2}\,dx &= \int \frac{1}{(a+bx^2)^2}\,dx\\ &= \frac{x}{2a\,(a+bx^2)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2a}\\ &= \frac{x}{2a\,(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x}{2a\,(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.37, size = 36, normalized size = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\;a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\;a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 35, normalized size = 0.78

$$\frac{x}{2\,(abx^2+a^2)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Fricas [A]
time = 3.03, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx - (bx^2+a)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(a^2b^2x^2+a^3b)},\ \frac{abx+(bx^2+a)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^2b^2x^2+a^3b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.11, size = 78, normalized size = 1.73

$$\frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Giac [A]
time = 1.43, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a} + \frac{x}{2\,(bx^2+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B]
time = 4.96, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3)^2,x)
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

3.22 $\int \frac{x}{(ax+bx^3)^2} dx$

Optimal. Leaf size=38

$$\frac{1}{2a(a+bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx^2)}{2a^2}$$

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 46}

$$-\frac{\log(a+bx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3)^2,x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \int \frac{1}{x(a+bx^2)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{2a(a+bx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx^2)}{2a^2}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2} + 2\log(x) - \log(a+bx^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.36, size = 42, normalized size = 1.11

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)} + \frac{\ln(x)}{a^2} - \frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)} + \frac{\ln(x)}{a^2} - \frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$-\frac{b\left(-\frac{a}{b(b\,x^2+a)} + \frac{\ln(b\,x^2+a)}{b}\right)}{2a^2} + \frac{\ln(x)}{a^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a^2*(-a/b/(b*x^2+a)+ln(b*x^2+a)/b)+ln(x)/a^2
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.89

$$\frac{1}{2(abx^2+a^2)} - \frac{\log(bx^2+a)}{2a^2} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2
```

Fricas [A]
time = 2.39, size = 47, normalized size = 1.24

$$-\frac{(bx^2+a)\log(bx^2+a)-2(bx^2+a)\log(x)-a}{2(a^2bx^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

Sympy [A]
time = 0.13, size = 34, normalized size = 0.89

$$\frac{1}{2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**2,x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Giac [A]
time = 1.46, size = 47, normalized size = 1.24

$$\frac{\log(x^2)}{2a^2}-\frac{\log(|bx^2+a|)}{2a^2}+\frac{bx^2+2a}{2(bx^2+a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{2a\left(bx^2+a\right)}-\frac{\ln\left(bx^2+a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3)^2,x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

3.23 $\int \frac{1}{(ax+bx^3)^2} dx$

Optimal. Leaf size=57

$$-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1607, 296, 331, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx^3)^2}\,dx &= \int \frac{1}{x^2(a+bx^2)^2}\,dx\\ &= \frac{1}{2ax(a+bx^2)} + \frac{3\int \frac{1}{x^2(a+bx^2)}\,dx}{2a}\\ &= -\frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)} - \frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\ &= -\frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.95

$$-\frac{1}{a^2x} - \frac{bx}{2a^2(a+bx^2)} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-2),x]
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

Maple [A]
time = 0.35, size = 45, normalized size = 0.79

method	result	size
default	$-\frac{b\left(\frac{x}{2b\,x^2+2a}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2x}$	45

risch	$\frac{-\frac{3bx^2}{2a^2}-\frac{1}{a}}{x(bx^2+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2+b\right)}\_R\ln\left(\left(3a^5\_R^2+2b\right)x+a^3\_R\right)\right)}{4}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] -b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/a^2/x
```

Maxima [A]
time = 0.51, size = 49, normalized size = 0.86

$$-\frac{3\,bx^2+2\,a}{2\,(a^2bx^3+a^3x)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

Fricas [A]
time = 1.82, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2-3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\,(a^2bx^3+a^3x)},-\frac{3\,bx^2+3\,(bx^3+ax)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\,(a^2bx^3+a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A]
time = 0.14, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A]
time = 1.17, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

Mupad [B]
time = 4.98, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3)^2,x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

3.24 $\int \frac{1}{x(ax+bx^3)^2} dx$

Optimal. Leaf size=49

$$-\frac{1}{2a^2x^2}-\frac{b}{2a^2(a+bx^2)}-\frac{2b\log(x)}{a^3}+\frac{b\log(a+bx^2)}{a^3}$$

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Rubi [A]

time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 272, 46}

$$\frac{b\log(a+bx^2)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2(a+bx^2)}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] -1/2*1/(a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*
x^2])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x(ax+bx^3)^2}\,dx = \int \frac{1}{x^3(a+bx^2)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{1}{2a^2x^2} - \frac{b}{2a^2(a+bx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx^2)}{a^3}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log(a+bx^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)^2),x]
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

Maple [A]
time = 0.36, size = 55, normalized size = 1.12

method	result	size
norman	$\frac{\frac{b^2x^4}{a^3}-\frac{1}{2a}}{x^2(bx^2+a)}+\frac{b\ln(bx^2+a)}{a^3}-\frac{2b\ln(x)}{a^3}$	52
risch	$\frac{-\frac{bx^2}{a^2}-\frac{1}{2a}}{x^2(bx^2+a)}-\frac{2b\ln(x)}{a^3}+\frac{b\ln(-bx^2-a)}{a^3}$	54
default	$\frac{b^2\left(-\frac{a}{b(bx^2+a)}+\frac{2\ln(bx^2+a)}{b}\right)}{2a^3}-\frac{1}{2a^2x^2}-\frac{2b\ln(x)}{a^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b^2/a^3*(-a/b/(b*x^2+a)+2*ln(b*x^2+a)/b)-1/2/a^2/x^2-2*b*ln(x)/a^3
```

Maxima [A]
time = 0.29, size = 50, normalized size = 1.02

$$-\frac{2\,bx^2+a}{2\,(a^2bx^4+a^3x^2)}+\frac{b\log(bx^2+a)}{a^3}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(x
)/a^3
```

Fricas [A]
time = 1.35, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2+a^2-2\,(b^2x^4+abx^2)\log\left(bx^2+a\right)+4\,(b^2x^4+abx^2)\log\left(x\right)}{2\,(a^3bx^4+a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A]
time = 0.18, size = 51, normalized size = 1.04

$$\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**2,x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

Giac [A]
time = 1.37, size = 51, normalized size = 1.04

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(|bx^2+a|\right)}{a^3}-\frac{2\,bx^2+a}{2\,(bx^4+ax^2)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)^2),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

3.25 $\int \frac{1}{x^2(ax+bx^3)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] -5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/a^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1598, 296, 331, 211}

$$\frac{5b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 (ax+bx^3)^2}\,dx &= \int \frac{1}{x^4 (a+bx^2)^2}\,dx\\ &= \frac{1}{2ax^3(a+bx^2)} + \frac{5\int \frac{1}{x^4(a+bx^2)}\,dx}{2a}\\ &= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3(a+bx^2)} - \frac{(5b)\int \frac{1}{x^2(a+bx^2)}\,dx}{2a^2}\\ &= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3(a+bx^2)} + \frac{(5b^2)\int \frac{1}{a+bx^2}\,dx}{2a^3}\\ &= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3(a+bx^2)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.99

$$-\frac{1}{3a^2x^3} + \frac{2b}{a^3x} + \frac{b^2x}{2a^3(a+bx^2)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A]
time = 0.36, size = 55, normalized size = 0.81

method	result	size

default	$\frac{b^2\left(\frac{x}{2b\,x^2+2a}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}$	55
risch	$\frac{\frac{5b^2x^4}{2a^3}+\frac{5b\,x^2}{3a^2}-\frac{1}{3a}}{x^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{4a^4}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^4}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3/a^2/x
^3+2*b/a^3/x
```

Maxima [A]
time = 0.51, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\,(a^3bx^5+a^4x^3)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 1.76, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4+20\,abx^2+15\,(b^2x^5+abx^3)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\,(a^3bx^5+a^4x^3)},\frac{15\,b^2x^4+10\,abx^2+15\,(b^2x^5+abx^3)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)-2\,a^2}{6\,(a^3bx^5+a^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A]
time = 0.16, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A]
time = 1.78, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2x}{2\,(bx^2+a)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B]
time = 5.03, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)^2),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

3.26 $\int \frac{x^5}{x-x^3}\,dx$

Optimal. Leaf size=13

$$-x-\frac{x^3}{3}+\tanh^{-1}(x)$$

```
[Out] -x-1/3*x^3+arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1598, 308, 212}

$$-\frac{x^3}{3}-x+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{x-x^3}\,dx &= \int \frac{x^4}{1-x^2}\,dx \\
&= \int \left(-1-x^2+\frac{1}{1-x^2}\right)dx \\
&= -x-\frac{x^3}{3}+\int \frac{1}{1-x^2}\,dx \\
&= -x-\frac{x^3}{3}+\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.
time = 0.00, size = 29, normalized size = 2.23

$$
-x-\frac{x^3}{3}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [**A**]
time = 0.35, size = 22, normalized size = 1.69

method	result	size
meijerg	$-\frac{i\left(-\frac{2ix\left(5x^2+15\right)}{15}+2i\,\mathrm{arctanh}(x)\right)}{2}$	21
default	$-\frac{x^3}{3}-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	22
norman	$-\frac{x^3}{3}-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	22
risch	$-\frac{x^3}{3}-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3-x-1/2*ln(x-1)+1/2*ln(x+1)
```

Maxima [**A**]
time = 0.30, size = 21, normalized size = 1.62

$$
-\frac{1}{3}x^3-x+\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="maxima")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A]
time = 1.60, size = 21, normalized size = 1.62

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="fricas")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.03, size = 19, normalized size = 1.46

$$-\frac{x^3}{3} - x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+x),x)
[Out] -x**3/3 - x - log(x - 1)/2 + log(x + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 1.08, size = 23, normalized size = 1.77

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="giac")
[Out] -1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 4.98, size = 11, normalized size = 0.85

$$\operatorname{atanh}(x) - x - \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x - x^3),x)
[Out] atanh(x) - x - x^3/3
```

3.27 $\int \frac{x^4}{x-x^3} \, dx$

Optimal. Leaf size=20

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

[Out] -1/2*x^2-1/2*ln(-x^2+1)

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 45}

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^4/(x - x^3),x]

[Out] -1/2*x^2 - Log[1 - x^2]/2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{x-x^3}\,dx &= \int \frac{x^3}{1-x^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1-x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)\,dx, x, x^2\right)\\ &= -\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 0.90

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(-1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(x - x^3),x]
[Out] -1/2*x^2 - Log[-1 + x^2]/2
```

Maple [A]
time = 0.34, size = 19, normalized size = 0.95

method	result	size
risch	$-\frac{x^2}{2}-\frac{\ln(x^2-1)}{2}$	15
meijerg	$-\frac{x^2}{2}-\frac{\ln(-x^2+1)}{2}$	17
default	$-\frac{x^2}{2}-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	19
norman	$-\frac{x^2}{2}-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2-1/2*ln(x-1)-1/2*ln(x+1)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.90

$$-\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A]
time = 2.10, size = 14, normalized size = 0.70

$$-\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/2*log(x^2 - 1)
```

Sympy [A]
time = 0.02, size = 14, normalized size = 0.70

$$-\frac{x^2}{2} - \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+x),x)
```

```
[Out] -x**2/2 - log(x**2 - 1)/2
```

Giac [A]
time = 1.02, size = 15, normalized size = 0.75

$$-\frac{1}{2}x^2 - \frac{1}{2}\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/2*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.70

$$-\frac{\ln\left(x^2 - 1\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x - x^3),x)
```

```
[Out] - log(x^2 - 1)/2 - x^2/2
```

3.28 $\int \frac{x^3}{x-x^3} \, dx$

Optimal. Leaf size=6

$$-x + \tanh^{-1}(x)$$

```
[Out] -x+arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1598, 327, 212}

$$\tanh^{-1}(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(x - x^3),x]
```

```
[Out] -x + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{x-x^3} \, dx &= \int \frac{x^2}{1-x^2} \, dx\\ &= -x + \int \frac{1}{1-x^2} \, dx\\ &= -x + \tanh^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 22 vs. $2(6) = 12$.
time = 0.00, size = 22, normalized size = 3.67

$$-x - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(x - x^3),x]
```

```
[Out] -x - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.35, size = 17, normalized size = 2.83

method	result	size
meijerg	$\frac{i(2ix-2i\,\mathrm{arctanh}(x))}{2}$	14
default	$-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	17
norman	$-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	17
risch	$-x-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] -x-1/2*ln(x-1)+1/2*ln(x+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.30, size = 16, normalized size = 2.67

$$-x + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 1.16, size = 16, normalized size = 2.67

$$-x + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.
time = 0.03, size = 14, normalized size = 2.33

$$-x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+x),x)
```

```
[Out] -x - log(x - 1)/2 + log(x + 1)/2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.
time = 1.72, size = 18, normalized size = 3.00

$$-x + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="giac")
```

```
[Out] -x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [**B**]
time = 0.06, size = 6, normalized size = 1.00

$$\operatorname{atanh}(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x - x^3),x)
```

```
[Out] atanh(x) - x
```

3.29 $\int \frac{x^2}{x-x^3}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1598, 266}

$$-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(x - x^3),x]
```

```
[Out] -1/2*Log[1 - x^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{x-x^3}\,dx &= \int \frac{x}{1-x^2}\,dx\\ &= -\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(x - x^3),x]
[Out] -1/2*Log[1 - x^2]
```

Maple [A]
time = 0.37, size = 14, normalized size = 1.17

method	result	size
risch	$-\frac{\ln(x^2-1)}{2}$	9
meijerg	$-\frac{\ln(-x^2+1)}{2}$	11
default	$-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	14
norman	$-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 1.08

$$-\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A]
time = 1.71, size = 8, normalized size = 0.67

$$-\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.67

$$-\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+x),x)
```

```
[Out] -log(x**2 - 1)/2
```

Giac [A]
time = 1.26, size = 15, normalized size = 1.25

$$-\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.67

$$-\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x - x^3),x)
```

```
[Out] -log(x^2 - 1)/2
```

3.30 $\int \frac{x}{x-x^3} \, dx$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

```
[Out] arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1598, 212}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - x^3),x]
```

```
[Out] ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{x-x^3} \, dx &= \int \frac{1}{1-x^2} \, dx \\ &= \tanh^{-1}(x) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(2) = 4.
time = 0.00, size = 19, normalized size = 9.50

$$-\frac{1}{2} \log(1-x) + \frac{1}{2} \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - x^3),x]
```

```
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

Maple [A]
time = 0.36, size = 3, normalized size = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
risch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 0.28, size = 13, normalized size = 6.50

$$\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 1.35, size = 13, normalized size = 6.50

$$\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.
time = 0.03, size = 12, normalized size = 6.00

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.
time = 1.15, size = 15, normalized size = 7.50

$$\frac{1}{2}\log\left(|x+1|\right) - \frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - x^3),x)
```

```
[Out] atanh(x)
```

3.31 $\int \frac{1}{x-x^3} \, dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

```
[Out] ln(x)-1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1607, 272, 36, 31, 29}

$$\log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-x^3}\,dx &= \int \frac{1}{x\left(1-x^2\right)}\,dx \\
&= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{(1-x)x}\,dx,x,x^2\right) \\
&= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx,x,x^2\right)+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right) \\
&= \log(x)-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Maple [A]
time = 0.36, size = 16, normalized size = 1.07

method	result	size
risch	$\ln(x)-\frac{\ln(x^2-1)}{2}$	12
default	$-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}+\ln(x)$	16
norman	$-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}+\ln(x)$	16
meijerg	$-\frac{\ln(-x^2+1)}{2}+\ln(x)+\frac{i\pi}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x+1)-1/2*ln(x-1)+ln(x)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 1.00

$$-\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Fricas [A]
time = 3.93, size = 11, normalized size = 0.73

$$-\frac{1}{2}\log\left(x^2-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 1) + log(x)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.67

$$\log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2
```

Giac [A]
time = 1.26, size = 16, normalized size = 1.07

$$\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B]
time = 4.96, size = 11, normalized size = 0.73

$$\ln\left(x\right)-\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^3),x)
```

```
[Out] log(x) - log(x^2 - 1)/2
```

3.32 $\int \frac{1}{x(x-x^3)} dx$

Optimal. Leaf size=8

$$-\frac{1}{x}+\tanh^{-1}(x)$$

[Out] -1/x+arctanh(x)

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1598, 331, 212}

$$\tanh^{-1}(x)-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(x - x^3)),x]

[Out] -x^(-1) + ArcTanh[x]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x(x-x^3)}\,dx = \int \frac{1}{x^2(1-x^2)}\,dx$$
$$= -\frac{1}{x} + \int \frac{1}{1-x^2}\,dx$$
$$= -\frac{1}{x} + \tanh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 24 vs. $2(8) = 16$.
time = 0.00, size = 24, normalized size = 3.00

$$-\frac{1}{x} - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.
time = 0.35, size = 19, normalized size = 2.38

method	result	size
meijerg	$\frac{i\left(\frac{2i}{x}-2i\,\mathrm{arctanh}(x)\right)}{2}$	16
default	$\frac{\ln(x+1)}{2} - \frac{\ln(x-1)}{2} - \frac{1}{x}$	19
norman	$\frac{\ln(x+1)}{2} - \frac{\ln(x-1)}{2} - \frac{1}{x}$	19
risch	$\frac{\ln(x+1)}{2} - \frac{\ln(x-1)}{2} - \frac{1}{x}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x+1)-1/2*ln(x-1)-1/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.
time = 0.29, size = 18, normalized size = 2.25

$$-\frac{1}{x} + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.
time = 2.50, size = 20, normalized size = 2.50

$$\frac{x \log (x+1)-x \log (x-1)-2}{2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.
time = 0.03, size = 15, normalized size = 1.88

$$-\frac{\log (x-1)}{2}+\frac{\log (x+1)}{2}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 1/x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.
time = 1.32, size = 20, normalized size = 2.50

$$-\frac{1}{x}+\frac{1}{2} \log (|x+1|)-\frac{1}{2} \log (|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.03, size = 8, normalized size = 1.00

$$\operatorname{atanh}(x)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x - x^3)),x)
```

```
[Out] atanh(x) - 1/x
```

3.33 $\int \frac{1}{x^2(x-x^3)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1598, 272, 46}

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(x - x^3)),x]
```

```
[Out] -1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^2(x-x^3)}\,dx = \int \frac{1}{x^3(1-x^2)}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(x - x^3)),x]
[Out] -1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

Maple [A]
time = 0.37, size = 21, normalized size = 0.95

method	result	size
risch	$-\frac{1}{2x^2}+\ln(x)-\frac{\ln(x^2-1)}{2}$	17
default	$-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}-\frac{1}{2x^2}+\ln(x)$	21
norman	$-\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}-\frac{1}{2x^2}+\ln(x)$	21
meijerg	$-\frac{\ln(-x^2+1)}{2}+\ln(x)+\frac{i\pi}{2}-\frac{1}{2x^2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x+1)-1/2*ln(x-1)-1/2/x^2+ln(x)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.91

$$-\frac{1}{2\,x^2}-\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Fricas [A]
time = 2.81, size = 24, normalized size = 1.09

$$-\frac{x^2 \log(x^2-1) - 2\,x^2 \log(x) + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.77

$$\log(x) - \frac{\log(x^2-1)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2 - 1/(2*x**2)
```

Giac [A]
time = 1.00, size = 26, normalized size = 1.18

$$-\frac{x^2+1}{2\,x^2} + \frac{1}{2}\,\log\left(x^2\right) - \frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.73

$$\ln(x) - \frac{\ln(x^2-1)}{2} - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x - x^3)),x)
```

```
[Out] log(x) - log(x^2 - 1)/2 - 1/(2*x^2)
```

3.34 $\int \frac{1}{x^3(x-x^3)}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

```
[Out] -1/3/x^3-1/x+arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1598, 331, 212}

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(x - x^3)),x]
```

```
[Out] -1/3*1/x^3 - x^(-1) + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^3 (x - x^3)} \, dx = \int \frac{1}{x^4 (1 - x^2)} \, dx$$
$$= -\frac{1}{3x^3} + \int \frac{1}{x^2 (1 - x^2)} \, dx$$
$$= -\frac{1}{3x^3} - \frac{1}{x} + \int \frac{1}{1 - x^2} \, dx$$
$$= -\frac{1}{3x^3} - \frac{1}{x} + \tanh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.
time = 0.00, size = 31, normalized size = 2.07

$$-\frac{1}{3x^3} - \frac{1}{x} - \frac{1}{2} \log(1 - x) + \frac{1}{2} \log(1 + x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(x - x^3)),x]
```

```
[Out] -1/3*1/x^3 - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [A]
time = 0.36, size = 24, normalized size = 1.60

method	result	size
meijerg	$-\frac{i\left(-\frac{2i}{x}-\frac{2i}{3x^3}+2i\operatorname{arctanh}(x)\right)}{2}$	22
default	$\frac{\ln(x+1)}{2}-\frac{\ln(x-1)}{2}-\frac{1}{3x^3}-\frac{1}{x}$	24
norman	$\frac{-\frac{1}{3}-x^2}{x^3}-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	25
risch	$\frac{-\frac{1}{3}-x^2}{x^3}-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x+1)-1/2*ln(x-1)-1/3/x^3-1/x
```

Maxima [A]
time = 0.28, size = 25, normalized size = 1.67

$$-\frac{3\,x^2 + 1}{3\,x^3} + \frac{1}{2}\,\log\left(x + 1\right) - \frac{1}{2}\,\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.
time = 2.57, size = 30, normalized size = 2.00

$$\frac{3\,x^3\log\left(x+1\right)-3\,x^3\log\left(x-1\right)-6\,x^2-2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^3*log(x + 1) - 3*x^3*log(x - 1) - 6*x^2 - 2)/x^3
```

Sympy [A]
time = 0.04, size = 24, normalized size = 1.60

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}-\frac{3x^2+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.
time = 1.04, size = 27, normalized size = 1.80

$$-\frac{3\,x^2+1}{3\,x^3}+\frac{1}{2}\,\log\left(\left|x+1\right|\right)-\frac{1}{2}\,\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 4.92, size = 13, normalized size = 0.87

$$\mathrm{atanh}(x)-\frac{x^2+\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x - x^3)),x)
```

```
[Out] atanh(x) - (x^2 + 1/3)/x^3
```

3.35 $\int \frac{1}{x^4(x-x^3)}\,dx$

Optimal. Leaf size=29

$$-\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

[Out] -1/4/x^4-1/2/x^2+ln(x)-1/2*ln(-x^2+1)

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 46}

$$-\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(x - x^3)),x]

[Out] -1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^4 (x - x^3)} \, dx = \int \frac{1}{x^5 (1 - x^2)} \, dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^3} \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x} + \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x}\right) \, dx, x, x^2\right)$$
$$= -\frac{1}{4x^4} - \frac{1}{2x^2} + \log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$-\frac{1}{4x^4} - \frac{1}{2x^2} + \log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(x - x^3)),x]
```

```
[Out] -1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Maple [A]
time = 0.36, size = 26, normalized size = 0.90

method	result	size
risch	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4} + \ln(x) - \frac{\ln(x^2-1)}{2}$	23
default	$-\frac{\ln(x+1)}{2} - \frac{\ln(x-1)}{2} - \frac{1}{4x^4} - \frac{1}{2x^2} + \ln(x)$	26
norman	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4} - \frac{\ln(x+1)}{2} - \frac{\ln(x-1)}{2} + \ln(x)$	27
meijerg	$-\frac{\ln(-x^2+1)}{2} + \ln(x) + \frac{i\pi}{2} - \frac{1}{4x^4} - \frac{1}{2x^2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x+1)-1/2*ln(x-1)-1/4/x^4-1/2/x^2+ln(x)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.93

$$-\frac{2\,x^2 + 1}{4\,x^4} - \frac{1}{2}\,\log\left(x + 1\right) - \frac{1}{2}\,\log\left(x - 1\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Fricas [A]
time = 2.70, size = 30, normalized size = 1.03

$$-\frac{2\,x^4\log\left(x^2-1\right)-4\,x^4\log\left(x\right)+2\,x^2+1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="fricas")
[Out] -1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.76

$$\log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}-\frac{2x^2+1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+x),x)
[Out] log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)
```

Giac [A]
time = 1.55, size = 33, normalized size = 1.14

$$-\frac{3\,x^4+2\,x^2+1}{4\,x^4}+\frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="giac")
[Out] -1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.79

$$\ln\left(x\right)-\frac{\ln\left(x^2-1\right)}{2}-\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x - x^3)),x)
[Out] log(x) - log(x^2 - 1)/2 - (x^2/2 + 1/4)/x^4
```

3.36 $\int \frac{1}{x+bx^3}\,dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(1 + bx^2\right)$$

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1607, 272, 36, 29, 31}

$$\log(x) - \frac{1}{2}\log\left(bx^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + b*x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+bx^3}\,dx &= \int \frac{1}{x\left(1+bx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}b\text{Subst}\left(\int \frac{1}{1+bx}\,dx,x,x^2\right)\\
&= \log(x)-\frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\log(x)-\frac{1}{2}\log\left(1+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + b*x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Maple [A]
time = 0.35, size = 14, normalized size = 0.93

method	result	size
default	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
norman	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
risch	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
meijerg	$-\frac{\ln\left(b\,x^{2}+1\right)}{2}+\ln\left(x\right)+\frac{\ln\left(b\right)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

Fricas [A]
time = 2.41, size = 13, normalized size = 0.87

$$-\frac{1}{2} \log \left(bx^2 + 1\right) + \log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="fricas")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

Sympy [A]
time = 0.06, size = 12, normalized size = 0.80

$$\log \left(x\right) - \frac{\log \left(x^2 + \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+x),x)
[Out] log(x) - log(x**2 + 1/b)/2
```

Giac [A]
time = 1.80, size = 18, normalized size = 1.20

$$\frac{1}{2} \log \left(x^2\right) - \frac{1}{2} \log \left(\left|bx^2 + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

Mupad [B]
time = 4.95, size = 14, normalized size = 0.93

$$\ln \left(x\right) - \frac{\ln \left(\frac{3\,b\,x^2}{2} + \frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + b*x^3),x)
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2
```

3.37 $\int \frac{1}{-x+bx^3} dx$

Optimal. Leaf size=18

$$-\log(x)+\frac{1}{2}\log\left(1-bx^2\right)$$

```
[Out] -ln(x)+1/2*ln(-b*x^2+1)
```

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1607, 272, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + b*x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-x+bx^3}\,dx &= \int \frac{1}{x\left(-1+bx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)\right)+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{-1+bx}\,dx,x,x^2\right)\\
&= -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\log(x)+\frac{1}{2}\log\left(1-bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + b*x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Maple [A]
time = 0.35, size = 16, normalized size = 0.89

method	result	size
default	$\frac{\ln\left(b\,x^{2}-1\right)}{2}-\ln\left(x\right)$	16
norman	$\frac{\ln\left(b\,x^{2}-1\right)}{2}-\ln\left(x\right)$	16
risch	$-\ln\left(x\right)+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	17
meijerg	$\frac{\ln\left(-b\,x^{2}+1\right)}{2}-\ln\left(x\right)-\frac{\ln\left(-b\right)}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3-x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(b*x^2-1)-ln(x)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

Fricas [A]
time = 1.61, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.67

$$-\log\left(x\right)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

Giac [A]
time = 1.09, size = 18, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

Mupad [B]
time = 0.05, size = 16, normalized size = 0.89

$$\frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2}-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x - b*x^3),x)
```

```
[Out] log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

3.38 $\int x^3 \sqrt{ax + bx^3}\, dx$

Optimal. Leaf size=163

$$-\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(}{231b^{9/4}\sqrt{ax+bx^3}}$$

```
[Out] -20/231*a^2*(b*x^3+a*x)^(1/2)/b^2+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b+2/11*x^4*(
b*x^3+a*x)^(1/2)+10/231*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^
(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2046, 2049, 2036, 335, 226}

$$\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}-\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-20*a^2*Sqrt[a*x + b*x^3])/(231*b^2) + (4*a*x^2*Sqrt[a*x + b*x^3])/(77*b)
+ (2*x^4*Sqrt[a*x + b*x^3])/11 + (10*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{ax+bx^3}\,dx = \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{1}{11}(2a)\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} - \frac{(10a^2)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{77b}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{(10a^3)\int \frac{1}{\sqrt{ax+bx^3}}\,d\ldots}{231b^2}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{\left(10a^3\sqrt{x}\sqrt{a+bx^2}\right)\ldots}{231b^2\sqrt{ax\ldots}}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{\left(20a^3\sqrt{x}\sqrt{a+bx^2}\right)\ldots}{231\ldots}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\ldots\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 95, normalized size = 0.58

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(\sqrt{1+\frac{bx^2}{a}}\left(-5a^2+2abx^2+7b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 + 7*b^2*x^4
) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b^2*Sqrt[1
+ (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 168, normalized size = 1.03

method	result
risch	$-\frac{2\left(-21b^2x^4-6ab\,x^2+10a^2\right)x\left(b\,x^2+a\right)}{231b^2\sqrt{x\left(b\,x^2+a\right)}}+\frac{10a^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \text{Ellip}}{231b^3\sqrt{b\,x^3+ax}}$
default	$\frac{2x^4\sqrt{b\,x^3+ax}}{11}+\frac{4a\,x^2\sqrt{b\,x^3+ax}}{77b}-\frac{20a^2\sqrt{b\,x^3+ax}}{231b^2}+\frac{10a^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{231}$
elliptic	$\frac{2x^4\sqrt{b\,x^3+ax}}{11}+\frac{4a\,x^2\sqrt{b\,x^3+ax}}{77b}-\frac{20a^2\sqrt{b\,x^3+ax}}{231b^2}+\frac{10a^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{231}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*x^4*(b*x^3+a*x)^(1/2)+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b-20/231*a^2*(b*x^3
+a*x)^(1/2)/b^2+10/231*a^3/b^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^
(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
```

```
/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 59, normalized size = 0.36

$$\frac{2\left(10\,a^3\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(21\,b^3x^4+6\,ab^2x^2-10\,a^2b\right)\sqrt{bx^3+ax}\right)}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] 2/231*(10*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (21*b^3*x^4 + 6*a
*b^2*x^2 - 10*a^2*b)*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{x\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a*x)**(1/2),x)
[Out] Integral(x**3*sqrt(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{b\,x^3+a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^3)^(1/2), x)
```

3.39 $\int x^2 \sqrt{ax + bx^3}\, dx$

Optimal. Leaf size=281

$$-\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{15b^{7/4}}$$

```
[Out] -4/15*a^2*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+4/45*a*
x*(b*x^3+a*x)^(1/2)/b+2/9*x^3*(b*x^3+a*x)^(1/2)+4/15*a^(9/4)*(cos(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*E
llipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(
1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(
1/2)-2/15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}-\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{2}{9}x^3\sqrt{ax+bx^3}+\frac{4ax\sqrt{ax+bx^3}}{45b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-4*a^2*x*(a + b*x^2))/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (4*a*x*Sqrt[a*x + b*x^3])/(45*b) + (2*x^3*Sqrt[a*x + b*x^3])/9 + (4*a^(9
/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x
+ b*x^3]) - (2*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(1
5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt{ax+bx^3}\,dx = \frac{2}{9}x^3\sqrt{ax+bx^3}+\frac{1}{9}(2a)\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{(2a^2)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{15b}$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{\left(2a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15b\sqrt{ax+bx^3}}$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x\right.}{15b\sqrt{ax+bx^3}}$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{\left(4a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx\right.}{15b^{3/2}\sqrt{ax+bx^3}}$$

$$= -\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}+\frac{4a^{9/4}\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 80, normalized size = 0.28

$$\frac{2x\sqrt{x(a+bx^2)}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}-a\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometri
c2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.36, size = 197, normalized size = 0.70

method	result

default	$\frac{2x^3\sqrt{b\,x^3+ax}}{9}+\frac{4ax\sqrt{b\,x^3+ax}}{45b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\cdots}{\cdots}$
elliptic	$\frac{2x^3\sqrt{b\,x^3+ax}}{9}+\frac{4ax\sqrt{b\,x^3+ax}}{45b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\cdots}{\cdots}$
risch	$\frac{2x^2(5b\,x^2+2a)(b\,x^2+a)}{45b\sqrt{x\,(b\,x^2+a)}}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{Elliptic}\cdots}{\cdots}\right.}{15b^2\sqrt{b\,x^3+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^3*(b*x^3+a*x)^(1/2)+4/45*a*x*(b*x^3+a*x)^(1/2)/b-2/15/b^2*a^2*(-a*b)^
(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*
b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a
*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 57, normalized size = 0.20

$$\frac{2\left(6\,a^2\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(5\,b^2x^3+2\,abx\right)\sqrt{bx^3+ax}\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
0, x)) + (5*b^2*x^3 + 2*a*b*x)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{b\,x^3+a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^2*(a*x + b*x^3)^(1/2), x)
```

3.40 $\int x\sqrt{ax+bx^3}\,dx$

Optimal. Leaf size=137

$$\frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] 4/21*a*(b*x^3+a*x)^(1/2)/b+2/7*x^2*(b*x^3+a*x)^(1/2)-2/21*a^(7/4)*(cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a
*x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2046, 2049, 2036, 335, 226}

$$-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}+\frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/(21*b) + (2*x^2*Sqrt[a*x + b*x^3])/7 - (2*a^(7/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[a*x + b*x
^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x\sqrt{ax+bx^3}\,dx = \frac{2}{7}x^2\sqrt{ax+bx^3} + \frac{1}{7}(2a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{(2a^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{21b}$$

$$= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{\left(2a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b\sqrt{ax+bx^3}}$$

$$= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x,\right.}{21b\sqrt{ax+bx^3}}$$

$$= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(}{21b^{5/4}\sqrt{ax+bx^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 9.56, size = 79, normalized size = 0.58

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}-a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometric2
F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 146, normalized size = 1.07

method	result
default	$\frac{2x^2\sqrt{b\,x^3+ax}}{7}+\frac{4a\sqrt{b\,x^3+ax}}{21b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{E}}{21b^2\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x^2\sqrt{b\,x^3+ax}}{7}+\frac{4a\sqrt{b\,x^3+ax}}{21b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{E}}{21b^2\sqrt{b\,x^3+ax}}$
risch	$\frac{2(3b\,x^2+2a)x(b\,x^2+a)}{21b\sqrt{x\left(b\,x^2+a\right)}}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{21b^2\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)+4/21*a*(b*x^3+a*x)^(1/2)/b-2/21/b^2*a^2*(-a*b)^(1
/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF((
(x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 49, normalized size = 0.36

$$-\frac{2\left(2\,a^{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\left(3\,b^{2}x^{2}+2\,ab\right)\sqrt{bx^{3}+ax}\right)}{21\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*(2*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (3*b^2*x^2 + 2*a*b
)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x\left(a+bx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{b\,x^{3}+a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x*(a*x + b*x^3)^(1/2), x)
```

3.41 $\int \sqrt{ax+bx^3}\, dx$

Optimal. Leaf size=255

$$\frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\right.}{5b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 4/5*a*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*x^
3+a*x)^(1/2)-4/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*
b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)+2/5*a^(5/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipt
icF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2029, 2057, 335, 311, 226, 1210}

$$\frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}+\frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*x*(a + b*x^2))/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (
2*x*Sqrt[a*x + b*x^3])/5 - (4*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3]) + (2*a^(5/4)*Sqrt[x]*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{ax+bx^3}\,dx = \frac{2}{5}x\sqrt{ax+bx^3} + \frac{1}{5}(2a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{2}{5}x\sqrt{ax+bx^3} + \frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}$$

$$= \frac{2}{5}x\sqrt{ax+bx^3} + \frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{ax+bx^3}}$$

$$= \frac{2}{5}x\sqrt{ax+bx^3} + \frac{\left(4a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{b}\sqrt{ax+bx^3}} - \frac{\left(4a^{3/2}\sqrt{x}\right.}{}$$

$$= \frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{2}{5}x\sqrt{ax+bx^3} - \frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{5b^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.78, size = 51, normalized size = 0.20

$$\frac{2x\sqrt{x(a+bx^2)}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)])/(
3*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 175, normalized size = 0.69

method	result

default	$\frac{2x\sqrt{b\,x^3+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\cdots\right.}{\cdots}\right.}{5b\sqrt{b\,x^3+ax}\cdots}$
elliptic	$\frac{2x\sqrt{b\,x^3+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\cdots\right.}{\cdots}\right.}{5b\sqrt{b\,x^3+ax}\cdots}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5\sqrt{x\left(b\,x^2+a\right)}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\cdots}{\cdots}\right.}{5b\sqrt{b\,x^3+ax}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)+2/5*a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)
^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(
1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b
*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 43, normalized size = 0.17

$$\frac{2\left(\sqrt{bx^3+ax}\,bx-2\,a\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(sqrt(b*x^3 + a*x)*b*x - 2*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierst
rassPInverse(-4*a/b, 0, x)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2),x)
[Out] Integral(sqrt(a*x + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

Mupad [B]
time = 5.05, size = 40, normalized size = 0.16

$$\frac{2\,x\,\sqrt{b\,x^3+a\,x}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{b\,x^2}{a}\right)}{3\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2),x)
[Out] (2*x*(a*x + b*x^3)^(1/2)*hypergeom([-1/2, 3/4], 7/4, -(b*x^2)/a))/(3*((b*x^
2)/a + 1)^(1/2))
```

3.42 $\int \frac{\sqrt{ax+bx^3}}{x}\,dx$

Optimal. Leaf size=113

$$\frac{2}{3}\sqrt{ax+bx^3}+\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

```
[Out] 2/3*(b*x^3+a*x)^(1/2)+2/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a
^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2046, 2036, 335, 226}

$$\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x}\,dx &= \frac{2}{3}\sqrt{ax+bx^3}+\frac{1}{3}(2a)\int \frac{1}{\sqrt{ax+bx^3}}\,dx \\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}} \\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}} \\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{3\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.86, size = 48, normalized size = 0.42

$$\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x,x]
[Out] (2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sqr
t[1 + (b*x^2)/a]
```

Maple [A]
time = 0.35, size = 124, normalized size = 1.10

method	result
default	$\frac{2\sqrt{b\,x^{3}+ax}}{3}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-a}}}\right.}{3b\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{2\sqrt{b\,x^{3}+ax}}{3}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-a}}}\right.}{3b\sqrt{b\,x^{3}+ax}}$
risch	$\frac{2x\left(b\,x^{2}+a\right)}{3\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\sqrt{\right.}{\sqrt{-}}}\right.}{3b\sqrt{b\,x^{3}+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x^3+a*x)^(1/2)+2/3*a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(
1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 34, normalized size = 0.30

$$\frac{2\left(2\,a\sqrt{b}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^{3}+ax}\,b\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*(2*a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*b)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(a+bx^2)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^3+a\,x}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x,x)
```

```
[Out] int((a*x + b*x^3)^(1/2)/x, x)
```

3.43 $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$

Optimal. Leaf size=248

$$\frac{4\sqrt{b}\,x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}-\frac{4\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{\sqrt{ax+bx^3}}$$

```
[Out] 4*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)^(
1/2)/x-4*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*
b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+2*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)
*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x
^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2045, 2057, 335, 311, 226, 1210}

$$\frac{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}-\frac{4\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}+\frac{4\sqrt{b}\,x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (4*Sqrt[b]*x*(a + b*x^2))/((Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sq
rt[a*x + b*x^3])/x - (4*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a
^(1/4)], 1/2])/Sqrt[a*x + b*x^3] + (2*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sq
rt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1
/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = -\frac{2\sqrt{ax+bx^3}}{x} + (2b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(4b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} - \frac{(4\sqrt{\ }\cdots}{\cdots}$$

$$= \frac{4\sqrt{b}\,x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} - \frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a\cdots}{\left(\sqrt{a}\cdots\right.}}}{\sqrt{ax\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 51, normalized size = 0.21

$$-\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/(
x*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 177, normalized size = 0.71

method	result

default	$-\frac{2(b\,x^2+a)}{\sqrt{x\,(b\,x^2+a)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\right.}{}\right.}{\sqrt{b\,x^3+ax}}$
risch	$-\frac{2(b\,x^2+a)}{\sqrt{x\,(b\,x^2+a)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\right.}{}\right.}{\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{2(b\,x^2+a)}{\sqrt{x\,(b\,x^2+a)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\right.}{}\right.}{\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a
*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b
)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+
1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 40, normalized size = 0.16

$$-\frac{2\left(2\sqrt{b}\,x\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -2*(2*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x
)) + sqrt(b*x^3 + a*x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(a+bx^2)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a\,x}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x^2,x)
```

```
[Out] int((a*x + b*x^3)^(1/2)/x^2, x)
```

3.44 $\int \frac{\sqrt{ax+bx^3}}{x^3} \, dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{ax+bx^3}}$$

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+
a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2045, 2036, 335, 226}

$$\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^3,x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*x^2) + (2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx &= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{1}{3}(2b)\int \frac{1}{\sqrt{ax+bx^3}}\,dx \\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(2b\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(4b\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{3\sqrt[4]{a}\,\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 53, normalized size = 0.46

$$-\frac{2\sqrt{x\left(a+bx^2\right)}\ {}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^3,x]
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(
3*x^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.39, size = 123, normalized size = 1.06

method	result
default	$-\frac{2\sqrt{b\,x^{3}+a x}}{3x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}\right.}{3\sqrt{b\,x^{3}+a x}}$
elliptic	$-\frac{2\sqrt{b\,x^{3}+a x}}{3x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}\right.}{3\sqrt{b\,x^{3}+a x}}$
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3x\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\sqrt{\right.}\right.}{\sqrt{}}}\right.}{3\sqrt{b\,x^{3}+a x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^
(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 36, normalized size = 0.31

$$\frac{2\left(2\sqrt{b}\,x^{2}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^{3}+ax}\right)}{3\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/3*(2*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x))/x
^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(a+bx^2)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^3+a\,x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^3)^(1/2)/x^3, x)
```

3.45 $\int \frac{\sqrt{ax+bx^3}}{x^4} \, dx$

Optimal. Leaf size=283

$$\frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}}{5a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 4/5*b^(3/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*x^3+
a*x)^(1/2)/x^3-4/5*b*(b*x^3+a*x)^(1/2)/a/x-4/5*b^(5/4)*(cos(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipti
cE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*
x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+2
/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$\frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}+\frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{4b\sqrt{ax+bx^3}}{5ax}-\frac{2\sqrt{ax+bx^3}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (4*b^(3/2)*x*(a + b*x^2))/(5*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (
2*Sqrt[a*x + b*x^3])/(5*x^3) - (4*b*Sqrt[a*x + b*x^3])/(5*a*x) - (4*b^(5/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a*x + b*
x^3]) + (2*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(
3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\sqrt{ax+bx^3}}{5x^3} + \frac{1}{5}(2b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} + \frac{(2b^2)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} + \frac{\left(2b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} + \frac{\left(4b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right.}{5a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} + \frac{\left(4b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{\right.}{5\sqrt{a}\sqrt{ax+bx^3}}$$

$$= \frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} - \frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 53, normalized size = 0.19

$$-\frac{2\sqrt{x(a+bx^2)}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/
(5*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.36, size = 201, normalized size = 0.71

method	result

risch	$-\frac{2(bx^2+a)(2bx^2+a)}{5x^2\sqrt{x(bx^2+a)}\,a}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{E}}{}\right.}{5a\sqrt{bx^3}}$
default	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{}$
elliptic	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/x^3*(b*x^3+a*x)^(1/2)-4/5*(b*x^2+a)*b/a/(x*(b*x^2+a))^(1/2)+2/5*b/a*(-
a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1
/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/
b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 54, normalized size = 0.19

$$-\frac{2\left(2\,b^{\frac{3}{2}}x^{3}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^{3}+ax}\,\left(2\,bx^{2}+a\right)\right)}{5\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/5*(2*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
0, x)) + sqrt(b*x^3 + a*x)*(2*b*x^2 + a))/(a*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(a+bx^{2})}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^{3}+a\,x}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x^4,x)
```

```
[Out] int((a*x + b*x^3)^(1/2)/x^4, x)
```

3.46 $\int x^2(ax+bx^3)^{3/2}\,dx$

Optimal. Leaf size=186

$$-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{}}{}$$

```
[Out] 2/15*x^3*(b*x^3+a*x)^(3/2)-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+8/385*a^2*x^2*(b
*x^3+a*x)^(1/2)/b+4/55*a*x^4*(b*x^3+a*x)^(1/2)+4/231*a^(15/4)*(cos(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)^
(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2046, 2049, 2036, 335, 226}

$$\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{4}{55}ax^4\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*Sqrt[a*x + b*x^3])/(231*b^2) + (8*a^2*x^2*Sqrt[a*x + b*x^3])/(385*b
) + (4*a*x^4*Sqrt[a*x + b*x^3])/55 + (2*x^3*(a*x + b*x^3)^(3/2))/15 + (4*a^
(15/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[
a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x^2 (ax+bx^3)^{3/2}\,dx = \frac{2}{15}x^3(ax+bx^3)^{3/2} + \frac{1}{5}(2a)\int x^3\sqrt{ax+bx^3}\,dx$$
$$= \frac{4}{55}ax^4\sqrt{ax+bx^3} + \frac{2}{15}x^3(ax+bx^3)^{3/2} + \frac{1}{55}(4a^2)\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$$
$$= \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3} + \frac{2}{15}x^3(ax+bx^3)^{3/2} - \frac{(4a^3)\int \frac{x^2}{\sqrt{ax+bx^3}}}{77b}$$
$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2} + \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3} + \frac{2}{15}x^3(ax+bx^3)^{3/2}$$
$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2} + \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3} + \frac{2}{15}x^3(ax+bx^3)^{3/2}$$
$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2} + \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3} + \frac{2}{15}x^3(ax+bx^3)^{3/2}$$
$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2} + \frac{8a^2x^2\sqrt{ax+bx^3}}{385b} + \frac{4}{55}ax^4\sqrt{ax+bx^3} + \frac{2}{15}x^3(ax+bx^3)^{3/2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 94, normalized size = 0.51

$$\frac{2\sqrt{x(a+bx^2)}\left(-\left((5a-11bx^2)(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}\right)+5a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a
]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqrt[
1 + (b*x^2)/a])
```

Maple [A]
time = 0.36, size = 188, normalized size = 1.01

method	result

risch	$-\frac{2(-77b^3x^6-119a\,b^2x^4-12a^2b\,x^2+20a^3)x(b\,x^2+a)}{1155b^2\sqrt{x\,(b\,x^2+a)}}+\frac{4a^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{}{\sqrt{-}}}}{231b^3\sqrt{b\,x^3+ax}}$
default	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{}$
elliptic	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b*x^6*(b*x^3+a*x)^(1/2)+34/165*a*x^4*(b*x^3+a*x)^(1/2)+8/385*a^2*x^2*(
b*x^3+a*x)^(1/2)/b-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+4/231*a^4/b^3*(-a*b)^(1/
2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((
x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 70, normalized size = 0.38

$$\frac{2\left(20\,a^4\sqrt{b}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(77\,b^4x^6+119\,ab^3x^4+12\,a^2b^2x^2-20\,a^3b\right)\sqrt{bx^3+ax}\right)}{1155\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*a^4*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (77*b^4*x^6 + 11
9*a*b^3*x^4 + 12*a^2*b^2*x^2 - 20*a^3*b)*sqrt(b*x^3 + a*x))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(x \left(a + bx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**(3/2),x)
[Out] Integral(x**2*(x*(a + b*x**2))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(b\,x^3+a\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3)^(3/2),x)
[Out] int(x^2*(a*x + b*x^3)^(3/2), x)
```

3.47 $\int x(ax+bx^3)^{3/2}\,dx$

Optimal. Leaf size=304

$$-\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{8a^{13/4}\sqrt{x}\,(\ \cdots}{\cdots}$$

```
[Out] 2/13*x^2*(b*x^3+a*x)^(3/2)-8/65*a^3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))
/(b*x^3+a*x)^(1/2)+8/195*a^2*x*(b*x^3+a*x)^(1/2)/b+4/39*a*x^3*(b*x^3+a*x)^(
1/2)+8/65*a^(13/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2
))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-4/65*a^(13/4)*(cos(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(
sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(
1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}-\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*x*(a + b*x^2))/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (8*a^2*x*Sqrt[a*x + b*x^3])/(195*b) + (4*a*x^3*Sqrt[a*x + b*x^3])/39 + (
2*x^2*(a*x + b*x^3)^(3/2))/13 + (8*a^(13/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3]) - (4*a^(13/4)*Sqrt[x]*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x\left(ax+bx^3\right)^{3/2}\,dx=\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{13}(6a)\int x^2\sqrt{ax+bx^3}\,dx$$

$$=\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{39}\left(4a^2\right)\int\frac{x^3}{\sqrt{ax+bx^3}}\,dx$$

$$=\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{x}{\sqrt{ax+bx^3}}\,d}{65b}$$

$$=\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)}{65b\sqrt{ax+}}$$

$$=\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)}{65}$$

$$=\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^{7/2}\sqrt{x}\sqrt{a+bx^2}\right.}{65}$$

$$=-\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 84, normalized size = 0.28

$$\frac{2x\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{13b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeom
etric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.36, size = 217, normalized size = 0.71

method	result
risch	$\frac{2x^2(15b^2x^4+25abx^2+4a^2)(bx^2+a)}{195b\sqrt{x(bx^2+a)}} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{}}{}\right.}{65}$
default	$\frac{2bx^5\sqrt{bx^3+ax}}{13} + \frac{10ax^3\sqrt{bx^3+ax}}{39} + \frac{8a^2x\sqrt{bx^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\right.}{\sqrt{}}}}{}$
elliptic	$\frac{2bx^5\sqrt{bx^3+ax}}{13} + \frac{10ax^3\sqrt{bx^3+ax}}{39} + \frac{8a^2x\sqrt{bx^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\right.}{\sqrt{}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*b*x^5*(b*x^3+a*x)^(1/2)+10/39*a*x^3*(b*x^3+a*x)^(1/2)+8/195*a^2*x*(b*x
^3+a*x)^(1/2)/b-4/65/b^2*a^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1
/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2
))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-
a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 68, normalized size = 0.22

$$\frac{2\left(12\,a^3\sqrt{b}\,\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(15\,b^3x^5+25\,ab^2x^3+4\,a^2bx\right)\sqrt{bx^3+ax}\right)}{195\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/195*(12*a^3*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) + (15*b^3*x^5 + 25*a*b^2*x^3 + 4*a^2*b*x)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x*(x*(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(b\,x^3+a\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x*(a*x + b*x^3)^(3/2), x)
```

3.48 $\int \left(ax+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=158

$$\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(}{77b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/11*x*(b*x^3+a*x)^(3/2)+8/77*a^2*(b*x^3+a*x)^(1/2)/b+12/77*a*x^2*(b*x^3+a*
x)^(1/2)-4/77*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2029, 2046, 2049, 2036, 335, 226}

$$-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}+\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{12}{77}ax^2\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (8*a^2*Sqrt[a*x + b*x^3])/(77*b) + (12*a*x^2*Sqrt[a*x + b*x^3])/77 + (2*x*(
a*x + b*x^3)^(3/2))/11 - (4*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(77*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \left(ax+bx^3\right)^{3/2}\,dx = \frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{11}(6a)\int x\sqrt{ax+bx^3}\,dx$$

$$=\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{77}\left(12a^2\right)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$$

$$=\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{77b}$$

$$=\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)\int}{77b\sqrt{ax+}}$$

$$=\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Su}}{77b}$$

$$=\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 83, normalized size = 0.53

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{11b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeomet
ric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.37, size = 166, normalized size = 1.05

method	result

risch	$\frac{2(7b^2x^4+13abx^2+4a^2)x(bx^2+a)}{77b\sqrt{x(bx^2+a)}} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\right.}{77b^2\sqrt{bx^3+ax}}$
default	$\frac{2bx^4\sqrt{bx^3+ax}}{11} + \frac{26ax^2\sqrt{bx^3+ax}}{77} + \frac{8a^2\sqrt{bx^3+ax}}{77b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{77}$
elliptic	$\frac{2bx^4\sqrt{bx^3+ax}}{11} + \frac{26ax^2\sqrt{bx^3+ax}}{77} + \frac{8a^2\sqrt{bx^3+ax}}{77b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{77}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*b*x^4*(b*x^3+a*x)^(1/2)+26/77*a*x^2*(b*x^3+a*x)^(1/2)+8/77*a^2*(b*x^3+
a*x)^(1/2)/b-4/77/b^2*a^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^
(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 60, normalized size = 0.38

$$-\frac{2\left(4a^3\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-(7b^3x^4+13ab^2x^2+4a^2b)\sqrt{bx^3+ax}\right)}{77b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (7*b^3*x^4 + 13*a*
b^2*x^2 + 4*a^2*b)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax+bx^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

Mupad [B]
time = 5.00, size = 40, normalized size = 0.25

$$\frac{2\,x\,{\left(b\,x^3+a\,x\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{5}{4};\ \frac{9}{4};\ -\frac{b\,x^2}{a}\right)}{5\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2),x)
```

```
[Out] (2*x*(a*x + b*x^3)^(3/2)*hypergeom([-3/2, 5/4], 9/4, -(b*x^2)/a))/(5*((b*x^
2)/a + 1)^(3/2))
```

3.49 $\int \frac{(ax+bx^3)^{3/2}}{x}\,dx$

Optimal. Leaf size=275

$$\frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+}{\left(\sqrt{a}+\right.}}}{15b^{3/4}\sqrt{}}$$

```
[Out] 2/9*(b*x^3+a*x)^(3/2)+8/15*a^2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x
^3+a*x)^(1/2)+4/15*a*x*(b*x^3+a*x)^(1/2)-8/15*a^(9/4)*(cos(2*arctan(b^(1/4)
*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
E(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x
^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)+4/
15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2046, 2029, 2057, 335, 311, 226, 1210}

$$\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}+\frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (8*a^2*x*(a + b*x^2))/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
+ (4*a*x*Sqrt[a*x + b*x^3])/15 + (2*(a*x + b*x^3)^(3/2))/9 - (8*a^(9/4)*Sqr
t[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3
]) + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/
4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \frac{2}{9}(ax+bx^3)^{3/2} + \frac{1}{3}(2a)\int \sqrt{ax+bx^3}\,dx$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}(ax+bx^3)^{3/2} + \frac{1}{15}(4a^2)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}(ax+bx^3)^{3/2} + \frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15\sqrt{ax+bx^3}}$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}(ax+bx^3)^{3/2} + \frac{\left(8a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x,\right.}{15\sqrt{ax+bx^3}}$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}(ax+bx^3)^{3/2} + \frac{\left(8a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,\right.}{15\sqrt{b}\sqrt{ax+bx^3}}$$

$$= \frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{4}{15}ax\sqrt{ax+bx^3} + \frac{2}{9}(ax+bx^3)^{3/2} - \frac{8a^{9/4}\sqrt{x}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 52, normalized size = 0.19

$$\frac{2ax\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*a*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)])
/(3*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 195, normalized size = 0.71

method	result

default	$\frac{2b\,x^{3}\sqrt{b\,x^{3}+ax}}{9}+\frac{22ax\sqrt{b\,x^{3}+ax}}{45}+\frac{4a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a}}}}{}$
elliptic	$\frac{2b\,x^{3}\sqrt{b\,x^{3}+ax}}{9}+\frac{22ax\sqrt{b\,x^{3}+ax}}{45}+\frac{4a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a}}}}{}$
risch	$\frac{2x^{2}\left(5b\,x^{2}+11a\right)\left(b\,x^{2}+a\right)}{45\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{4a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{Elli}}{}\right.}{15b\sqrt{b\,x^{3}-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*b*x^3*(b*x^3+a*x)^(1/2)+22/45*a*x*(b*x^3+a*x)^(1/2)+4/15*a^2/b*(-a*b)^(
1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*
b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 58, normalized size = 0.21

$$-\frac{2\left(12\,a^2\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\left(5\,b^2x^3+11\,abx\right)\sqrt{bx^3+ax}\right)}{45\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -2/45*(12*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) - (5*b^2*x^3 + 11*a*b*x)*sqrt(b*x^3 + a*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x, x)
```

3.50 $\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=134

$$\frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}+\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

```
[Out] 2/7*(b*x^3+a*x)^(3/2)/x+4/7*a*(b*x^3+a*x)^(1/2)+4/7*a^(7/4)*(cos(2*arctan(b
^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*El
lipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1
/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x)^(1
/2)
```

Rubi [A]
time = 0.19, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2046, 2036, 335, 226}

$$\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/7 + (2*(a*x + b*x^3)^(3/2))/(7*x) + (4*a^(7/4)*Sqrt
[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx &= \frac{2(ax+bx^3)^{3/2}}{7x}+\frac{1}{7}(6a)\int \frac{\sqrt{ax+bx^3}}{x}\,dx\\ &= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}+\frac{1}{7}(4a^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\ &= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}+\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}}\\ &= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}+\frac{\left(8a^2\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{\cdots}\right.}{7\sqrt{ax+bx^3}}\\ &= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}+\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(2\cdots}{7\sqrt[4]{b}\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.21, size = 49, normalized size = 0.37

$$\frac{2a\sqrt{x\,(a+bx^2)}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^2,x]
```

[Out] (2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/S
qrt[1 + (b*x^2)/a]

Maple [A]
time = 0.35, size = 144, normalized size = 1.07

method	result
risch	$\frac{2(b x^2+3a)x(b x^2+a)}{7\sqrt{x(b x^2+a)}}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{7b\sqrt{b x^3+ax}}$
default	$\frac{2b x^2\sqrt{b x^3+ax}}{7}+\frac{6a\sqrt{b x^3+ax}}{7}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{7b\sqrt{b x^3+ax}}$
elliptic	$\frac{2b x^2\sqrt{b x^3+ax}}{7}+\frac{6a\sqrt{b x^3+ax}}{7}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{7b\sqrt{b x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] 2/7*b*x^2*(b*x^3+a*x)^(1/2)+6/7*a*(b*x^3+a*x)^(1/2)+4/7*a^2/b*(-a*b)^(1/2)*
((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*
b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1
/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 47, normalized size = 0.35

$$\frac{2\left(4\,a^2\sqrt{b}\;\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(b^2x^2+3\,ab)\sqrt{bx^3+ax}\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/7*(4*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (b^2*x^2 + 3*a*b)*sq
rt(b*x^3 + a*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^2,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^2, x)
```

3.51 $\int \frac{(ax+bx^3)^{3/2}}{x^3} dx$

Optimal. Leaf size=274

$$\frac{24a\sqrt{b}\,x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2(ax+bx^3)^{3/2}}{x^2}-\frac{24a^{5/4}\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a\cdots}{\left(\sqrt{a}\cdots\right.}}\cdots}{5\sqrt{ax+\cdots}}$$

```
[Out] -2*(b*x^3+a*x)^(3/2)/x^2+24/5*a*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)+12/5*b*x*(b*x^3+a*x)^(1/2)-24/5*a^(5/4)*b^(1/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+1
2/5*a^(5/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2045, 2029, 2057, 335, 311, 226, 1210}

$$\frac{12a^{5/4}\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24a^{5/4}\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}+\frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2(ax+bx^3)^{3/2}}{x^2}+\frac{24a\sqrt{b}\,x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (24*a*Sqrt[b]*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(12*b*x*Sqrt[a*x + b*x^3])/5 - (2*(a*x + b*x^3)^(3/2))/x^2 - (24*a^(5/4)*b^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3
]) + (12*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(5*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = -\frac{2(ax+bx^3)^{3/2}}{x^2} + (6b)\int \sqrt{ax+bx^3}\,dx$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} + \frac{1}{5}(12ab)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} + \frac{\left(12ab\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} + \frac{\left(24ab\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,\right.}{5\sqrt{ax+bx^3}}$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} + \frac{\left(24a^{3/2}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}}\right.}{5\sqrt{ax+bx^3}}$$

$$= \frac{24a\sqrt{b}\,x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} + \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} - \frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 52, normalized size = 0.19

$$-\frac{2a\sqrt{x\,(a+bx^2)}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])
/(x*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 194, normalized size = 0.71

method	result

risch	$-\frac{2(bx^2+a)(-bx^2+5a)}{5\sqrt{x(bx^2+a)}}+\frac{12a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{Ellipti}\cdots}{\cdots}\right.}{5\sqrt{bx^3+a\cdots}}$
default	$-\frac{2(bx^2+a)a}{\sqrt{x(bx^2+a)}}+\frac{2bx\sqrt{bx^3+ax}}{5}+\frac{12a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\cdots}{\cdots}$
elliptic	$-\frac{2(bx^2+a)a}{\sqrt{x(bx^2+a)}}+\frac{2bx\sqrt{bx^3+ax}}{5}+\frac{12a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^2+a)*a/(x*(b*x^2+a))^(1/2)+2/5*b*x*(b*x^3+a*x)^(1/2)+12/5*a*(-a*b)^
(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*
b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a
*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 52, normalized size = 0.19

$$-\frac{2\left(12\,a\sqrt{b}\,x\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\sqrt{bx^3+ax}\,(bx^2-5\,a)\right)}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -2/5*(12*a*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) - sqrt(b*x^3 + a*x)*(b*x^2 - 5*a))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x(a+bx^2)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\,x\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^3,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^3, x)
```

3.52 $\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=134

$$\frac{4}{3}b\sqrt{ax+bx^3}-\frac{2(ax+bx^3)^{3/2}}{3x^3}+\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}$$

```
[Out] -2/3*(b*x^3+a*x)^(3/2)/x^3+4/3*b*(b*x^3+a*x)^(1/2)+4/3*a^(3/4)*b^(3/4)*(cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)
^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2045, 2046, 2036, 335, 226}

$$\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}-\frac{2(ax+bx^3)^{3/2}}{3x^3}+\frac{4}{3}b\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (4*b*Sqrt[a*x + b*x^3])/3 - (2*(a*x + b*x^3)^(3/2))/(3*x^3) + (4*a^(3/4)*b^
(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*Sqrt[a*x + b*x^3
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx &= -\frac{2(ax+bx^3)^{3/2}}{3x^3} + (2b)\int \frac{\sqrt{ax+bx^3}}{x}\,dx \\
&= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{1}{3}(4ab)\int \frac{1}{\sqrt{ax+bx^3}}\,dx \\
&= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{\left(4ab\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}} \\
&= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{\left(8ab\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{}\right.}{3\sqrt{ax+bx^3}} \\
&= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}}{3\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 54, normalized size = 0.40

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\ {}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])
/(3*x^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 139, normalized size = 1.04

method	result
default	$-\frac{2a\sqrt{b\,x^3+ax}}{3x^2}+\frac{2b\sqrt{b\,x^3+ax}}{3}+\frac{4a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{E}}{3\sqrt{b\,x^3+ax}}$
risch	$-\frac{2(b\,x^2+a)(-b\,x^2+a)}{3x\sqrt{x\,(b\,x^2+a)}}+\frac{4a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{3\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{3x^2}+\frac{2b\sqrt{b\,x^3+ax}}{3}+\frac{4a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{E}}{3\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a*(b*x^3+a*x)^(1/2)/x^2+2/3*b*(b*x^3+a*x)^(1/2)+4/3*a*(-a*b)^(1/2)*((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^
(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*
(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 45, normalized size = 0.34

$$\frac{2\left(4\,a\sqrt{b}\,x^2\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}\,(bx^2-a)\right)}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/3*(4*a*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*
(b*x^2 - a))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^3+a\,x\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^4, x)
```

3.53 $\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=277

$$\frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2(ax+bx^3)^{3/2}}{5x^4}-\frac{24\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+b\cdots}{\left(\sqrt{a}+\cdots\right.}}}{5\sqrt{ax+b\cdots}}$$

```
[Out] -2/5*(b*x^3+a*x)^(3/2)/x^4+24/5*b^(3/2)*x*(b*x^2+a)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)-12/5*b*(b*x^3+a*x)^(1/2)/x-24/5*a^(1/4)*b^(5/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+1
2/5*a^(1/4)*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2045, 2057, 335, 311, 226, 1210}

$$\frac{12\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}+\frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2(ax+bx^3)^{3/2}}{5x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (24*b^(3/2)*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (1
2*b*Sqrt[a*x + b*x^3])/(5*x) - (2*(a*x + b*x^3)^(3/2))/(5*x^4) - (24*a^(1/4
)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b
*x^3]) + (12*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(5*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = -\frac{2(ax+bx^3)^{3/2}}{5x^4} + \frac{1}{5}(6b)\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$$
$$= -\frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} + \frac{1}{5}\left(12b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$
$$= -\frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} + \frac{\left(12b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}$$
$$= -\frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} + \frac{\left(24b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x \ldots\right)}{5\sqrt{ax+bx^3}}$$

$$= -\frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} + \frac{\left(24\sqrt{a}\,b^{3/2}\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}} \ldots\right)}{5\sqrt{ax+bx^3}}$$
$$= \frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} - \frac{24\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\ldots\right)}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.19

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\;{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)]
)/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 196, normalized size = 0.71

method	result

risch	$-\frac{2(bx^2+a)(7bx^2+a)}{5x^2\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{El}\cdots}{\cdots}\right.}{5\sqrt{bx^3\cdots}}$
default	$-\frac{2a\sqrt{bx^3+ax}}{5x^3}-\frac{14(bx^2+a)b}{5\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a\cdots}}}\cdots}{\cdots}$
elliptic	$-\frac{2a\sqrt{bx^3+ax}}{5x^3}-\frac{14(bx^2+a)b}{5\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a\cdots}}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*a*(b*x^3+a*x)^(1/2)/x^3-14/5*(b*x^2+a)*b/(x*(b*x^2+a))^(1/2)+12/5*b*(-
a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1
/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/
b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 51, normalized size = 0.18

$$-\frac{2\left(12\,b^{\frac{3}{2}}x^{3}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^{3}+ax}\,\left(7\,bx^{2}+a\right)\right)}{5\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -2/5*(12*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) + sqrt(b*x^3 + a*x)*(7*b*x^2 + a))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**5,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(b\,x^{3}+a\,x\right)}^{3/2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^5, x)
```

3.54 $\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=137

$$-\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2(ax+bx^3)^{3/2}}{7x^5}+\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}$$

```
[Out] -2/7*(b*x^3+a*x)^(3/2)/x^5-4/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b^(7/4)*(cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^3+
a*x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2045, 2036, 335, 226}

$$\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2(ax+bx^3)^{3/2}}{7x^5}-\frac{4b\sqrt{ax+bx^3}}{7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-4*b*Sqrt[a*x + b*x^3])/(7*x^2) - (2*(a*x + b*x^3)^(3/2))/(7*x^5) + (4*b^(
7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2
]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*a^(1/4)*Sqrt[a*x
+ b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = -\frac{2(ax+bx^3)^{3/2}}{7x^5}+\frac{1}{7}(6b)\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx$$

$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2(ax+bx^3)^{3/2}}{7x^5}+\frac{1}{7}(4b^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2(ax+bx^3)^{3/2}}{7x^5}+\frac{\left(4b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}}$$

$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2(ax+bx^3)^{3/2}}{7x^5}+\frac{\left(8b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{\right.}}{7\sqrt{ax+bx^3}}$$

$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2(ax+bx^3)^{3/2}}{7x^5}+\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(\right.}{7\sqrt[4]{a}\,\sqrt{ax+bx^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 54, normalized size = 0.39

$$-\frac{2a\sqrt{x\,(a+bx^2)}\;{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7x^4\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)]
)/(7*x^4*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.36, size = 142, normalized size = 1.04

method	result
risch	$-\frac{2(bx^2+a)(3bx^2+a)}{7x^3\sqrt{x(bx^2+a)}}+\frac{4b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x\right.}{}}\right.}{7\sqrt{bx^3+ax}}$
default	$-\frac{2a\sqrt{bx^3+ax}}{7x^4}-\frac{6b\sqrt{bx^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{7\sqrt{bx^3+ax}}$
elliptic	$-\frac{2a\sqrt{bx^3+ax}}{7x^4}-\frac{6b\sqrt{bx^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{7\sqrt{bx^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*a*(b*x^3+a*x)^(1/2)/x^4-6/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b*(-a*b)^(1/2)
*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+
1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 44, normalized size = 0.32

$$\frac{2\left(4\,b^{\frac{3}{2}}x^{4}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^{3}+ax}\,\left(3\,bx^{2}+a\right)\right)}{7\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 2/7*(4*b^(3/2)*x^4*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*(3
*b*x^2 + a))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**6,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^6,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^6, x)
```

3.55 $\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=306

$$\frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2(ax+bx^3)^{3/2}}{9x^6}-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right.}{}$$

```
[Out] -2/9*(b*x^3+a*x)^(3/2)/x^6+8/15*b^(5/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/(
b*x^3+a*x)^(1/2)-4/15*b*(b*x^3+a*x)^(1/2)/x^3-8/15*b^2*(b*x^3+a*x)^(1/2)/a/
x-8/15*b^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+4/15*b^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$\frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}+\frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{2(ax+bx^3)^{3/2}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (8*b^(5/2)*x*(a + b*x^2))/(15*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) -
(4*b*Sqrt[a*x + b*x^3])/(15*x^3) - (8*b^2*Sqrt[a*x + b*x^3])/(15*a*x) - (2*
(a*x + b*x^3)^(3/2))/(9*x^6) - (8*b^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3]) + (4*b^(9/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = -\frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{1}{3}(2b)\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{1}{15}\left(4b^2\right)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{(4b^3)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{15a}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{}}{\sqrt{a+}}}{15a\sqrt{ax+bx^3}}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\Big(}{15a\sqrt{ax-}}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{\left(8b^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}}{15\sqrt{a}\sqrt{a}}$$
$$= \frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/}}{9x^6}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.18

$$-\frac{2a\sqrt{x(a+bx^2)}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9x^5\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)]
)/(9*x^5*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.37, size = 223, normalized size = 0.73

method	result
risch	$-\frac{2(b\,x^2+a)(12b^2x^4+11ab\,x^2+5a^2)}{45x^4\sqrt{x\,(b\,x^2+a)}\,a}+\frac{4b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-\cdots}}{\cdots}\right.}{15a\cdots}$
default	$-\frac{2a\sqrt{b\,x^3+ax}}{9x^5}-\frac{22b\sqrt{b\,x^3+ax}}{45x^3}-\frac{8(b\,x^2+a)b^2}{15a\sqrt{x\,(b\,x^2+a)}}+\frac{4b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{\cdots}}{\cdots}\right.}{\sqrt{-\cdots}}}}{\cdots}$
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{9x^5}-\frac{22b\sqrt{b\,x^3+ax}}{45x^3}-\frac{8(b\,x^2+a)b^2}{15a\sqrt{x\,(b\,x^2+a)}}+\frac{4b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{\cdots}}{\cdots}\right.}{\sqrt{-\cdots}}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*a*(b*x^3+a*x)^(1/2)/x^5-22/45*b*(b*x^3+a*x)^(1/2)/x^3-8/15*(b*x^2+a)*b
^2/a/(x*(b*x^2+a))^(1/2)+4/15*b^2/a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-
a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*
b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 67, normalized size = 0.22

$$-\frac{2\left(12\,b^{\frac{5}{2}}x^{5}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(12\,b^{2}x^{4}+11\,abx^{2}+5\,a^{2}\right)\sqrt{bx^{3}+ax}\right)}{45\,ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")
[Out] -2/45*(12*b^(5/2)*x^5*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) + (12*b^2*x^4 + 11*a*b*x^2 + 5*a^2)*sqrt(b*x^3 + a*x))/(a*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**7,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^7,x)
[Out] int((a*x + b*x^3)^(3/2)/x^7, x)
```

3.56 $\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=163

$$-\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2(ax+bx^3)^{3/2}}{11x^7}-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan\right.}{77a^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] -2/11*(b*x^3+a*x)^(3/2)/x^7-12/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3+a
*x)^(1/2)/a/x^2-4/77*b^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/
2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2045, 2050, 2036, 335, 226}

$$-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2(ax+bx^3)^{3/2}}{11x^7}-\frac{12b\sqrt{ax+bx^3}}{77x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-12*b*Sqrt[a*x + b*x^3])/(77*x^4) - (8*b^2*Sqrt[a*x + b*x^3])/(77*a*x^2) -
 (2*(a*x + b*x^3)^(3/2))/(11*x^7) - (4*b^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[x])/a^(1/4)], 1/2])/(77*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx &= -\frac{2(ax+bx^3)^{3/2}}{11x^7}+\frac{1}{11}(6b)\int \frac{\sqrt{ax+bx^3}}{x^5}\,dx \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{2(ax+bx^3)^{3/2}}{11x^7}+\frac{1}{77}(12b^2)\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2(ax+bx^3)^{3/2}}{11x^7}-\frac{(4b^3)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{77a} \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2(ax+bx^3)^{3/2}}{11x^7}-\frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{}{\sqrt{x}}}{77a\sqrt{ax+bx^3}} \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2(ax+bx^3)^{3/2}}{11x^7}-\frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}(}{77a\sqrt{ax}} \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2(ax+bx^3)^{3/2}}{11x^7}-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{}}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 54, normalized size = 0.33

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\;{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11x^6\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a)
])/(11*x^6*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.35, size = 169, normalized size = 1.04

method	result
risch	$-\frac{2(b\,x^2+a)\left(4b^2x^4+13ab\,x^2+7a^2\right)}{77x^5\sqrt{x\left(b\,x^2+a\right)}\,a}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\right.}{77a\sqrt{b\,x^3+ax}}$
default	$-\frac{2a\sqrt{b\,x^3+ax}}{11x^6}-\frac{26b\sqrt{b\,x^3+ax}}{77x^4}-\frac{8b^2\sqrt{b\,x^3+ax}}{77a\,x^2}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}}{77a}$
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{11x^6}-\frac{26b\sqrt{b\,x^3+ax}}{77x^4}-\frac{8b^2\sqrt{b\,x^3+ax}}{77a\,x^2}-\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}}{77a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -2/11*a*(b*x^3+a*x)^(1/2)/x^6-26/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3
+a*x)^(1/2)/a/x^2-4/77*b^2/a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1
/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 59, normalized size = 0.36

$$-\frac{2\left(4\,b^{\frac{5}{2}}x^{6}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(4\,b^{2}x^{4}+13\,abx^{2}+7\,a^{2}\right)\sqrt{bx^{3}+ax}\right)}{77\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -2/77*(4*b^(5/2)*x^6*weierstrassPInverse(-4*a/b, 0, x) + (4*b^2*x^4 + 13*a*
b*x^2 + 7*a^2)*sqrt(b*x^3 + a*x))/(a*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**8,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^8,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^8, x)
```

3.57 $\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=140

$$-\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}+\frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\right.}{21b^{9/4}\sqrt{ax+bx^3}}$$

```
[Out] -10/21*a*(b*x^3+a*x)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x)^(1/2)/b+5/21*a^(7/4)*(co
s(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b
*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2049, 2036, 335, 226}

$$\frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}-\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-10*a*Sqrt[a*x + b*x^3])/(21*b^2) + (2*x^2*Sqrt[a*x + b*x^3])/(7*b) + (5*a
^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[a
*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{2x^2\sqrt{ax+bx^3}}{7b} - \frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{7b}$$

$$= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{(5a^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{21b^2}$$

$$= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{\left(5a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b^2\sqrt{ax+bx^3}}$$

$$= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{\left(10a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x\right.}{21b^2\sqrt{ax+bx^3}}$$

$$= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(}{21b^{9/4}\sqrt{ax+bx^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 80, normalized size = 0.57

$$\frac{2x\left(-5a^2-2abx^2+3b^2x^4+5a^2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{21b^2\sqrt{x\,(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.35, size = 149, normalized size = 1.06

method	result
risch	$-\frac{2(-3b\,x^2+5a)x(b\,x^2+a)}{21b^2\sqrt{x\,(b\,x^2+a)}}+\frac{5a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{21b^3\sqrt{b\,x^3+ax}}$
default	$\frac{2x^2\sqrt{b\,x^3+ax}}{7b}-\frac{10a\sqrt{b\,x^3+ax}}{21b^2}+\frac{5a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ldots}{21b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x^2\sqrt{b\,x^3+ax}}{7b}-\frac{10a\sqrt{b\,x^3+ax}}{21b^2}+\frac{5a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ldots}{21b^3\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)/b-10/21*a*(b*x^3+a*x)^(1/2)/b^2+5/21*a^2/b^3*(-a*
b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2
))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*Ellipt
icF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 48, normalized size = 0.34

$$\frac{2\left(5\,a^{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(3\,b^{2}x^{2}-5\,ab\right)\sqrt{bx^{3}+ax}\right)}{21\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (3*b^2*x^2 - 5*a*b)
*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{4}}{\sqrt{x\left(a+bx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{4}}{\sqrt{b\,x^{3}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^3)^(1/2), x)
```

3.58 $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=258

$$-\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2\tan}{5b^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] -6/5*a*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*x
^3+a*x)^(1/2)/b+6/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-3/5*a^(5/4)*(cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/
2)
```

Rubi [A]
time = 0.15, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2049, 2057, 335, 311, 226, 1210}

$$-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}-\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-6*a*x*(a + b*x^2))/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(2*x*Sqrt[a*x + b*x^3])/(5*b) + (6*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x]
)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3]) - (3*a^(5/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{(3a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5b}$$

$$= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{\left(3a\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5b\sqrt{ax+bx^3}}$$

$$= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{\left(6a\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b\sqrt{ax+bx^3}}$$

$$= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{\left(6a^{3/2}\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{ax+bx^3}} + \frac{\left(6a^{3/2}\sqrt{\right.}{}$$

$$= -\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} + \frac{2x\sqrt{ax+bx^3}}{5b} + \frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{}{(}}}{5b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 66, normalized size = 0.26

$$\frac{2x^2\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{5b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4,
-((b*x^2)/a)]))/(5*b*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.34, size = 178, normalized size = 0.69

method	result

default	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\ldots}\right.}{\ldots}\right.}{5b^2\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\ldots}\right.}{\ldots}\right.}{5b^2\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5b\sqrt{x\left(b\,x^2+a\right)}}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\ldots\right.}{\ldots}\right.}{5b^2\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)/b-3/5*a/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-
a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*
b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 43, normalized size = 0.17

$$\frac{2\left(\sqrt{bx^3+ax}\,bx+3\,a\sqrt{b}\,\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(b*x^3 + a*x)*b*x + 3*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierst
rassPInverse(-4*a/b, 0, x)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x\,(a+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^3)^(1/2), x)
```

3.59 $\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=116

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/
(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2049, 2036, 335, 226}

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx &= \frac{2\sqrt{ax+bx^3}}{3b} - \frac{a\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{3b} \\
&= \frac{2\sqrt{ax+bx^3}}{3b} - \frac{\left(a\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{3b\sqrt{ax+bx^3}} \\
&= \frac{2\sqrt{ax+bx^3}}{3b} - \frac{\left(2a\sqrt{x}\,\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{3b\sqrt{ax+bx^3}} \\
&= \frac{2\sqrt{ax+bx^3}}{3b} - \frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\tfrac{1}{2}\right.}{3b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 64, normalized size = 0.55

$$\frac{2x\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b\sqrt{x\,(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x + b*x^3],x]
```

[Out] (2*x*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[x*(a + b*x^2)])

Maple [A]
time = 0.39, size = 127, normalized size = 1.09

method	result
default	$\frac{2\sqrt{b\,x^3+ax}}{3b}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{3b^2\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2\sqrt{b\,x^3+ax}}{3b}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{3b^2\sqrt{b\,x^3+ax}}$
risch	$\frac{2x\left(b\,x^2+a\right)}{3b\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}}{b}\right)}{\sqrt{-}}}\right.}{3b^2\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3*a/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 34, normalized size = 0.29

$$-\frac{2\left(a\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}\,b\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*b)/b^
2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x\,(a+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x*(a + b*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^3)^(1/2), x)
```

3.60 $\int \frac{x}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=229

$$\frac{2x(a+bx^2)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{a}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*a^(1/4)*(cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x
^3+a*x)^(1/2)+a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(
1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2057, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{a}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{a}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}+\frac{2x(a+bx^2)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(a + b*x^2))/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*a^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a*x +
 b*x^3]) + (a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3
/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}}$$

$$= \frac{\left(2\sqrt{a}\,\sqrt{x}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,\sqrt{ax+bx^3}} - \frac{\left(2\sqrt{a}\,\sqrt{x}\,\sqrt{a+bx^2}\right)\mathrm{Su}}{\sqrt{b}\,\sqrt{}}$$

$$= \frac{2x(a+bx^2)}{\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2}{b^{3/4}\sqrt{ax+bx^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 53, normalized size = 0.23

$$\frac{2x^2\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])/
(3*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.38, size = 158, normalized size = 0.69

method	result
default	$$\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{b}\right.}{b\sqrt{b\,x^3+ax}}$$
elliptic	$$\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{b}\right.}{b\sqrt{b\,x^3+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2
)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),
```

```
1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 22, normalized size = 0.10

$$-\frac{2\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] -2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))/sqrt(b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\,(a+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(1/2),x)
[Out] Integral(x/sqrt(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^3)^(1/2), x)
```

3.61 $\int \frac{1}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

```
[Out] (cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)
/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2036, 335, 226}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^3],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a*
x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax+bx^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}\\ &= \frac{\left(2\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{ax+bx^3}}\\ &= \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 49, normalized size = 0.53

$$\frac{2x\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^3],x]
[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

Maple [A]
time = 0.35, size = 108, normalized size = 1.17

method	result	size
default	$\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^3+ax}}$	108

elliptic	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2
)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 14, normalized size = 0.15

$$\frac{2\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassPInverse(-4*a/b, 0, x)/sqrt(b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

Mupad [**B**]
time = 5.03, size = 40, normalized size = 0.43

$$\frac{2\,x\,\sqrt{\frac{b\,x^2}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^2}{a}\right)}{\sqrt{b\,x^3+a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3)^(1/2),x)
```

```
[Out] (2*x*((b*x^2)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^2)/a))/(a*x + b
*x^3)^(1/2)
```

3.62 $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=253

$$\frac{2\sqrt{b}\,x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}-\frac{2\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{}}{}\right.\right.}{a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 2*x*(b*x^2+a)*b^(1/2)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)
^(1/2)/a/x-2*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1
/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)
*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2050, 2057, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}+\frac{2\sqrt{b}\,x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (2*Sqrt[b]*x*(a + b*x^2))/(a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*
Sqrt[a*x + b*x^3])/(a*x) - (2*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3]) + (b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{ax+bx^3}}\,dx &= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{b\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{a}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(b\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2b\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2\sqrt{b}\,\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\,\sqrt{ax+bx^3}}-\frac{\left(2\sqrt{b}\right.}{}\\
&= \frac{2\sqrt{b}\,x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}-\frac{2\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+}{\left(\sqrt{a}+\right.}}}{a^{3/4}\sqrt{a}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 48, normalized size = 0.19

$$
-\frac{2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x\,(a+bx^2)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

Maple [A]
time = 0.35, size = 182, normalized size = 0.72

method	result

default	$-\frac{2(bx^2+a)}{a\sqrt{x(bx^2+a)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\cdots\right.}{\cdots}\right.}{a\sqrt{bx^3+ax}\cdots}$
risch	$-\frac{2(bx^2+a)}{a\sqrt{x(bx^2+a)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\cdots\right.}{\cdots}\right.}{a\sqrt{bx^3+ax}\cdots}$
elliptic	$-\frac{2(bx^2+a)}{a\sqrt{x(bx^2+a)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\cdots\right.}{\cdots}\right.}{a\sqrt{bx^3+ax}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^2+a)/a/(x*(b*x^2+a))^(1/2)+1/a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b
/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(
-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(
((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 42, normalized size = 0.17

$$-\frac{2\left(\sqrt{b}\,x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))
 + sqrt(b*x^3 + a*x))/(a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^3)^(1/2)), x)
```

3.63 $\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=119

$$-\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2050, 2036, 335, 226}

$$-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx &= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{b\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{3a}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{\left(b\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{3a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{\left(2b\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{3a^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 53, normalized size = 0.45

$$-\frac{2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/(3 *x*Sqrt[x*(a + b*x^2)])

Maple [A]
time = 0.36, size = 129, normalized size = 1.08

method	result
default	$-\frac{2\sqrt{b\,x^3+ax}}{3a\,x^2}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{3a\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{2\sqrt{b\,x^3+ax}}{3a\,x^2}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{3a\sqrt{b\,x^3+ax}}$
risch	$-\frac{2(b\,x^2+a)}{3ax\sqrt{x\,(b\,x^2+a)}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{3a\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3/a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a *b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 36, normalized size = 0.30

$$-\frac{2\left(\sqrt{b}\,x^2\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}\right)}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x))/(a
*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x\,(a+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**2))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^3)^(1/2)), x)
```

3.64 $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=286

$$-\frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}}{5a^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] -6/5*b^(3/2)*x*(b*x^2+a)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*x
^3+a*x)^(1/2)/a/x^3+6/5*b*(b*x^3+a*x)^(1/2)/a^2/x+6/5*b^(5/4)*(cos(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^
(1/2)-3/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2050, 2057, 335, 311, 226, 1210}

$$-\frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}-\frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{2\sqrt{ax+bx^3}}{5ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-6*b^(3/2)*x*(a + b*x^2))/(5*a^2*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
- (2*Sqrt[a*x + b*x^3])/(5*a*x^3) + (6*b*Sqrt[a*x + b*x^3])/(5*a^2*x) + (6*
b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a
*x + b*x^3]) - (3*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])
/(5*a^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{ax+bx^3}}{5ax^3} - \frac{(3b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{5a}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{(3b^2)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a^2}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{\left(3b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a^2\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{\left(6b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{\right.}{5a^2\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{\left(6b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x,\right.}{5a^{3/2}\sqrt{ax+bx^3}}$$

$$= -\frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} + \frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 53, normalized size = 0.19

$$-\frac{2\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^2\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])/(
5*x^2*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.37, size = 204, normalized size = 0.71

method	result

risch	$-\frac{2(b\,x^{2}+a)(-3b\,x^{2}+a)}{5a^{2}x^{2}\sqrt{x\,(b\,x^{2}+a)}}-\frac{3b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{E}}{}\right.}{5a^{2}\sqrt{b\,x}}$
default	$-\frac{2\sqrt{b\,x^{3}+ax}}{5a\,x^{3}}+\frac{6(b\,x^{2}+a)b}{5a^{2}\sqrt{x\,(b\,x^{2}+a)}}-\frac{3b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}}{}$
elliptic	$-\frac{2\sqrt{b\,x^{3}+ax}}{5a\,x^{3}}+\frac{6(b\,x^{2}+a)b}{5a^{2}\sqrt{x\,(b\,x^{2}+a)}}-\frac{3b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^3+a*x)^(1/2)/a/x^3+6/5*(b*x^2+a)*b/a^2/(x*(b*x^2+a))^(1/2)-3/5*b/
a^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2
)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 56, normalized size = 0.20

$$\frac{2\left(3\,b^{\frac{3}{2}}x^{3}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^{3}+ax}\,(3\,bx^{2}-a)\right)}{5\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0
, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 - a))/(a^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{3}\sqrt{x\,(a+bx^{2})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{x^{3}\,\sqrt{b\,x^{3}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^3)^(1/2)), x)
```

3.65 $\int \frac{x^7}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(2\tan}{14b^{13/4}\sqrt{ax+bx^3}}$$

```
[Out] -x^5/b/(b*x^3+a*x)^(1/2)-15/7*a*(b*x^3+a*x)^(1/2)/b^3+9/7*x^2*(b*x^3+a*x)^(
1/2)/b^2+15/14*a^(7/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(13/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2047, 2049, 2036, 335, 226}

$$\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^5/(b*Sqrt[a*x + b*x^3])) - (15*a*Sqrt[a*x + b*x^3])/(7*b^3) + (9*x^2*Sq
rt[a*x + b*x^3])/(7*b^2) + (15*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(14*b^(13/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^5}{b\sqrt{ax+bx^3}} + \frac{9\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx}{2b}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} - \frac{(45a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{14b^2}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{(15a^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{14b^3}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{\left(15a^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{}{\sqrt{x}}}{14b^3\sqrt{ax+bx^3}}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{\left(15a^2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}}{7b^3\sqrt{ax}}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{}}{1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 80, normalized size = 0.50

$$\frac{x\left(-15a^2-6abx^2+2b^2x^4+15a^2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b^3\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(-15*a^2 - 6*a*b*x^2 + 2*b^2*x^4 + 15*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(7*b^3*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.41, size = 172, normalized size = 1.07

method	result

default	$-\frac{x\,a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b\,x^3+ax}}{7b^2}-\frac{8a\sqrt{b\,x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{\ }}{}\right.}{\sqrt{-}}}}{14}$
elliptic	$-\frac{x\,a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b\,x^3+ax}}{7b^2}-\frac{8a\sqrt{b\,x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{\ }}{}\right.}{\sqrt{-}}}}{14}$
risch	$-\frac{2(-b\,x^2+4a)\,(b\,x^2+a)\,x}{7b^3\sqrt{x\,(b\,x^2+a)}}+\frac{a^2\left(\frac{11\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(x}{}}\right.}{b\sqrt{b\,x^3+ax}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*x*a^2/((x^2+a/b)*b*x)^(1/2)+2/7*x^2*(b*x^3+a*x)^(1/2)/b^2-8/7*a*(b*x
^3+a*x)^(1/2)/b^3+15/14*a^2/b^4*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)
^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 80, normalized size = 0.50

$$\frac{15\,(a^2bx^2+a^3)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(2\,b^3x^4-6\,ab^2x^2-15\,a^2b)\sqrt{bx^3+ax}}{7\,(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*(15*(a^2*b*x^2 + a^3)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (2*b^
3*x^4 - 6*a*b^2*x^2 - 15*a^2*b)*sqrt(b*x^3 + a*x))/(b^5*x^2 + a*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**7/(x*(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{{\left(b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^7/(a*x + b*x^3)^(3/2), x)
```

3.66 $\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{x^4}{b\sqrt{ax+bx^3}}-\frac{21ax(a+bx^2)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx\cdots}{\left(\sqrt{a}+\sqrt{\cdots}\right.}}\cdots}{5b^{11/4}\sqrt{ax\cdots}}$$

```
[Out] -x^4/b/(b*x^3+a*x)^(1/2)-21/5*a*x*(b*x^2+a)/b^(5/2)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)+7/5*x*(b*x^3+a*x)^(1/2)/b^2+21/5*a^(5/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipt
icE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^3+a*x)^(1/2)
-21/10*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/b^(11/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2047, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}}-\frac{21ax(a+bx^2)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}-\frac{x^4}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^4/(b*Sqrt[a*x + b*x^3])) - (21*a*x*(a + b*x^2))/(5*b^(5/2)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[a*x + b*x^3]) + (7*x*Sqrt[a*x + b*x^3])/(5*b^2) + (21*a^(5/4
)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[a*x +
b*x^3]) - (21*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10
*b^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx}{2b}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{(21a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{10b^2}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{10b^2\sqrt{ax+bx^3}}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{\ }\right.}{5b^2\sqrt{ax+bx^3}}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x\right.}{5b^{5/2}\sqrt{ax+bx^3}}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} - \frac{21ax(a+bx^2)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} + \frac{21a^{5/4}\sqrt{x}\left(\sqrt{\ }\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 68, normalized size = 0.24

$$\frac{2x^2\left(-7a+bx^2+7a\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*(-7*a + b*x^2 + 7*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2,
7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.40, size = 200, normalized size = 0.72

method	result
default	$\frac{x^2 a}{b^2 \sqrt{\left(x^2+\frac{a}{b}\right) b x}}+\frac{2 x \sqrt{b\,x^3+a x}}{5 b^2}-\frac{21 a \sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}}{}$
elliptic	$\frac{x^2 a}{b^2 \sqrt{\left(x^2+\frac{a}{b}\right) b x}}+\frac{2 x \sqrt{b\,x^3+a x}}{5 b^2}-\frac{21 a \sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}}{}$
risch	$\frac{2 x^2 \left(b\,x^2+a\right)}{5 b^2 \sqrt{x \left(b\,x^2+a\right)}}-\frac{a \left(\frac{8 \sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\left(-\frac{2 \sqrt{-a b}\ \mathrm{Ellipt}}{}\right.}{b \sqrt{b\,x^3+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/b^2*x^2*a/((x^2+a/b)*b*x)^(1/2)+2/5*x*(b*x^3+a*x)^(1/2)/b^2-21/10*a/b^3*(
```

```
-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(
1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2
/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 76, normalized size = 0.27

$$\frac{21\,(abx^2+a^2)\sqrt{b}\;\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(2\,b^2x^3+7\,abx)\sqrt{bx^3+ax}}{5\,(b^4x^2+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(21*(a*b*x^2 + a^2)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInve
rse(-4*a/b, 0, x)) + (2*b^2*x^3 + 7*a*b*x)*sqrt(b*x^3 + a*x))/(b^4*x^2 + a*
b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**6/(x*(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(b\,x^3+a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^6/(a*x + b*x^3)^(3/2), x)
```

3.67 $\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}$$

```
[Out] -x^3/b/(b*x^3+a*x)^(1/2)+5/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a^(3/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)
^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2047, 2049, 2036, 335, 226}

$$-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a*x + b*x^3])) + (5*Sqrt[a*x + b*x^3])/(3*b^2) - (5*a^(3/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[a*x + b*x^
3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{2b}$$

$$= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{(5a)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{6b^2}$$

$$= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6b^2\sqrt{ax+bx^3}}$$

$$= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right.}{3b^2\sqrt{ax+bx^3}}$$

$$= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\right.}{6b^{9/4}\sqrt{ax+bx^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 67, normalized size = 0.49

$$\frac{x\left(5a+2bx^2-5a\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, -((b*x^2)/a)]))/(3*b^2*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.38, size = 147, normalized size = 1.07

method	result
default	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{E}}{6b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{E}}{6b^3\sqrt{b\,x^3+ax}}$
risch	$\frac{2\left(b\,x^2+a\right)x}{3b^2\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\left(\frac{4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\sqrt{\cdots}\right.}{\sqrt{\cdots}}}\right.}{b\sqrt{b\,x^3+ax}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*x*a/((x^2+a/b)*b*x)^(1/2)+2/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a/b^3*(-a*b)^
(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*
b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF
(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 68, normalized size = 0.50

$$-\frac{5\left(abx^2+a^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\left(2b^2x^2+5ab\right)\sqrt{bx^3+ax}}{3\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(a*b*x^2 + a^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (2*b^2*
x^2 + 5*a*b)*sqrt(b*x^3 + a*x))/(b^4*x^2 + a*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**5/(x*(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^5/(a*x + b*x^3)^(3/2), x)
```

3.68 $\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=253

$$-\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2\tan^{-1}}{b^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] -x^2/b/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a
*x)^(1/2)-3*a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)+3/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2047, 2057, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{a}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{ax+bx^3}}+\frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^2/(b*Sqrt[a*x + b*x^3])) + (3*x*(a + b*x^2))/(b^(3/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[a*x + b*x^3]) - (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(b^(7/4)*Sqrt[a*x + b*x^3]) + (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^2}{b\sqrt{ax+bx^3}} + \frac{3\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2b}$$

$$= -\frac{x^2}{b\sqrt{ax+bx^3}} + \frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}$$

$$= -\frac{x^2}{b\sqrt{ax+bx^3}} + \frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{ax+bx^3}}$$

$$= -\frac{x^2}{b\sqrt{ax+bx^3}} + \frac{\left(3\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{b^{3/2}\sqrt{ax+bx^3}} - \frac{\left(3\sqrt{a}\right.}{}$$

$$= -\frac{x^2}{b\sqrt{ax+bx^3}} + \frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\ldots}}{b^{7/\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 57, normalized size = 0.23

$$-\frac{2x^2\left(-1+\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{b\sqrt{x\,(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2
)/a)]))/(b*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.35, size = 182, normalized size = 0.72

method	result

default	$-\frac{x^2}{b\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\cdots\right.}{\cdots}\right.}{2b^2\sqrt{b\,x^3+a x}}$
elliptic	$-\frac{x^2}{b\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\cdots\right.}{\cdots}\right.}{2b^2\sqrt{b\,x^3+a x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*x^2/((x^2+a/b)*b*x)^(1/2)+3/2/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b
/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(
-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(
((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 61, normalized size = 0.24

$$-\frac{\sqrt{bx^3+ax}\,bx+3\left(bx^2+a\right)\sqrt{b}\ \mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b^3x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*x^3 + a*x)*b*x + 3*(b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0,
weierstrassPInverse(-4*a/b, 0, x)))/(b^3*x^2 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x(a+bx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(x*(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(b\,x^3+a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^4/(a*x + b*x^3)^(3/2), x)
```

3.69 $\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{x}{b\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] -x/b/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1
/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+
x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2047, 2036, 335, 226}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a*x + b*x^3])) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1
/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx &= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{ax+bx^3}}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{2\sqrt[4]{a}\,b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 54, normalized size = 0.47

$$\frac{x\left(-1+\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3)^(3/2),x]
```

[Out] (x*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]
))/(b*Sqrt[x*(a + b*x^2)])

Maple [A]
time = 0.34, size = 130, normalized size = 1.13

method	result
default	$-\frac{x}{b\sqrt{\left(x^{2}+\frac{a}{b}\right) b x}}+\frac{\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-\cdots}}{\cdots}\right)\cdots}{\sqrt{-\cdots}}}\cdots\right.}{2 b^{2}\sqrt{b\,x^{3}+a x}}$
elliptic	$-\frac{x}{b\sqrt{\left(x^{2}+\frac{a}{b}\right) b x}}+\frac{\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-\cdots}}{\cdots}\right)\cdots}{\sqrt{-\cdots}}}\cdots\right.}{2 b^{2}\sqrt{b\,x^{3}+a x}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/b*x/((x^2+a/b)*b*x)^(1/2)+1/2/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(
-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 51, normalized size = 0.44

$$\frac{\left(bx^{2}+a\right)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^{3}+ax}\ b}{b^{3}x^{2}+ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*
b)/(b^3*x^2 + a*b^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(x*(a + b*x**2))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^3)^(3/2), x)
```

3.70 $\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=254

$$\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{x(a+bx^2)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{\cdots}}\right.\right.}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] x^2/a/(b*x^3+a*x)^(1/2)-x*(b*x^2+a)/a/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*
x)^(1/2)+(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*
2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2
)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)-1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2
+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2048, 2057, 335, 311, 226, 1210}

$$-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{x(a+bx^2)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x^2/(a*Sqrt[a*x + b*x^3]) - (x*(a + b*x^2))/(a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(
a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3]) - (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a}$$

$$= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}}$$

$$= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^3}}$$

$$= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{ax+bx^3}} + \frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right.}{}$$

$$= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{x(a+bx^2)}{a\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+b}{\left(\sqrt{a}+\right.}}}{a^{3/4}b^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.22

$$\frac{2x^2\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt{x\,(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)])/
(3*a*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.34, size = 184, normalized size = 0.72

method	result

default	$\frac{x^{2}}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\right.}{}\right.}{2ab\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{x^{2}}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\right.}{}\right.}{2ab\sqrt{b\,x^{3}+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2/a/((x^2+a/b)*b*x)^(1/2)-1/2/a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-
a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*
b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 60, normalized size = 0.24

$$\frac{\sqrt{bx^{3}+ax}\,bx+(bx^{2}+a)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)}{ab^{2}x^{2}+a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^3 + a*x)*b*x + (b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0, wei
erstrassPInverse(-4*a/b, 0, x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(x*(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^3)^(3/2), x)
```

3.71 $\int \frac{x}{(ax+bx^3)^{3/2}} \, dx$

Optimal. Leaf size=114

$$\frac{x}{a\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

```
[Out] x/a/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/a^(5/4)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2048, 2036, 335, 226}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2]
)/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(ax+bx^3)^{3/2}}\,dx &= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.99, size = 54, normalized size = 0.47

$$
\frac{x+x\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/
(a*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.34, size = 132, normalized size = 1.16

method	result
default	$\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}\right.}{2ab\sqrt{b\,x^3+ax}}$
elliptic	$\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}\right.}{2ab\sqrt{b\,x^3+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/a/((x^2+a/b)*b*x)^(1/2)+1/2/a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*
b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1
/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 51, normalized size = 0.45

$$\frac{(bx^2+a)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}\,b}{ab^2x^2+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*
b)/(a*b^2*x^2 + a^2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x/(x*(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x/(a*x + b*x^3)^(3/2), x)
```

3.72 $\int \frac{1}{(ax+bx^3)^{3/2}}\,dx$

<u>**Optimal**</u>. Leaf size=273

$$\frac{1}{a\sqrt{ax+bx^3}}+\frac{3\sqrt{b}\,x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}-\frac{3\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}}{a^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] 1/a/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)*b^(1/2)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3+
a*x)^(1/2)-3*(b*x^3+a*x)^(1/2)/a^2/x-3*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)+3/2*b^(1/
4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7
/4)/(b*x^3+a*x)^(1/2)
```

<u>**Rubi**</u> [A]

time = 0.16, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2031, 2050, 2057, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{3\sqrt{b}\,x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{1}{a\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] 1/(a*Sqrt[a*x + b*x^3]) + (3*Sqrt[b]*x*(a + b*x^2))/(a^2*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[a*x + b*x^3]) - (3*Sqrt[a*x + b*x^3])/(a^2*x) - (3*b^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[
2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a*x + b*x^3]) + (3
*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*
x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[
a*x + b*x^3])
```

<u>Rule 226</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

<u>Rule 311</u>

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{2a}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{(3b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a^2}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a^2\sqrt{ax+bx^3}}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right.}{a^2\sqrt{ax+bx^3}}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \right.}{a^{3/2}\sqrt{ax+bx^3}}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\sqrt{b}\,x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} - \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.19

$$-\frac{2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/(a
*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.50, size = 206, normalized size = 0.75

method	result

default	$-\frac{b\,x^{2}}{a^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{2\left(b\,x^{2}+a\right)}{a^{2}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a}}}}{}$
elliptic	$-\frac{b\,x^{2}}{a^{2}\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}-\frac{2\left(b\,x^{2}+a\right)}{a^{2}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a}}}}{}$

risch	$-\frac{2(b\,x^2+a)}{a^2\sqrt{x\,(b\,x^2+a)}}+\frac{b^2\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{Elli}}{}\right.}{b^2\sqrt{b\,x^3}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*x^2/a^2/((x^2+a/b)*b*x)^(1/2)-2*(b*x^2+a)/a^2/(x*(b*x^2+a))^(1/2)+3/2/a^
2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b
)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*
(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 72, normalized size = 0.26

$$-\frac{3\,(bx^3+ax)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\,(3\,bx^2+2\,a)}{a^2bx^3+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(b*x^3 + a*x)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4
*a/b, 0, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 + 2*a))/(a^2*b*x^3 + a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ax+bx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

Mupad [B]
time = 5.17, size = 40, normalized size = 0.15

$$-\frac{2\,x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}\,{{}}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{3}{4};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^3+a\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3)^(3/2),x)
```

```
[Out] -(2*x*((b*x^2)/a + 1)^(3/2)*hypergeom([-1/4, 3/2], 3/4, -(b*x^2)/a))/(a*x +
 b*x^3)^(3/2)
```

3.73 $\int \frac{1}{x(ax+bx^3)^{3/2}} dx$

Optimal. Leaf size=139

$$\frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}$$

```
[Out] 1/a/x/(b*x^3+a*x)^(1/2)-5/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6*b^(3/4)*(cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/(b*x^3+a
*x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2048, 2050, 2036, 335, 226}

$$-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}+\frac{1}{ax\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x*Sqrt[a*x + b*x^3]) - (5*Sqrt[a*x + b*x^3])/(3*a^2*x^2) - (5*b^(3/4)*
Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*a^(9/4)*Sqrt[a*x + b*x
^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = \frac{1}{ax\sqrt{ax+bx^3}} + \frac{5\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx}{2a}$$

$$= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{(5b)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{6a^2}$$

$$= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6a^2\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \ldots\right.}{3a^2\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(\ldots}{6a^{9/4}\sqrt{ax+bx^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.40

$$-\frac{2\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/(3
*a*x*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.41, size = 150, normalized size = 1.08

method	result
default	$-\frac{bx}{a^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^3+ax}}{3a^2x^2}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{6a^2\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{bx}{a^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^3+ax}}{3a^2x^2}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{6a^2\sqrt{b\,x^3+ax}}$
risch	$-\frac{2\left(b\,x^2+a\right)}{3a^2x\sqrt{x\left(b\,x^2+a\right)}}-\frac{b\left(\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{b\sqrt{b\,x^3+ax}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*x/a^2/((x^2+a/b)*b*x)^(1/2)-2/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6/a^2*(-a*b)
^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))
```

```
*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*Elliptic
F(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 68, normalized size = 0.49

$$-\frac{5\,(bx^4+ax^2)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}\,(5\,bx^2+2\,a)}{3\,(a^2bx^4+a^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(b*x^4 + a*x^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*
x^3 + a*x)*(5*b*x^2 + 2*a))/(a^2*b*x^4 + a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(x*(a + b*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,(b\,x^3+a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)^(3/2)),x)
[Out] int(1/(x*(a*x + b*x^3)^(3/2)), x)
```

3.74 $\int \frac{1}{x^2(ax+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=306

$$\frac{1}{ax^2\sqrt{ax+bx^3}}-\frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{7\sqrt{ax+bx^3}}{5a^2x^3}+\frac{21b\sqrt{ax+bx^3}}{5a^3x}+\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\right.}{}$$

```
[Out] 1/a/x^2/(b*x^3+a*x)^(1/2)-21/5*b^(3/2)*x*(b*x^2+a)/a^3/(a^(1/2)+x*b^(1/2))/
(b*x^3+a*x)^(1/2)-7/5*(b*x^3+a*x)^(1/2)/a^2/x^3+21/5*b*(b*x^3+a*x)^(1/2)/a^
3/x+21/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))
^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(1/2)-21/10*b^(5/4)*(cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2048, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}+\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+bx^3}}-\frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{21b\sqrt{ax+bx^3}}{5a^3x}-\frac{7\sqrt{ax+bx^3}}{5a^2x^3}+\frac{1}{ax^2\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x^2*Sqrt[a*x + b*x^3]) - (21*b^(3/2)*x*(a + b*x^2))/(5*a^3*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (7*Sqrt[a*x + b*x^3])/(5*a^2*x^3) + (21*b*S
qrt[a*x + b*x^3])/(5*a^3*x) + (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[a*x + b*x^3]) - (21*b^(5/4)*Sqrt[x]*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10*a^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^2 (ax+bx^3)^{3/2}} dx = \frac{1}{ax^2\sqrt{ax+bx^3}} + \frac{7\int \frac{1}{x^3\sqrt{ax+bx^3}} dx}{2a}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} - \frac{(21b)\int \frac{1}{x\sqrt{ax+bx^3}} dx}{10a^2}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{(21b^2)\int \frac{x}{\sqrt{ax+bx^3}} dx}{10a^3}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{}{\sqrt{a}}}{10a^3\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}}{5a^3\sqrt{ax}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Sub}}{5a^{5/2}\sqrt{a}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+b}}{5a^3x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.18

$$-\frac{2\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^2\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])/(
5*a*x^2*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.41, size = 228, normalized size = 0.75

method	result
default	$\frac{b^2x^2}{a^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^3+ax}}{5a^2x^3}+\frac{16(b\,x^2+a)b}{5a^3\sqrt{x\,(b\,x^2+a)}}-\frac{21b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{}\right.}{\sqrt{-a}}}}{}$
elliptic	$\frac{b^2x^2}{a^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^3+ax}}{5a^2x^3}+\frac{16(b\,x^2+a)b}{5a^3\sqrt{x\,(b\,x^2+a)}}-\frac{21b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{}\right.}{\sqrt{-a}}}}{}$
risch	$-\frac{2(b\,x^2+a)\,(-8b\,x^2+a)}{5a^3x^2\sqrt{x\,(b\,x^2+a)}}-\frac{b^2\left(\frac{8\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}}{}\right.}{b\sqrt{b\,x^{}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] b^2*x^2/a^3/((x^2+a/b)*b*x)^(1/2)-2/5*(b*x^3+a*x)^(1/2)/a^2/x^3+16/5*(b*x^2
```

```
+a)/a^3*b/(x*(b*x^2+a))^(1/2)-21/10*b/a^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/
b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*Ellipti
cF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 90, normalized size = 0.29

$$\frac{21\left(b^2x^5+abx^3\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(21\,b^2x^4+14\,abx^2-2\,a^2\right)\sqrt{bx^3+ax}}{5\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(21*(b^2*x^5 + a*b*x^3)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassP
Inverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 14*a*b*x^2 - 2*a^2)*sqrt(b*x^3 + a*x
))/(a^3*b*x^5 + a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x*(a + b*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)^(3/2)),x)
[Out] int(1/(x^2*(a*x + b*x^3)^(3/2)), x)
```

3.75 $\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{x^{25/2}}{7b\,(ax+bx^3)^{7/2}}-\frac{9x^{19/2}}{35b^2\,(ax+bx^3)^{5/2}}-\frac{3x^{13/2}}{5b^3\,(ax+bx^3)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\,\sqrt{ax+bx^3}}{2b^5}-\frac{9a\tanh^{-1}\left(\right.}{2b}$$

```
[Out] -1/7*x^(25/2)/b/(b*x^3+a*x)^(7/2)-9/35*x^(19/2)/b^2/(b*x^3+a*x)^(5/2)-3/5*x
^(13/2)/b^3/(b*x^3+a*x)^(3/2)-9/2*a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(1/
2))/b^(11/2)-3*x^(7/2)/b^4/(b*x^3+a*x)^(1/2)+9/2*x^(1/2)*(b*x^3+a*x)^(1/2)/
b^5
```

Rubi [A]
time = 0.16, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2047, 2049, 2054, 212}

$$-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}+\frac{9\sqrt{x}\,\sqrt{ax+bx^3}}{2b^5}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}-\frac{3x^{13/2}}{5b^3\,(ax+bx^3)^{3/2}}-\frac{9x^{19/2}}{35b^2\,(ax+bx^3)^{5/2}}-\frac{x^{25/2}}{7b\,(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(25/2)/(b*(a*x + b*x^3)^(7/2)) - (9*x^(19/2))/(35*b^2*(a*x + b*x^3)^
(5/2)) - (3*x^(13/2))/(5*b^3*(a*x + b*x^3)^(3/2)) - (3*x^(7/2))/(b^4*Sqrt[a
*x + b*x^3]) + (9*Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b
]*x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx &= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}+\frac{9\int \frac{x^{23/2}}{(ax+bx^3)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}+\frac{9\int \frac{x^{17/2}}{(ax+bx^3)^{5/2}}\,dx}{5b^2}\\
&= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}}+\frac{3\int \frac{x^{11/2}}{(ax+bx^3)^{3/2}}\,dx}{b^3}\\
&= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\int \cdots}{\cdots}\\
&= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\cdots}{\cdots}\\
&= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\cdots}{\cdots}\\
&= -\frac{x^{25/2}}{7b(ax+bx^3)^{7/2}}-\frac{9x^{19/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{3x^{13/2}}{5b^3(ax+bx^3)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\cdots}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 122, normalized size = 0.77

$$\frac{\sqrt{x}\left(\sqrt{b}\,x(315a^4+1050a^3bx^2+1218a^2b^2x^4+528ab^3x^6+35b^4x^8)+315a(a+bx^2)^{7/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{70b^{11/2}(a+bx^2)^3\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^
3*x^6 + 35*b^4*x^8) + 315*a*(a + b*x^2)^(7/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]]))/(70*b^(11/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.42, size = 212, normalized size = 1.33

method	result
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(-35x^{9}b^{\frac{9}{2}}+315\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)a\,b^{3}x^{6}\sqrt{b\,x^{2}+a}-528b^{\frac{7}{2}}a\,x^{7}+945\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+\right.\right.}{}$
risch	$\frac{x^{\frac{3}{2}}\left(b\,x^{2}+a\right)}{2b^{5}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{\left(-\frac{9a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{11}{2}}}+\frac{a^{3}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right.}}{112b^{7}\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)^{4}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(29/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/70*(x*(b*x^2+a))^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))*a*b^3*x^6*(b*x^2+a)^(1/2)-528*b^(7/2)*a*x^7+945*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*b^(5/2)*a^2*x^5+945*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))*a^3*b*x^2*(b*x^2+a)^(1/2)-1050*b^(3/2)*a^3*x^3+315*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*a^4*x)/x^(1/2)/
(b*x^2+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 2.84, size = 376, normalized size = 2.36

$$\left[\frac{315\,(ab^{4}x^{8}+4\,a^{2}b^{3}x^{6}+6\,a^{3}b^{2}x^{4}+4\,a^{4}bx^{2}+a^{5})\sqrt{b}\,\log\left(2\,bx^{2}-2\,\sqrt{bx^{3}+ax}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(35\,b^{5}x^{8}+528\,ab^{4}x^{6}+1218\,a^{2}b^{3}x^{4}+1050\,a^{3}b^{2}x^{2}+315\,a^{4}b)\sqrt{bx^{3}+ax}\,\sqrt{x}}{140\,(b^{10}x^{8}+4\,ab^{9}x^{6}+6\,a^{2}b^{8}x^{4}+4\,a^{3}b^{7}x^{2}+a^{4}b^{6})},\ \frac{315\,(ab^{4}x^{8}+4\,a^{2}b^{3}x^{6}+6\,a^{3}b^{2}x^{4}+4\,a^{4}bx^{2}+a^{5})\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^{3}+ax}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+(35\,b^{5}x^{8}+528\,ab^{4}x^{6}+1218\,a^{2}b^{3}x^{4}+1050\,a^{3}b^{2}x^{2}+315\,a^{4}b)\sqrt{bx^{3}+ax}\,\sqrt{x}}{70\,(b^{10}x^{8}+4\,ab^{9}x^{6}+6\,a^{2}b^{8}x^{4}+4\,a^{3}b^{7}x^{2}+a^{4}b^{6})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b^5
*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*sqr
t(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7
*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a
^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (
35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*
b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a
^3*b^7*x^2 + a^4*b^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.51, size = 100, normalized size = 0.63

$$\frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\,(bx^2+a)^{\frac{7}{2}}}+\frac{9\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{2\,b^{\frac{11}{2}}}-\frac{9\,a\log\left(|a|\right)}{4\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2) - 9/4*a*log(abs(a))/b^(11/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{29/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(29/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(29/2)/(a*x + b*x^3)^(9/2), x)
```

3.76 $\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}} dx$

Optimal. Leaf size=126

$$-\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}-\frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}$$

```
[Out] -1/7*x^(23/2)/b/(b*x^3+a*x)^(7/2)-8/35*x^(17/2)/b^2/(b*x^3+a*x)^(5/2)-16/35
*x^(11/2)/b^3/(b*x^3+a*x)^(3/2)-64/35*x^(5/2)/b^4/(b*x^3+a*x)^(1/2)+128/35*
(b*x^3+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}-\frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}}-\frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}}-\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(23/2)/(b*(a*x + b*x^3)^(7/2)) - (8*x^(17/2))/(35*b^2*(a*x + b*x^3)^
(5/2)) - (16*x^(11/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (64*x^(5/2))/(35*b^4*
Sqrt[a*x + b*x^3]) + (128*Sqrt[a*x + b*x^3])/(35*b^5*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{8\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} + \frac{48\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{12}{}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{12}{}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.54

$$\frac{x^{7/2}\left(128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8\right)}{35b^5\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^
4*x^8))/(35*b^5*(x*(a + b*x^2))^(7/2))
```

Maple [A]
time = 0.39, size = 72, normalized size = 0.57

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)\left(35b^{4}x^{8}+280a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+448a^{3}b\,x^{2}+128a^{4}\right)x^{\frac{9}{2}}}{35b^{5}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	70
default	$\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(35b^{4}x^{8}+280a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+448a^{3}b\,x^{2}+128a^{4}\right)}{35\sqrt{x}\,\left(b\,x^{2}+a\right)^{4}b^{5}}$	72
risch	$\frac{\left(b\,x^{2}+a\right)\sqrt{x}}{b^{5}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{\left(b\,x^{2}+a\right)\left(140b^{3}x^{6}+350a\,b^{2}x^{4}+308a^{2}b\,x^{2}+93a^{3}\right)a\sqrt{x}}{35b^{5}\left(b^{4}x^{8}+4a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}+4a^{3}b\,x^{2}+a^{4}\right)\sqrt{x\left(b\,x^{2}+a\right)}}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(27/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35/x^(1/2)*(x*(b*x^2+a))^(1/2)*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+
448*a^3*b*x^2+128*a^4)/(b*x^2+a)^4/b^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [**A**]
time = 2.14, size = 108, normalized size = 0.86

$$\frac{\left(35\,b^{4}x^{8}+280\,ab^{3}x^{6}+560\,a^{2}b^{2}x^{4}+448\,a^{3}bx^{2}+128\,a^{4}\right)\sqrt{bx^{3}+ax}\,\sqrt{x}}{35\,\left(b^{9}x^{9}+4\,ab^{8}x^{7}+6\,a^{2}b^{7}x^{5}+4\,a^{3}b^{6}x^{3}+a^{4}b^{5}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^9*x^9 + 4*a*b^8*x^7 + 6*a^2*b^7*x^5 + 4*a^3
*b^6*x^3 + a^4*b^5*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

Giac [**A**]
time = 1.49, size = 80, normalized size = 0.63

$$\frac{\sqrt{bx^{2}+a}}{b^{5}}-\frac{128\,\sqrt{a}}{35\,b^{5}}+\frac{140\,{\left(bx^{2}+a\right)}^{3}a-70\,{\left(bx^{2}+a\right)}^{2}a^{2}+28\,{\left(bx^{2}+a\right)}a^{3}-5\,a^{4}}{35\,{\left(bx^{2}+a\right)}^{\frac{7}{2}}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] sqrt(b*x^2 + a)/b^5 - 128/35*sqrt(a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*(
b*x^2 + a)^2*a^2 + 28*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{27/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(27/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(27/2)/(a*x + b*x^3)^(9/2), x)
```

3.77 $\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}} dx$

Optimal. Leaf size=130

$$-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}$$

```
[Out] -1/7*x^(21/2)/b/(b*x^3+a*x)^(7/2)-1/5*x^(15/2)/b^2/(b*x^3+a*x)^(5/2)-1/3*x^
(9/2)/b^3/(b*x^3+a*x)^(3/2)+arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(1/2))/b^(9
/2)-x^(3/2)/b^4/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2047, 2054, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(21/2)/(b*(a*x + b*x^3)^(7/2)) - x^(15/2)/(5*b^2*(a*x + b*x^3)^(5/2)
) - x^(9/2)/(3*b^3*(a*x + b*x^3)^(3/2)) - x^(3/2)/(b^4*Sqrt[a*x + b*x^3]) +
 ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}} + \frac{\int \frac{x^{19/2}}{(ax+bx^3)^{7/2}}\,dx}{b}$$

$$= -\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax+bx^3)^{5/2}} + \frac{\int \frac{x^{13/2}}{(ax+bx^3)^{5/2}}\,dx}{b^2}$$

$$= -\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax+bx^3)^{3/2}} + \frac{\int \frac{x^{7/2}}{(ax+bx^3)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\int \frac{\sqrt{\ }}{\sqrt{ax}}}{}$$

$$= -\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\text{Subst}\Big(}{}$$

$$= -\frac{x^{21/2}}{7b(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\tanh^{-1}}{}$$

Mathematica [A]
time = 0.16, size = 110, normalized size = 0.85

$$-\frac{\sqrt{x}\left(\sqrt{b}\,x(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6)+105(a+bx^2)^{7/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{105b^{9/2}(a+bx^2)^3\sqrt{x(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(25/2)/(a*x + b*x^3)^(9/2),x]
[Out] -1/105*(Sqrt[x]*(Sqrt[b]*x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b
^3*x^6) + 105*(a + b*x^2)^(7/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(b^(9
/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

Maple [A]
time = 0.35, size = 198, normalized size = 1.52

method	result
default	$\frac{\sqrt{x\,(b\,x^2+a)}\,\left(105\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)b^3x^6\sqrt{b\,x^2+a}-176x^7b^{\frac{7}{2}}+315\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a\,b^2x^4\sqrt{b\,x}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(25/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(x*(b*x^2+a))^(1/2)/b^(9/2)*(105*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*b^3*x^
6*(b*x^2+a)^(1/2)-176*x^7*b^(7/2)+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a*b^2*x
^4*(b*x^2+a)^(1/2)-406*b^(5/2)*a*x^5+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a^2*
b*x^2*(b*x^2+a)^(1/2)-350*b^(3/2)*a^2*x^3+105*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
*a^3*(b*x^2+a)^(1/2)-105*b^(1/2)*a^3*x)/x^(1/2)/(b*x^2+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(25/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 3.43, size = 348, normalized size = 2.68

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\,\log\left(2\,bx^2+2\,\sqrt{bx^3+ax}\,\sqrt{b}\,\sqrt{x}+a\right)-2\left(176\,b^4x^6+406\,ab^3x^4+350\,a^2b^2x^2+105\,a^3b\right)\sqrt{bx^3+ax}\,\sqrt{x}}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)},\ -\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^3+ax}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\left(176\,b^4x^6+406\,ab^3x^4+350\,a^2b^2x^2+105\,a^3b\right)\sqrt{bx^3+ax}\,\sqrt{x}}{105\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*x^
6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))
/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*
(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*a
rctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (176*b^4*x^6 + 406*a*b^3*x^
4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8 + 4*a*
b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.45, size = 86, normalized size = 0.66

$$-\frac{\left(2\left(x^{2}\left(\frac{88\,x^{2}}{b}+\frac{203\,a}{b^{2}}\right)+\frac{175\,a^{2}}{b^{3}}\right)x^{2}+\frac{105\,a^{3}}{b^{4}}\right)x}{105\,(bx^{2}+a)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{b^{\frac{9}{2}}}+\frac{\log\left(|a|\right)}{2\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2) + 1/2*log
(abs(a))/b^(9/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{25/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(25/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(25/2)/(a*x + b*x^3)^(9/2), x)
```

3.78 $\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}$$

```
[Out] -1/7*x^(19/2)/b/(b*x^3+a*x)^(7/2)-6/35*x^(13/2)/b^2/(b*x^3+a*x)^(5/2)-8/35*
x^(7/2)/b^3/(b*x^3+a*x)^(3/2)-16/35*x^(1/2)/b^4/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(19/2)/(b*(a*x + b*x^3)^(7/2)) - (6*x^(13/2))/(35*b^2*(a*x + b*x^3)^
(5/2)) - (8*x^(7/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (16*Sqrt[x])/(35*b^4*Sq
rt[a*x + b*x^3])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{19/2}}{7b\,(ax+bx^3)^{7/2}} + \frac{6\int \frac{x^{17/2}}{(ax+bx^3)^{7/2}}\,dx}{7b}$$
$$= -\frac{x^{19/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2\,(ax+bx^3)^{5/2}} + \frac{24\int \frac{x^{11/2}}{(ax+bx^3)^{5/2}}\,dx}{35b^2}$$
$$= -\frac{x^{19/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2\,(ax+bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3\,(ax+bx^3)^{3/2}} + \frac{16\int \frac{x^{5/2}}{(ax+bx^3)^{3/2}}\,dx}{35b^3}$$
$$= -\frac{x^{19/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2\,(ax+bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3\,(ax+bx^3)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.63

$$\frac{x^{9/2}(a+bx^2)\left(-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6\right)}{35b^4\left(x\left(a+bx^2\right)\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)*(-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6))/
(35*b^4*(x*(a + b*x^2))^(9/2))
```

Maple [A]
time = 0.35, size = 61, normalized size = 0.60

method	result	size
gosper	$-\frac{(b\,x^2+a)\left(35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3\right)x^{\frac{9}{2}}}{35b^4(b\,x^3+ax)^{\frac{9}{2}}}$	59
default	$-\frac{\sqrt{x\,(b\,x^2+a)}\,\left(35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3\right)}{35\sqrt{x}\,(b\,x^2+a)^4b^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35/x^(1/2)*(x*(b*x^2+a))^(1/2)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*
a^3)/(b*x^2+a)^4/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.85, size = 97, normalized size = 0.96

$$-\frac{(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3)\sqrt{bx^3+ax}\,\sqrt{x}}{35\,(b^8x^9+4\,ab^7x^7+6\,a^2b^6x^5+4\,a^3b^5x^3+a^4b^4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^3 + a*x)
*sqrt(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4*x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.29, size = 64, normalized size = 0.63

$$\frac{16}{35\,\sqrt{a}\,b^4}-\frac{35\,(bx^2+a)^3-35\,(bx^2+a)^2a+21\,(bx^2+a)a^2-5\,a^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 16/35/(sqrt(a)*b^4) - 1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x
^2 + a)*a^2 - 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{23/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(23/2)/(a*x + b*x^3)^(9/2), x)
```

3.79 $\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=25

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

[Out] 1/7*x^(21/2)/a/(b*x^3+a*x)^(7/2)

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^(21/2)/(a*x + b*x^3)^(9/2),x]

[Out] x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Mathematica [A]
time = 0.08, size = 25, normalized size = 1.00

$$\frac{x^{21/2}}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]

```
[Out] x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))
```

Maple [A]
time = 0.35, size = 29, normalized size = 1.16

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{23}{2}}}{7a(b\,x^3+ax)^{\frac{9}{2}}}$	27
default	$\frac{x^{\frac{13}{2}}\sqrt{x\,(b\,x^2+a)}}{7a(b\,x^2+a)^4}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/7*x^(13/2)*(x*(b*x^2+a))^(1/2)/a/(b*x^2+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.
time = 1.26, size = 61, normalized size = 2.44

$$\frac{\sqrt{bx^3+ax}\,x^{\frac{13}{2}}}{7\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 +
 4*a^4*b*x^2 + a^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.39, size = 17, normalized size = 0.68

$$\frac{x^7}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{21/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(21/2)/(a*x + b*x^3)^(9/2), x)
```

3.80 $\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}$$

```
[Out] -1/7*x^(15/2)/b/(b*x^3+a*x)^(7/2)-4/35*x^(9/2)/b^2/(b*x^3+a*x)^(5/2)-8/105*
x^(3/2)/b^3/(b*x^3+a*x)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(15/2)/(b*(a*x + b*x^3)^(7/2)) - (4*x^(9/2))/(35*b^2*(a*x + b*x^3)^(
5/2)) - (8*x^(3/2))/(105*b^3*(a*x + b*x^3)^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}} + \frac{4\int \frac{x^{13/2}}{(ax+bx^3)^{7/2}}\,dx}{7b}$$
$$= -\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{4x^{9/2}}{35b^2\,(ax+bx^3)^{5/2}} + \frac{8\int \frac{x^{7/2}}{(ax+bx^3)^{5/2}}\,dx}{35b^2}$$
$$= -\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{4x^{9/2}}{35b^2\,(ax+bx^3)^{5/2}} - \frac{8x^{3/2}}{105b^3\,(ax+bx^3)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.61

$$\frac{x^{7/2}(-8a^2-28abx^2-35b^2x^4)}{105b^3\,(x\,(a+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*(-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4))/(105*b^3*(x*(a + b*x^2))^(7/2)
)
```

Maple [A]
time = 0.37, size = 50, normalized size = 0.66

method	result	size
gosper	$-\frac{(b\,x^2+a)\,(35b^2x^4+28ab\,x^2+8a^2)\,x^{\frac{9}{2}}}{105b^3\,(b\,x^3+ax)^{\frac{9}{2}}}$	48
default	$-\frac{\sqrt{x\,(b\,x^2+a)}\,(35b^2x^4+28ab\,x^2+8a^2)}{105\sqrt{x}\,(b\,x^2+a)^4\,b^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/x^(1/2)*(x*(b*x^2+a))^(1/2)*(35*b^2*x^4+28*a*b*x^2+8*a^2)/(b*x^2+a)^
4/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 2.23, size = 86, normalized size = 1.13

$$-\frac{(35\,b^2x^4+28\,abx^2+8\,a^2)\sqrt{bx^3+ax}\,\sqrt{x}}{105\,(b^7x^9+4\,ab^6x^7+6\,a^2b^5x^5+4\,a^3b^4x^3+a^4b^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^9
 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 10661 deep
```

Giac [A]
time = 1.53, size = 50, normalized size = 0.66

$$\frac{8}{105\,a^{\frac{3}{2}}b^3}-\frac{35\,(bx^2+a)^2-42\,(bx^2+a)a+15\,a^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 8/105/(a^(3/2)*b^3) - 1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/
((b*x^2 + a)^(7/2)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(19/2)/(a*x + b*x^3)^(9/2), x)
```

3.81 $\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}$$

```
[Out] 1/7*x^(17/2)/a/(b*x^3+a*x)^(7/2)+2/35*x^(15/2)/a^2/(b*x^3+a*x)^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5/
2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{2\int \frac{x^{15/2}}{(ax+bx^3)^{7/2}}\,dx}{7a}$$
$$= \frac{x^{17/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{2x^{15/2}}{35a^2\,(ax+bx^3)^{5/2}}$$

Mathematica [A]
time = 0.08, size = 38, normalized size = 0.75

$$\frac{x^{7/2}(7ax^5+2bx^7)}{35a^2\,(x\,(a+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(a*x + b*x^3)^(9/2),x]
[Out] (x^(7/2)*(7*a*x^5 + 2*b*x^7))/(35*a^2*(x*(a + b*x^2))^(7/2))
```

Maple [A]
time = 0.36, size = 39, normalized size = 0.76

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{19}{2}}(2b\,x^2+7a)}{35a^2(b\,x^3+ax)^{\frac{9}{2}}}$	37
default	$\frac{x^{\frac{9}{2}}\sqrt{x\,(b\,x^2+a)}\,(2b\,x^2+7a)}{35a^2(b\,x^2+a)^4}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/35*x^(9/2)*(x*(b*x^2+a))^(1/2)*(2*b*x^2+7*a)/a^2/(b*x^2+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.27, size = 76, normalized size = 1.49

$$\frac{(2\,bx^6+7\,ax^4)\sqrt{bx^3+ax}\,\sqrt{x}}{35\,(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^3
*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7771 deep
```

Giac [A]
time = 1.37, size = 29, normalized size = 0.57

$$\frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\,(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{17/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(17/2)/(a*x + b*x^3)^(9/2), x)
```

3.82 $\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}$$

```
[Out] -1/7*x^(11/2)/b/(b*x^3+a*x)^(7/2)-2/35*x^(5/2)/b^2/(b*x^3+a*x)^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(11/2)/(b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^(
5/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}} + \frac{2\int \frac{x^{9/2}}{(ax+bx^3)^{7/2}}\,dx}{7b}$$
$$= -\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{2x^{5/2}}{35b^2\,(ax+bx^3)^{5/2}}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.69

$$\frac{x^{7/2}(-2a-7bx^2)}{35b^2\,(x\,(a+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a*x + b*x^3)^(9/2),x]
[Out] (x^(7/2)*(-2*a - 7*b*x^2))/(35*b^2*(x*(a + b*x^2))^(7/2))
```

Maple [A]
time = 0.36, size = 39, normalized size = 0.76

method	result	size
gosper	$-\frac{(b\,x^2+a)\,(7b\,x^2+2a)\,x^{\frac{9}{2}}}{35b^2\,(b\,x^3+ax)^{\frac{9}{2}}}$	37
default	$-\frac{\sqrt{x\,(b\,x^2+a)}\,(7b\,x^2+2a)}{35\sqrt{x}\,(b\,x^2+a)^4\,b^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/35/x^(1/2)*(x*(b*x^2+a))^(1/2)*(7*b*x^2+2*a)/(b*x^2+a)^4/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.75, size = 75, normalized size = 1.47

$$-\frac{\sqrt{bx^3+ax}\,(7\,bx^2+2\,a)\sqrt{x}}{35\,(b^6x^9+4\,ab^5x^7+6\,a^2b^4x^5+4\,a^3b^3x^3+a^4b^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6*
a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Giac [A]
time = 1.62, size = 33, normalized size = 0.65

$$-\frac{7\,bx^2+2\,a}{35\,(bx^2+a)^{\frac{7}{2}}b^2}+\frac{2}{35\,a^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2) + 2/35/(a^(5/2)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{15/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(15/2)/(a*x + b*x^3)^(9/2), x)
```

3.83 $\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=76

$$\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}$$

```
[Out] 1/7*x^(13/2)/a/(b*x^3+a*x)^(7/2)+4/35*x^(11/2)/a^2/(b*x^3+a*x)^(5/2)+8/105*
x^(9/2)/a^3/(b*x^3+a*x)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*x^(11/2))/(35*a^2*(a*x + b*x^3)^(5/
2)) + (8*x^(9/2))/(105*a^3*(a*x + b*x^3)^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{13/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{4\int \frac{x^{11/2}}{(ax+bx^3)^{7/2}}\,dx}{7a}$$
$$= \frac{x^{13/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{4x^{11/2}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{8\int \frac{x^{9/2}}{(ax+bx^3)^{5/2}}\,dx}{35a^2}$$
$$= \frac{x^{13/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{4x^{11/2}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{8x^{9/2}}{105a^3\,(ax+bx^3)^{3/2}}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.74

$$\frac{x^{9/2}(a+bx^2)\,(35a^2x^3+28abx^5+8b^2x^7)}{105a^3\,(x\,(a+bx^2))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)*(35*a^2*x^3 + 28*a*b*x^5 + 8*b^2*x^7))/(105*a^3*(x*(a
+ b*x^2))^(9/2))
```

Maple [A]
time = 0.36, size = 50, normalized size = 0.66

method	result	size
gosper	$\frac{(b\,x^2+a)\,x^{\frac{15}{2}}\,(8b^2x^4+28ab\,x^2+35a^2)}{105a^3(b\,x^3+ax)^{\frac{9}{2}}}$	48
default	$\frac{x^{\frac{5}{2}}\,\sqrt{x\,(b\,x^2+a)}\,(8b^2x^4+28ab\,x^2+35a^2)}{105a^3(b\,x^2+a)^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*x^(5/2)*(x*(b*x^2+a))^(1/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^2+
a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.02, size = 87, normalized size = 1.14

$$\frac{(8\,b^2x^6 + 28\,abx^4 + 35\,a^2x^2)\sqrt{bx^3 + ax}\,\sqrt{x}}{105\,(a^3b^4x^8 + 4\,a^4b^3x^6 + 6\,a^5b^2x^4 + 4\,a^6bx^2 + a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3*
b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [A]
time = 0.99, size = 43, normalized size = 0.57

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3} + \frac{7\,b}{a^2}\right) + \frac{35}{a}\right)x^3}{105\,(bx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13/2}}{(b\,x^3 + a\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(13/2)/(a*x + b*x^3)^(9/2), x)
```

3.84 $\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(7/2)/b/(b*x^3+a*x)^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(7/2)/(b*(a*x + b*x^3)^(7/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-\frac{x^{7/2}}{7b\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(7/2)/(b*(x*(a + b*x^2))^(7/2))
```

Maple [A]
time = 0.37, size = 29, normalized size = 1.16

method	result	size
gosper	$-\frac{(b\,x^{2}+a)x^{\frac{9}{2}}}{7b(b\,x^{3}+ax)^{\frac{9}{2}}}$	27
default	$-\frac{\sqrt{x\,(b\,x^{2}+a)}}{7\sqrt{x}\,\,(b\,x^{2}+a)^{4}b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/x^(1/2)*(x*(b*x^2+a))^(1/2)/(b*x^2+a)^4/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.
time = 1.62, size = 63, normalized size = 2.52

$$-\frac{\sqrt{bx^3+ax}\,\sqrt{x}}{7\left(b^5x^9+4\,ab^4x^7+6\,a^2b^3x^5+4\,a^3b^2x^3+a^4bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*a
^3*b^2*x^3 + a^4*b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
[Out] Integral(x**(11/2)/(x*(a + b*x**2))**(9/2), x)
```

Giac [A]
time = 0.96, size = 23, normalized size = 0.92

$$-\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{1}{7\,a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] -1/7/((b*x^2 + a)^(7/2)*b) + 1/7/(a^(7/2)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{11/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(11/2)/(a*x + b*x^3)^(9/2), x)
```

3.85 $\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=101

$$\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}$$

```
[Out] 1/7*x^(9/2)/a/(b*x^3+a*x)^(7/2)+6/35*x^(7/2)/a^2/(b*x^3+a*x)^(5/2)+8/35*x^(
5/2)/a^3/(b*x^3+a*x)^(3/2)+16/35*x^(3/2)/a^4/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*x^(7/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (8*x^(5/2))/(35*a^3*(a*x + b*x^3)^(3/2)) + (16*x^(3/2))/(35*a^4*Sqrt[a*
x + b*x^3])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{6\int \frac{x^{7/2}}{(ax+bx^3)^{7/2}}\,dx}{7a}$$
$$= \frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{24\int \frac{x^{5/2}}{(ax+bx^3)^{5/2}}\,dx}{35a^2}$$
$$= \frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3\,(ax+bx^3)^{3/2}} + \frac{16\int \frac{x^{3/2}}{(ax+bx^3)^{3/2}}\,dx}{35a^3}$$
$$= \frac{x^{9/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{6x^{7/2}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{8x^{5/2}}{35a^3\,(ax+bx^3)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 0.57

$$\frac{x^{7/2}(35a^3x+70a^2bx^3+56ab^2x^5+16b^3x^7)}{35a^4\,(x\,(a+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7))/(35*a^4*(x*
(a + b*x^2))^(7/2))
```

Maple [A]
time = 0.36, size = 61, normalized size = 0.60

method	result	size
gosper	$\frac{(b\,x^2+a)x^{\frac{11}{2}}\,(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3)}{35a^4(b\,x^3+ax)^{\frac{9}{2}}}$	59
default	$\frac{\sqrt{x}\,\sqrt{x\,(b\,x^2+a)}\,(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3)}{35(b\,x^2+a)^4a^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x^(1/2)*(x*(b*x^2+a))^(1/2)*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a
^3)/(b*x^2+a)^4/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.36, size = 95, normalized size = 0.94

$$\frac{\left(16\, b^3 x^6 + 56\, a b^2 x^4 + 70\, a^2 b x^2 + 35\, a^3\right) \sqrt{b x^3 + a x}\, \sqrt{x}}{35\, \left(a^4 b^4 x^8 + 4\, a^5 b^3 x^6 + 6\, a^6 b^2 x^4 + 4\, a^7 b x^2 + a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*sqrt(b*x^3 + a*x)*
sqrt(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\left(x\left(a + b x^2\right)\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(9/2)/(x*(a + b*x**2))**(9/2), x)
```

Giac [A]
time = 0.97, size = 55, normalized size = 0.54

$$\frac{\left(2\left(4\, x^2 \left(\frac{2\, b^3 x^2}{a^4} + \frac{7\, b^2}{a^3}\right) + \frac{35\, b}{a^2}\right) x^2 + \frac{35}{a}\right) x}{35\, \left(b x^2 + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}}{\left(b\, x^3 + a\, x\right)^{9/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(9/2)/(a*x + b*x^3)^(9/2), x)
```

3.86 $\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=130

$$\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}$$

```
[Out] 1/7*x^(7/2)/a/(b*x^3+a*x)^(7/2)+1/5*x^(5/2)/a^2/(b*x^3+a*x)^(5/2)+1/3*x^(3/
2)/a^3/(b*x^3+a*x)^(3/2)-arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(9/2)
+x^(1/2)/a^4/(b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2048, 2054, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + x^(5/2)/(5*a^2*(a*x + b*x^3)^(5/2)) + x
^(3/2)/(3*a^3*(a*x + b*x^3)^(3/2)) + Sqrt[x]/(a^4*Sqrt[a*x + b*x^3]) - ArcT
anh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{\int \frac{x^{5/2}}{(ax+bx^3)^{7/2}}\,dx}{a}$$

$$= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{\int \frac{x^{3/2}}{(ax+bx^3)^{5/2}}\,dx}{a^2}$$

$$= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\int \frac{\sqrt{x}}{(ax+bx^3)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} + \frac{\int \frac{}{\sqrt{x}\sqrt{}}}{}$$

$$= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\text{Subst}\Big(}{}$$

$$= \frac{x^{7/2}}{7a(ax+bx^3)^{7/2}} + \frac{x^{5/2}}{5a^2(ax+bx^3)^{5/2}} + \frac{x^{3/2}}{3a^3(ax+bx^3)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\tanh^{-1}\Big(}{}$$

Mathematica [A]
time = 0.09, size = 106, normalized size = 0.82

$$\frac{\sqrt{x}\left(\sqrt{a}\,(176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6)-105(a+bx^2)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{105a^{9/2}\,(a+bx^2)^3\,\sqrt{x\,(a+bx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(Sqrt[a]*(176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6) -
 105*(a + b*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(105*a^(9/2)*(a +
 b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(100) = 200.
time = 0.50, size = 217, normalized size = 1.67

method	result
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(105\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)b^{3}x^{6}\sqrt{b\,x^{2}+a}-105\sqrt{a}\,b^{3}x^{6}+315\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(x*(b*x^2+a))^(1/2)/a^(9/2)*(105*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)
*b^3*x^6*(b*x^2+a)^(1/2)-105*a^(1/2)*b^3*x^6+315*ln(2*(a^(1/2)*(b*x^2+a)^(1
/2)+a)/x)*a*b^2*x^4*(b*x^2+a)^(1/2)-350*a^(3/2)*b^2*x^4+315*ln(2*(a^(1/2)*(
b*x^2+a)^(1/2)+a)/x)*a^2*b*x^2*(b*x^2+a)^(1/2)-406*b*x^2*a^(5/2)+105*ln(2*(
a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*(b*x^2+a)^(1/2)-176*a^(7/2))/x^(1/2)/(b*x
^2+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.44, size = 360, normalized size = 2.77

$$\left[\frac{105\,(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x)\sqrt{a}\,\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\,\sqrt{a}\,\sqrt{x}}{x^3}\right)+2\,(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3bx^2+176\,a^4)\sqrt{bx^3+ax}\,\sqrt{x}}{210\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x)},\ \frac{105\,(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax}\,\sqrt{-a}}{a\sqrt{x}}\right)+(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3bx^2+176\,a^4)\sqrt{bx^3+ax}\,\sqrt{x}}{105\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*s
qrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3) + 2*(
105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x
)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9
*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*
x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))/(a
^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\left(x\left(a+bx^{2}\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(7/2)/(x*(a + b*x**2))**(9/2), x)
```

Giac [A]
time = 0.80, size = 114, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}-\frac{105\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+176\,\sqrt{-a}}{105\,\sqrt{-a}\,a^{\frac{9}{2}}}+\frac{105\,(bx^2+a)^3+35\,(bx^2+a)^2a+21\,(bx^2+a)a^2+15\,a^3}{105\,(bx^2+a)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) - 1/105*(105*sqrt(a)*arctan
(sqrt(a)/sqrt(-a)) + 176*sqrt(-a))/(sqrt(-a)*a^(9/2)) + 1/105*(105*(b*x^2 +
 a)^3 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2
)*a^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(7/2)/(a*x + b*x^3)^(9/2), x)
```

3.87 $\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}} \, dx$

Optimal. Leaf size=126

$$\frac{x^{5/2}}{7a\,(ax+bx^3)^{7/2}}+\frac{8x^{3/2}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{16\sqrt{x}}{35a^3\,(ax+bx^3)^{3/2}}+\frac{64}{35a^4\sqrt{x}\,\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}$$

```
[Out] 1/7*x^(5/2)/a/(b*x^3+a*x)^(7/2)+8/35*x^(3/2)/a^2/(b*x^3+a*x)^(5/2)+16/35*x^
(1/2)/a^3/(b*x^3+a*x)^(3/2)+64/35/a^4/x^(1/2)/(b*x^3+a*x)^(1/2)-128/35*(b*x
^3+a*x)^(1/2)/a^5/x^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 2039}

$$-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{64}{35a^4\sqrt{x}\,\sqrt{ax+bx^3}}+\frac{16\sqrt{x}}{35a^3\,(ax+bx^3)^{3/2}}+\frac{8x^{3/2}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{x^{5/2}}{7a\,(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*x^(3/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (16*Sqrt[x])/(35*a^3*(a*x + b*x^3)^(3/2)) + 64/(35*a^4*Sqrt[x]*Sqrt[a*x
 + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(35*a^5*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}} + \frac{8\int \frac{x^{3/2}}{(ax+bx^3)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{48\int \frac{\sqrt{x}}{(ax+bx^3)^{5/2}}\,dx}{35a^2}$$

$$= \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax+bx^3)^{3/2}} + \frac{64\int \frac{1}{\sqrt{x}(ax+bx^3)^{3/2}}\,dx}{35a^3}$$

$$= \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax+bx^3)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} +$$

$$= \frac{x^{5/2}}{7a(ax+bx^3)^{7/2}} + \frac{8x^{3/2}}{35a^2(ax+bx^3)^{5/2}} + \frac{16\sqrt{x}}{35a^3(ax+bx^3)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} -$$

Mathematica [**A**]
time = 0.09, size = 68, normalized size = 0.54

$$\frac{x^{5/2}(-35a^4-280a^3bx^2-560a^2b^2x^4-448ab^3x^6-128b^4x^8)}{35a^5\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a*x + b*x^3)^(9/2),x]
[Out] (x^(5/2)*(-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b
^4*x^8))/(35*a^5*(x*(a + b*x^2))^(7/2))
```

Maple [**A**]
time = 0.46, size = 72, normalized size = 0.57

method	result	size
gosper	$-\frac{x^{\frac{7}{2}}\left(b\,x^{2}+a\right)\left(128b^{4}x^{8}+448a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+280a^{3}b\,x^{2}+35a^{4}\right)}{35a^{5}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	70
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(128b^{4}x^{8}+448a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+280a^{3}b\,x^{2}+35a^{4}\right)}{35x^{\frac{3}{2}}\left(b\,x^{2}+a\right)^{4}a^{5}}$	72
risch	$-\frac{b\,x^{2}+a}{a^{5}\sqrt{x}\,\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{\left(b\,x^{2}+a\right)x^{\frac{3}{2}}\left(93b^{3}x^{6}+308a\,b^{2}x^{4}+350a^{2}b\,x^{2}+140a^{3}\right)b}{35\left(b^{4}x^{8}+4a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}+4a^{3}b\,x^{2}+a^{4}\right)a^{5}\sqrt{x\left(b\,x^{2}+a\right)}}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35/x^(3/2)*(x*(b*x^2+a))^(1/2)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^
4+280*a^3*b*x^2+35*a^4)/(b*x^2+a)^4/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.57, size = 110, normalized size = 0.87

$$-\frac{\left(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6
 + 4*a^8*b*x^4 + a^9*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{5}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)
[Out] Integral(x**(5/2)/(x*(a + b*x**2))**(9/2), x)
```

Giac [A]
time = 0.68, size = 90, normalized size = 0.71

$$-\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^3)^(9/2), x)
```

3.88 $\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=159

$$\frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}}+\frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{3}{5a^3\sqrt{x}\,\left(ax+bx^3\right)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\tanh^{-1}}{}$$

```
[Out] 1/7*x^(3/2)/a/(b*x^3+a*x)^(7/2)+9/2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(
1/2))/a^(11/2)+3/5/a^3/(b*x^3+a*x)^(3/2)/x^(1/2)+9/35*x^(1/2)/a^2/(b*x^3+a*
x)^(5/2)+3/a^4/x^(3/2)/(b*x^3+a*x)^(1/2)-9/2*(b*x^3+a*x)^(1/2)/a^5/x^(5/2)
```

Rubi [A]
time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2048, 2050, 2054, 212}

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}+\frac{3}{5a^3\sqrt{x}\,\left(ax+bx^3\right)^{3/2}}+\frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*Sqrt[x])/(35*a^2*(a*x + b*x^3)^(5/2)
) + 3/(5*a^3*Sqrt[x]*(a*x + b*x^3)^(3/2)) + 3/(a^4*x^(3/2)*Sqrt[a*x + b*x^3
]) - (9*Sqrt[a*x + b*x^3])/(2*a^5*x^(5/2)) + (9*b*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^3]])/(2*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{9\int \frac{\sqrt{x}}{(ax+bx^3)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{9\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{5/2}}\,dx}{5a^2}$$

$$= \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}\,(ax+bx^3)^{3/2}} + \frac{3\int \frac{1}{x^{3/2}(ax+bx^3)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}\,(ax+bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} +$$

$$= \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}\,(ax+bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} -$$

$$= \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}\,(ax+bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} -$$

$$= \frac{x^{3/2}}{7a\,(ax+bx^3)^{7/2}} + \frac{9\sqrt{x}}{35a^2\,(ax+bx^3)^{5/2}} + \frac{3}{5a^3\sqrt{x}\,(ax+bx^3)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} -$$

Mathematica [A]
time = 0.15, size = 122, normalized size = 0.77

$$\frac{\sqrt{x\,(a+bx^2)}\left(-\sqrt{a}\,(35a^4+528a^3bx^2+1218a^2b^2x^4+1050ab^3x^6+315b^4x^8)+315bx^2(a+bx^2)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{70a^{11/2}x^{5/2}\,(a+bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x*(a + b*x^2)]*(-(Sqrt[a]*(35*a^4 + 528*a^3*b*x^2 + 1218*a^2*b^2*x^4
+ 1050*a*b^3*x^6 + 315*b^4*x^8)) + 315*b*x^2*(a + b*x^2)^(7/2)*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]]))/(70*a^(11/2)*x^(5/2)*(a + b*x^2)^4)
```

Maple [A]
time = 0.44, size = 234, normalized size = 1.47

method	result
default	$\frac{\sqrt{x\,(b\,x^2+a)}\,\left(315\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^4x^8\sqrt{b\,x^2+a}-315\sqrt{a}\,b^4x^8+945\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right.}{\cdots}$
risch	$-\frac{b\,x^2+a}{2a^5x^{\frac{3}{2}}\sqrt{x\,(b\,x^2+a)}}+\frac{\left(-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}}{112a^4b\left(x-\frac{\sqrt{-ab}}{b}\right)^4}+\frac{19\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right.\cdots}}{280a^4\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*(x*(b*x^2+a))^(1/2)/a^(11/2)*(315*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*
b^4*x^8*(b*x^2+a)^(1/2)-315*a^(1/2)*b^4*x^8+945*ln(2*(a^(1/2)*(b*x^2+a)^(1/
2)+a)/x)*a*b^3*x^6*(b*x^2+a)^(1/2)-1050*a^(3/2)*b^3*x^6+945*ln(2*(a^(1/2)*(
b*x^2+a)^(1/2)+a)/x)*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*a^(5/2)*b^2*x^4+315*l
n(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b*x^2*(b*x^2+a)^(1/2)-528*a^(7/2)*b*
x^2-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.55, size = 396, normalized size = 2.49

$$\left[\frac{315\,(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3)\sqrt{a}\,\log\left(\frac{bx^3+2\,ax+2\,\sqrt{bx^3+ax}\,\sqrt{a}\,\sqrt{x}}{x^3}\right)-2\,(315\,ab^4x^8+1050\,a^2b^3x^6+1218\,a^3b^2x^4+528\,a^4bx^2+35\,a^5)\sqrt{bx^3+ax}\,\sqrt{x}}{140\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3)},-\frac{315\,(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax}\,\sqrt{-a}}{a\sqrt{x}}\right)+(315\,ab^4x^8+1050\,a^2b^3x^6+1218\,a^3b^2x^4+528\,a^4bx^2+35\,a^5)\sqrt{bx^3+ax}\,\sqrt{x}}{70\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b
*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3
) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2
+ 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a
^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*s
qrt(-a)/(a*sqrt(x))) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4
 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7
*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(3/2)/(x*(a + b*x**2))**(9/2), x)
```

Giac [A]
time = 1.19, size = 104, normalized size = 0.65

$$-\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,a^5}-\frac{\sqrt{bx^2+a}}{2\,a^5x^2}-\frac{140\left(bx^2+a\right)^3b+35\left(bx^2+a\right)^2ab+14\left(bx^2+a\right)a^2b+5\,a^3b}{35\left(bx^2+a\right)^{\frac{7}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 + a
)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^2
+ a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^3)^(9/2), x)
```

3.89 $\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}+\frac{2}{7a^2\sqrt{x}\,\,(ax+bx^3)^{5/2}}+\frac{16}{21a^3x^{3/2}\,(ax+bx^3)^{3/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{2}{}$$

```
[Out] 16/21/a^3/x^(3/2)/(b*x^3+a*x)^(3/2)+2/7/a^2/(b*x^3+a*x)^(5/2)/x^(1/2)+1/7*x
^(1/2)/a/(b*x^3+a*x)^(7/2)+32/7/a^4/x^(5/2)/(b*x^3+a*x)^(1/2)-128/21*(b*x^3
+a*x)^(1/2)/a^5/x^(7/2)+256/21*b*(b*x^3+a*x)^(1/2)/a^6/x^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\,(ax+bx^3)^{3/2}}+\frac{2}{7a^2\sqrt{x}\,\,(ax+bx^3)^{5/2}}+\frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) +
 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x
^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])
/(21*a^6*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx &= \frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}+\frac{10\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{7/2}}\,dx}{7a}\\
&= \frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}+\frac{2}{7a^2\sqrt{x}\,(ax+bx^3)^{5/2}}+\frac{16\int \frac{1}{x^{3/2}(ax+bx^3)^{5/2}}\,dx}{7a^2}\\
&= \frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}+\frac{2}{7a^2\sqrt{x}\,(ax+bx^3)^{5/2}}+\frac{16}{21a^3x^{3/2}\,(ax+bx^3)^{3/2}}+\frac{32\int \frac{1}{x^{5/2}(ax+bx^3)^{3}}
\end{aligned}$$

$$= \frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}+\frac{2}{7a^2\sqrt{x}\,(ax+bx^3)^{5/2}}+\frac{16}{21a^3x^{3/2}\,(ax+bx^3)^{3/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+b}}$$

$$= \frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}+\frac{2}{7a^2\sqrt{x}\,(ax+bx^3)^{5/2}}+\frac{16}{21a^3x^{3/2}\,(ax+bx^3)^{3/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+b}}$$

$$= \frac{\sqrt{x}}{7a\,(ax+bx^3)^{7/2}}+\frac{2}{7a^2\sqrt{x}\,(ax+bx^3)^{5/2}}+\frac{16}{21a^3x^{3/2}\,(ax+bx^3)^{3/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+b}}$$

Mathematica [A]
time = 0.10, size = 79, normalized size = 0.52

$$\frac{\sqrt{x}\,(-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10})}{21a^6\,(x\,(a+bx^2))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*
a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*(x*(a + b*x^2))^(7/2))
```

Maple [A]
time = 0.39, size = 83, normalized size = 0.55

method	result	size
gosper	$-\frac{x^{\frac{3}{2}} (b\,x^{2}+a) (-256 b^{5} x^{10}-896 a\,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b\,x^{2}+7 a^{5})}{21 a^{6} (b\,x^{3}+a x)^{\frac{9}{2}}}$	81
default	$-\frac{\sqrt{x\,(b\,x^{2}+a)}\,(-256 b^{5} x^{10}-896 a\,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b\,x^{2}+7 a^{5})}{21 x^{\frac{7}{2}} (b\,x^{2}+a)^{4} a^{6}}$	83
risch	$-\frac{(b\,x^{2}+a) (-14 b\,x^{2}+a)}{3 a^{6} x^{\frac{5}{2}} \sqrt{x\,(b\,x^{2}+a)}}+\frac{(b\,x^{2}+a) x^{\frac{3}{2}} (158 b^{3} x^{6}+511 a\,b^{2} x^{4}+560 a^{2} b\,x^{2}+210 a^{3}) b^{2}}{21 a^{6} (b^{4} x^{8}+4 a\,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b\,x^{2}+a^{4}) \sqrt{x\,(b\,x^{2}+a)}}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/x^(7/2)*(x*(b*x^2+a))^(1/2)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3
*x^6-560*a^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/(b*x^2+a)^4/a^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)
```

Fricas [A]
time = 1.76, size = 121, normalized size = 0.80

$$\frac{(256\,b^{5}x^{10}+896\,ab^{4}x^{8}+1120\,a^{2}b^{3}x^{6}+560\,a^{3}b^{2}x^{4}+70\,a^{4}bx^{2}-7\,a^{5})\sqrt{bx^{3}+ax}\,\sqrt{x}}{21\,(a^{6}b^{4}x^{12}+4\,a^{7}b^{3}x^{10}+6\,a^{8}b^{2}x^{8}+4\,a^{9}bx^{6}+a^{10}x^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^6*b^4*x^12 + 4*a^7*b^3*x^
10 + 6*a^8*b^2*x^8 + 4*a^9*b*x^6 + a^10*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\left(x\left(a+bx^{2}\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2))**(9/2), x)
```

Giac [A]
time = 0.85, size = 147, normalized size = 0.97

$$\frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\,(bx^2+a)^{\frac{7}{2}}}-\frac{4\left(6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^3)^(9/2), x)
```

3.90 $\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx$

Optimal. Leaf size=189

$$\frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}}+\frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}}+\frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}$$

```
[Out] 11/35/a^2/x^(3/2)/(b*x^3+a*x)^(5/2)+33/35/a^3/x^(5/2)/(b*x^3+a*x)^(3/2)-99/
8*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(13/2)+1/7/a/(b*x^3+a*x)
^(7/2)/x^(1/2)+33/5/a^4/x^(7/2)/(b*x^3+a*x)^(1/2)-33/4*(b*x^3+a*x)^(1/2)/a^
5/x^(9/2)+99/8*b*(b*x^3+a*x)^(1/2)/a^6/x^(5/2)
```

Rubi [A]
time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2048, 2050, 2054, 212}

$$-\frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}}+\frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}}+\frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2
)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x +
 b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3]
)/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(
8*a^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11\int \frac{1}{x^{3/2}(ax+bx^3)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}} + \frac{99\int \frac{1}{x^{5/2}(ax+bx^3)^{5/2}}\,dx}{35a^2}$$

$$= \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}} + \frac{33\int \frac{}{x}}{}$$

$$= \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}} + \frac{}{5a^4x^{7/}}$$

$$= \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}} + \frac{}{5a^4x^{7/}}$$

$$= \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}} + \frac{}{5a^4x^{7/}}$$

$$= \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}} + \frac{}{5a^4x^{7/}}$$

$$= \frac{1}{7a\sqrt{x}\,(ax+bx^3)^{7/2}} + \frac{11}{35a^2x^{3/2}\,(ax+bx^3)^{5/2}} + \frac{33}{35a^3x^{5/2}\,(ax+bx^3)^{3/2}} + \frac{}{5a^4x^{7/}}$$

Mathematica [A]

time = 0.16, size = 134, normalized size = 0.71

$$\frac{\sqrt{x\left(a+bx^{2}\right)}\left(\sqrt{a}\left(-70a^{5}+385a^{4}bx^{2}+5808a^{3}b^{2}x^{4}+13398a^{2}b^{3}x^{6}+11550ab^{4}x^{8}+3465b^{5}x^{10}\right)-3465b^{2}x^{4}\left(a+bx^{2}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{2}}}{\sqrt{a}}\right)\right)}{280a^{13/2}x^{9/2}\left(a+bx^{2}\right)^{4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (Sqrt[x*(a + b*x^2)]*(Sqrt[a]*(-70*a^5 + 385*a^4*b*x^2 + 5808*a^3*b^2*x^4 +
 13398*a^2*b^3*x^6 + 11550*a*b^4*x^8 + 3465*b^5*x^10) - 3465*b^2*x^4*(a + b
*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(280*a^(13/2)*x^(9/2)*(a + b
*x^2)^4)
```

Maple [A]
time = 0.45, size = 247, normalized size = 1.31

method	result
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\left(3465\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)b^{5}x^{10}\sqrt{b\,x^{2}+a}-3465\sqrt{a}\,b^{5}x^{10}+10395\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}}}{x}\right.\right.}{}$
risch	$-\frac{\left(b\,x^{2}+a\right)\left(-19b\,x^{2}+2a\right)}{8a^{6}x^{\frac{7}{2}}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{\left(-\frac{13b\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}}{140a^{5}\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)^{3}}-\frac{711b\sqrt{\left(x-\frac{\sqrt{-}}{b}\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(x*(b*x^2+a))^(1/2)/a^(13/2)*(3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/
x)*b^5*x^10*(b*x^2+a)^(1/2)-3465*a^(1/2)*b^5*x^10+10395*ln(2*(a^(1/2)*(b*x^
2+a)^(1/2)+a)/x)*a*b^4*x^8*(b*x^2+a)^(1/2)-11550*a^(3/2)*b^4*x^8+10395*ln(2
*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^2*b^3*x^6*(b*x^2+a)^(1/2)-13398*a^(5/2)*b
^3*x^6+3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b^2*x^4*(b*x^2+a)^(1/2)
-5808*a^(7/2)*b^2*x^4-385*a^(9/2)*b*x^2+70*a^(11/2))/x^(9/2)/(b*x^2+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)
```

Fricas [A]
time = 1.51, size = 422, normalized size = 2.23

$$\left[\frac{3465\,(b^6x^{13}+4\,ab^5x^{11}+6\,a^2b^4x^9+4\,a^3b^3x^7+a^4b^2x^5)\sqrt{a}\,\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\,\sqrt{a}\,\sqrt{x}}{x^3}\right)+2\,(3465\,ab^5x^{10}+11550\,a^2b^4x^8+13398\,a^3b^3x^6+5808\,a^4b^2x^4+385\,a^5bx^2-70\,a^6)\sqrt{bx^3+ax}\,\sqrt{x}}{560\,(a^7b^4x^{13}+4\,a^8b^3x^{11}+6\,a^9b^2x^9+4\,a^{10}bx^7+a^{11}x^5)},\ \frac{3465\,(b^6x^{13}+4\,ab^5x^{11}+6\,a^2b^4x^9+4\,a^3b^3x^7+a^4b^2x^5)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax}\,\sqrt{-a}}{a\sqrt{x}}\right)+(3465\,ab^5x^{10}+11550\,a^2b^4x^8+13398\,a^3b^3x^6+5808\,a^4b^2x^4+385\,a^5bx^2-70\,a^6)\sqrt{bx^3+ax}\,\sqrt{x}}{280\,(a^7b^4x^{13}+4\,a^8b^3x^{11}+6\,a^9b^2x^9+4\,a^{10}bx^7+a^{11}x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4
*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))
/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a
^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^
13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*(3465
*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x^5)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (3465*a*b^5*x^10 +
11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*b*x^2 -
70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b
^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2))**(9/2)), x)
```

Giac [A]
time = 0.77, size = 138, normalized size = 0.73

$$\frac{99\,b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^6}+\frac{350\,(bx^2+a)^3b^2+70\,(bx^2+a)^2ab^2+21\,(bx^2+a)a^2b^2+5\,a^3b^2}{35\,(bx^2+a)^{\frac{7}{2}}a^6}+\frac{19\,(bx^2+a)^{\frac{3}{2}}b^2-21\,\sqrt{bx^2+a}\,ab^2}{8\,a^6b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 99/8*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 1/35*(350*(b*x^2
 + a)^3*b^2 + 70*(b*x^2 + a)^2*a*b^2 + 21*(b*x^2 + a)*a^2*b^2 + 5*a^3*b^2)/
((b*x^2 + a)^(7/2)*a^6) + 1/8*(19*(b*x^2 + a)^(3/2)*b^2 - 21*sqrt(b*x^2 + a
)*a*b^2)/(a^6*b^2*x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)), x)
```

3.91 $\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=180

$$\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+$$

```
[Out] 1/7/a/x^(3/2)/(b*x^3+a*x)^(7/2)+12/35/a^2/x^(5/2)/(b*x^3+a*x)^(5/2)+8/7/a^3
/x^(7/2)/(b*x^3+a*x)^(3/2)+64/7/a^4/x^(9/2)/(b*x^3+a*x)^(1/2)-384/35*(b*x^3
+a*x)^(1/2)/a^5/x^(11/2)+512/35*b*(b*x^3+a*x)^(1/2)/a^6/x^(7/2)-1024/35*b^2
*(b*x^3+a*x)^(1/2)/a^7/x^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2
)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b
*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^
3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12\int \frac{1}{x^{5/2}(ax+bx^3)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{1}{x^{7/2}(ax+bx^3)^{5/2}}\,dx}{7a^2}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64\int \cdots}{\cdots}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{\cdots}{7a^4x^{9/\cdots}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{\cdots}{7a^4x^{9/\cdots}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{\cdots}{7a^4x^{9/\cdots}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{\cdots}{7a^4x^{9/\cdots}}$$

Mathematica [A]
time = 0.12, size = 90, normalized size = 0.50

$$\frac{-7a^6+28a^5bx^2-280a^4b^2x^4-2240a^3b^3x^6-4480a^2b^4x^8-3584ab^5x^{10}-1024b^6x^{12}}{35a^7x^{3/2}\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (-7*a^6 + 28*a^5*b*x^2 - 280*a^4*b^2*x^4 - 2240*a^3*b^3*x^6 - 4480*a^2*b^4*
x^8 - 3584*a*b^5*x^10 - 1024*b^6*x^12)/(35*a^7*x^(3/2)*(x*(a + b*x^2))^(7/2
))
```

Maple [A]
time = 0.40, size = 94, normalized size = 0.52

method	result	size
gosper	$-\frac{(b\,x^{2}+a)\,(1024b^{6}x^{12}+3584b^{5}x^{10}a+4480b^{4}x^{8}a^{2}+2240b^{3}x^{6}a^{3}+280b^{2}x^{4}a^{4}-28b\,x^{2}a^{5}+7a^{6})}{35\sqrt{x}\;a^{7}(b\,x^{3}+ax)^{\frac{9}{2}}}$	92
default	$-\frac{\sqrt{x\,(b\,x^{2}+a)}\;(1024b^{6}x^{12}+3584b^{5}x^{10}a+4480b^{4}x^{8}a^{2}+2240b^{3}x^{6}a^{3}+280b^{2}x^{4}a^{4}-28b\,x^{2}a^{5}+7a^{6})}{35x^{\frac{11}{2}}\,(b\,x^{2}+a)^{4}a^{7}}$	94
risch	$-\frac{(b\,x^{2}+a)\,(66b^{2}x^{4}-8ab\,x^{2}+a^{2})}{5a^{7}x^{\frac{9}{2}}\sqrt{x\,(b\,x^{2}+a)}}-\frac{(b\,x^{2}+a)x^{\frac{3}{2}}\,(562b^{3}x^{6}+1792a\,b^{2}x^{4}+1925a^{2}b\,x^{2}+700a^{3})\,b^{3}}{35a^{7}(b^{4}x^{8}+4a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}+4a^{3}b\,x^{2}+a^{4})\sqrt{x\,(b\,x^{2}+a)}}$	150

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35/x^(11/2)*(x*(b*x^2+a))^(1/2)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*
b^4*x^8+2240*a^3*b^3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/(b*x^2+a)^4/a^
7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)
```

Fricas [A]
time = 1.97, size = 132, normalized size = 0.73

$$-\frac{\left(1024\,b^{6}x^{12}+3584\,ab^{5}x^{10}+4480\,a^{2}b^{4}x^{8}+2240\,a^{3}b^{3}x^{6}+280\,a^{4}b^{2}x^{4}-28\,a^{5}bx^{2}+7\,a^{6}\right)\sqrt{bx^{3}+ax}\,\sqrt{x}}{35\left(a^{7}b^{4}x^{14}+4\,a^{8}b^{3}x^{12}+6\,a^{9}b^{2}x^{10}+4\,a^{10}bx^{8}+a^{11}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x^
6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^7*
b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^{2}\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2))**(9/2)), x)
```

Giac [A]
time = 0.87, size = 202, normalized size = 1.12

$$-\frac{\left(\left(2\,x^2\left(\frac{281\,b^6x^2}{a^7}+\frac{896\,b^5}{a^6}\right)+\frac{1925\,b^4}{a^5}\right)x^2+\frac{700\,b^3}{a^4}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}}+\frac{4\left(25\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8b^{\frac{5}{2}}-120\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}+210\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{5}{2}}-140\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{5}{2}}+33\,a^4b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^3
/a^4)*x/(b*x^2 + a)^(7/2) + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)
 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2) +
 33*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)), x)
```

3.92 $\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=55

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

```
[Out] -1/3*a*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(3/2)+1/3*x*(b*x^4+a*x)^(1/
2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2049, 2054, 212}

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/
(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx &= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\int \frac{x}{\sqrt{ax+bx^4}}\,dx}{2b}\\ &= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{ax+bx^4}}\right)}{3b}\\ &= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 81, normalized size = 1.47

$$\frac{\sqrt{b}\,x^2(a+bx^3)-a\sqrt{x}\,\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{3b^{3/2}\sqrt{x\,(a+bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3
]/(Sqrt[b]*x^(3/2))])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 997, normalized size = 18.13

method	result	size
default	Expression too large to display	997
elliptic	Expression too large to display	997
risch	Expression too large to display	1006

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(b*x^4+a*x)^(1/2)/b-a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*
(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
```

```
3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*EllipticF(((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/b*(-a*b^2)^
(1/3)*EllipticPi(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2),(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(b*x^4 + a*x), x)
```

Fricas [A]
time = 1.52, size = 133, normalized size = 2.42

$$\left[\frac{4\,\sqrt{bx^4+ax}\,bx+a\sqrt{b}\,\log\left(-8\,b^2x^6-8\,abx^3-a^2+4\,(2\,bx^4+ax)\sqrt{bx^4+ax}\,\sqrt{b}\right)}{12\,b^2},\ \frac{2\,\sqrt{bx^4+ax}\,bx+a\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^4+ax}\,\sqrt{-b}\,x}{2\,bx^3+a}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] [1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2
 + 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 + a
*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)))/
b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

Giac [A]
time = 0.82, size = 45, normalized size = 0.82

$$\frac{\sqrt{bx^4+ax}\,x}{3\,b}+\frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-b
)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{b\,x^4+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^4)^(1/2), x)
```

3.93 $\int \frac{x}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2054, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^4],x]
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{ax+bx^4}}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 61, normalized size = 1.91

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{3\sqrt{b}\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))])/(3*S
qrt[b]*Sqrt[x*(a + b*x^3)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 979, normalized size = 30.59

method	result	size
default	Expression too large to display	979
elliptic	Expression too large to display	979

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(
1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*b/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/b*(-a*b^2)^(1/3)*EllipticPi(((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3
```

```
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x), x)
```

Fricas [A]
time = 2.08, size = 94, normalized size = 2.94

$$\left[\frac{\log\left(-8\,b^2x^6 - 8\,abx^3 - a^2 - 4\,(2\,bx^4 + ax)\sqrt{bx^4 + ax}\,\sqrt{b}\right)}{6\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^4 + ax}\,\sqrt{-b}\,x}{2\,bx^3 + a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)
*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b
*x^3 + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\,(a + bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**3)), x)
```

Giac [A]
time = 0.69, size = 23, normalized size = 0.72

$$-\frac{2\,\arctan\left(\frac{\sqrt{b + \frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{b\,x^4+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^4)^(1/2), x)
```

3.94 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

```
[Out] -2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2039}

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Mathematica [A]
time = 0.14, size = 23, normalized size = 1.00

$$-\frac{2\sqrt{x\left(a+bx^3\right)}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)
```

Maple [A]
time = 0.36, size = 20, normalized size = 0.87

method	result	size
default	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
trager	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
elliptic	$-\frac{2\sqrt{b\,x^{4}+a x}}{3 a\,x^{2}}$	20
gosper	$-\frac{2\left(b\,x^{3}+a\right)}{3 x a\sqrt{b\,x^{4}+a x}}$	27
risch	$-\frac{2\left(b\,x^{3}+a\right)}{3 a x\,\sqrt{x\left(b\,x^{3}+a\right)}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

Maxima [A]
time = 0.29, size = 26, normalized size = 1.13

$$-\frac{2\,(bx^{4}+ax)}{3\,\sqrt{bx^{3}+a}\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

Fricas [A]
time = 1.63, size = 19, normalized size = 0.83

$$-\frac{2\,\sqrt{bx^{4}+ax}}{3\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x^4 + a*x)/(a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{2}\,\sqrt{x\,(a+bx^{3})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)
```

Giac [A]
time = 0.86, size = 14, normalized size = 0.61

$$-\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)/a
```

Mupad [B]
time = 5.13, size = 19, normalized size = 0.83

$$-\frac{2\sqrt{b\,x^4+a\,x}}{3\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x + b*x^4)^(1/2))/(3*a*x^2)
```

3.95 $\int \frac{1}{x^5 \sqrt{ax+bx^4}}\, dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}$$

```
[Out] -2/9*(b*x^4+a*x)^(1/2)/a/x^5+4/9*b*(b*x^4+a*x)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2041, 2039}

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{ax+bx^4}}\, dx &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}-\frac{(2b)\int \frac{1}{x^2\sqrt{ax+bx^4}}\, dx}{3a}\\
&= -\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 31, normalized size = 0.65

$$-\frac{2(a-2bx^3)\sqrt{x(a+bx^3)}}{9a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)
```

Maple [A]
time = 0.36, size = 41, normalized size = 0.85

method	result	size
trager	$-\frac{2(-2b\,x^3+a)\sqrt{b\,x^4+ax}}{9a^2x^5}$	28
gosper	$-\frac{2(b\,x^3+a)(-2b\,x^3+a)}{9x^4a^2\sqrt{b\,x^4+ax}}$	35
risch	$-\frac{2(b\,x^3+a)(-2b\,x^3+a)}{9a^2x^4\sqrt{x(b\,x^3+a)}}$	35
default	$-\frac{2\sqrt{b\,x^4+ax}}{9a\,x^5}+\frac{4b\sqrt{b\,x^4+ax}}{9a^2x^2}$	41
elliptic	$-\frac{2\sqrt{b\,x^4+ax}}{9a\,x^5}+\frac{4b\sqrt{b\,x^4+ax}}{9a^2x^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(b*x^4+a*x)^(1/2)/a/x^5+4/9*b*(b*x^4+a*x)^(1/2)/a^2/x^2
```

Maxima [A]
time = 0.29, size = 38, normalized size = 0.79

$$\frac{2\,(2\,b^2x^7+abx^4-a^2x)}{9\,\sqrt{bx^3+a}\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

Fricas [A]
time = 1.42, size = 29, normalized size = 0.60

$$\frac{2\,\sqrt{bx^4+ax}\,(2\,bx^3-a)}{9\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{x\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)
```

Giac [A]
time = 0.89, size = 30, normalized size = 0.62

$$-\frac{2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\,\sqrt{b+\frac{a}{x^3}}\,b}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2
```

Mupad [B]
time = 5.13, size = 27, normalized size = 0.56

$$-\frac{2\,\sqrt{b\,x^4+a\,x}\,\left(a-2\,b\,x^3\right)}{9\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a*x + b*x^4)^(1/2)),x)
[Out] -(2*(a*x + b*x^4)^(1/2)*(a - 2*b*x^3))/(9*a^2*x^5)
```

3.96 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}$$

```
[Out] -2/15*(b*x^4+a*x)^(1/2)/a/x^8+8/45*b*(b*x^4+a*x)^(1/2)/a^2/x^5-16/45*b^2*(b
*x^4+a*x)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2041, 2039}

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(15*a*x^8) + (8*b*Sqrt[a*x + b*x^4])/(45*a^2*x^5) -
(16*b^2*Sqrt[a*x + b*x^4])/(45*a^3*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{15ax^8} - \frac{(4b)\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx}{5a}$$
$$= -\frac{2\sqrt{ax+bx^4}}{15ax^8} + \frac{8b\sqrt{ax+bx^4}}{45a^2x^5} + \frac{(8b^2)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{15a^2}$$
$$= -\frac{2\sqrt{ax+bx^4}}{15ax^8} + \frac{8b\sqrt{ax+bx^4}}{45a^2x^5} - \frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}$$

<u>**Mathematica**</u> [A]
time = 1.22, size = 44, normalized size = 0.59

$$-\frac{2\sqrt{x(a+bx^3)}\,(3a^2-4abx^3+8b^2x^6)}{45a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

<u>**Maple**</u> [A]
time = 0.36, size = 63, normalized size = 0.85

method	result	size
trager	$-\frac{2(8b^2x^6-4ab\,x^3+3a^2)\sqrt{b\,x^4+ax}}{45a^3x^8}$	41
gosper	$-\frac{2(b\,x^3+a)(8b^2x^6-4ab\,x^3+3a^2)}{45x^7a^3\sqrt{b\,x^4+ax}}$	48
risch	$-\frac{2(b\,x^3+a)(8b^2x^6-4ab\,x^3+3a^2)}{45a^3x^7\sqrt{x(b\,x^3+a)}}$	48
default	$-\frac{2\sqrt{b\,x^4+ax}}{15a\,x^8}+\frac{8b\sqrt{b\,x^4+ax}}{45a^2x^5}-\frac{16b^2\sqrt{b\,x^4+ax}}{45a^3x^2}$	63
elliptic	$-\frac{2\sqrt{b\,x^4+ax}}{15a\,x^8}+\frac{8b\sqrt{b\,x^4+ax}}{45a^2x^5}-\frac{16b^2\sqrt{b\,x^4+ax}}{45a^3x^2}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15*(b*x^4+a*x)^(1/2)/a/x^8+8/45*b*(b*x^4+a*x)^(1/2)/a^2/x^5-16/45*b^2*(b
*x^4+a*x)^(1/2)/a^3/x^2
```

<u>**Maxima**</u> [A]
time = 0.30, size = 50, normalized size = 0.68

$$-\frac{2\,(8\,b^3x^{10}+4\,ab^2x^7-a^2bx^4+3\,a^3x)}{45\,\sqrt{bx^3+a}\,a^3x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(8*b^3*x^10 + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^3
*x^(17/2))
```

Fricas [A]
time = 1.81, size = 40, normalized size = 0.54

$$-\frac{2\,(8\,b^2x^6-4\,abx^3+3\,a^2)\sqrt{bx^4+ax}}{45\,a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\sqrt{x\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)
```

Giac [A]
time = 0.81, size = 47, normalized size = 0.64

$$-\frac{2\sqrt{b+\frac{a}{x^3}}\,b^2}{3\,a^3}-\frac{2\left(3\left(b+\frac{a}{x^3}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}b\right)}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)*b^2/a^3 - 2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^(
3/2)*b)/a^3
```

Mupad [B]
time = 5.27, size = 40, normalized size = 0.54

$$-\frac{2\sqrt{b\,x^4+a\,x}\,\left(3\,a^2-4\,a\,b\,x^3+8\,b^2\,x^6\right)}{45\,a^3\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^6 - 4*a*b*x^3))/(45*a^3*x^8)
```

3.97 $\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\left(2+\right.\right.}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/12*a^(2/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*
x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2049, 2036, 335, 231}

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ell
ipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \frac{\sqrt{ax+bx^4}}{2b} - \frac{a\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{4b}$$

$$= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax+bx^4}}$$

$$= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax+bx^4}}$$

$$= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+(}{\sqrt[3]{a}+(}\right.\right.}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 64, normalized size = 0.29

$$\frac{x\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b
*x^3)/a)]))/(2*b*Sqrt[x*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 688, normalized size = 3.07 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/2*a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2
)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
```

```
I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)*x^2/(b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x**3/sqrt(x*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{b\,x^4+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^4)^(1/2), x)
```

3.98 $\int \frac{1}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=197

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}$$

```
[Out] 1/3*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3
)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2036, 335, 231}

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt
[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^
(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}}$$

$$= \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)\middle|\frac{1}{4}\left(\cdots\right.\right.}{\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 49, normalized size = 0.25

$$\frac{2x\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 671, normalized size = 3.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(
1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*b/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^4 + a*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 16, normalized size = 0.08

$$-\frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**4), x)
```

Giac [A]
time = 0.48, size = 39, normalized size = 0.20

$$\frac{1}{3}\,\sqrt{bx^4 + ax}\,x - \frac{a \arctan\left(\frac{\sqrt{b + \frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^4 + a*x)*x - 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

Mupad [B]
time = 5.16, size = 40, normalized size = 0.20

$$\frac{2\,x\,\sqrt{\frac{b\,x^3}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^4+a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^4)^(1/2),x)
```

```
[Out] (2*x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/6, 1/2], 7/6, -(b*x^3)/a))/(a*x + b
*x^4)^(1/2)
```

3.99 $\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=225

$$-\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\left(2+\right.\right.}{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}$$

```
[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-2/15*b*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2050, 2036, 335, 231}

$$-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*(a^(
1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]
)
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{5ax^3} - \frac{(2b)\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{5a}$$

$$= -\frac{2\sqrt{ax+bx^4}}{5ax^3} - \frac{\left(2b\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax+bx^4}}$$

$$= -\frac{2\sqrt{ax+bx^4}}{5ax^3} - \frac{\left(4b\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{5a\sqrt{ax+bx^4}}$$

$$= -\frac{2\sqrt{ax+bx^4}}{5ax^3} - \frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}\cdots}{\sqrt[3]{a}\cdots}\right.\right.}{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 53, normalized size = 0.24

$$-\frac{2\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{5x^2\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^2*Sqrt[x*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 696, normalized size = 3.09 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-4/5*b^2/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x
/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/
3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
```

```
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 43, normalized size = 0.19

$$\frac{2\left(2\sqrt{a}\,bx^3\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^4+ax}\,a\right)}{5\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(2*sqrt(a)*b*x^3*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^4 + a*x
)*a)/(a^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{b\,x^4+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^4)^(1/2)),x)
[Out] int(1/(x^3*(a*x + b*x^4)^(1/2)), x)
```

3.100 $\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=503

$$-\frac{5\left(1+\sqrt{3}\right)ax(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}}+\frac{x^2\sqrt{ax+bx^4}}{4b}+\frac{5\sqrt[4]{3}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots}{\left(\sqrt[3]{a}+\left(1+\cdots\right.\right.}}}{8b^{5/3}\sqrt{\frac{\sqrt[3]{\cdots}}{\left(\sqrt[3]{a}\cdots\right.}}}$$

```
[Out] -5/8*a*x*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x
^4+a*x)^(1/2)+1/4*x^2*(b*x^4+a*x)^(1/2)/b+5/8*3^(1/4)*a^(4/3)*x*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^4+a*x)^(1/2)/(b^(1
/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+5/48*a^(
4/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/b^(5/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2049, 2057, 335, 314, 231, 1895}

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}+\frac{5\sqrt[4]{3}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}-\frac{5\left(1+\sqrt{3}\right)ax(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}}+\frac{x^2\sqrt{ax+bx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a*x + b*x^4],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)*Sqrt[a*x + b*x^4]) + (x^2*Sqrt[a*x + b*x^4])/(4*b) + (5*3^(1/4)*a^(4
/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - S
```

```
qrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/
(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) + (5*(1 - Sqrt[3])*a^(4/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
```

```
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx}{8b}$$

$$= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax+bx^4}}$$

$$= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{4b\sqrt{ax+bx^4}}$$

$$= \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{\left(5a\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax+bx^4}} + \frac{\left(\right.}{}$$

$$= -\frac{5\left(1+\sqrt{3}\right)ax(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}} + \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{5\sqrt[4]{3}\,a^{4/3}x\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 66, normalized size = 0.13

$$\frac{x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -
((b*x^3)/a)]))/(4*b*Sqrt[x*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 1079, normalized size = 2.15

method	result	size
default	Expression too large to display	1079
elliptic	Expression too large to display	1079
risch	Expression too large to display	1086

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2*(b*x^4+a*x)^(1/2)/b-5/8*a/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1
/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a
*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1
```

```
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)*x^4/(b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x**5/sqrt(x*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{b\,x^4+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a*x + b*x^4)^(1/2), x)
```

3.101 $\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=474

$$\frac{\left(1+\sqrt{3}\right)x(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\Big(c}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right.}}}$$

```
[Out] x*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4+a*x)
^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/b^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2057, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}+\frac{\left(1+\sqrt{3}\right)x(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] ((1 + Sqrt[3])*x*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*
Sqrt[a*x + b*x^4]) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*
EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/
```

```
3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sq
rt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/
3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sq
rt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx &= \frac{\left(\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}}\\
&= \frac{\left(2\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}}\\
&= -\frac{\left(\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax+bx^4}} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}\sqrt{\cdots}\right.}{\cdots}\\
&= \frac{\left(1+\sqrt{3}\right)x(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}} - \frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\sqrt[3]{a}+\left(1\cdots\right.\right.}}}{b^{2/3}\sqrt{\frac{\cdots}{\left(\sqrt[3]{\cdots}\right.}}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 53, normalized size = 0.11

$$\frac{2x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)])
/(5*Sqrt[x*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 1054, normalized size = 2.22

method	result	size

default	Expression too large to display	1054
elliptic	Expression too large to display	1054

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3
)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1
/3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)*x/(b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x**2/sqrt(x*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{b\,x^4+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^4)^(1/2),x)
[Out] int(x^2/(a*x + b*x^4)^(1/2), x)
```

3.102 $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=497

$$\frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b\cdots}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\cdots\right.}}}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}\cdots\right.}{\left(\sqrt[3]{a}+\left(1+\cdots\right.\right.}}}$$

```
[Out] 2*b^(1/3)*x*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4+
a*x)^(1/2)-2*(b*x^4+a*x)^(1/2)/a/x-2*3^(1/4)*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/3*b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(
2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2050, 2057, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}+\frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)*Sqrt[a*x + b*x^4]) - (2*Sqrt[a*x + b*x^4])/(a*x) - (2*3^(1/4)*b^(1/3)*x
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3
```

```
])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x
 + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{(2b)\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx}{a}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{\left(2b\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{\left(4b\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}-\frac{\left(2\sqrt[3]{b}\,\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^4}} \\
&= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 48, normalized size = 0.10

$$-\frac{2\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 1083, normalized size = 2.18

method	result	size
default	Expression too large to display	1083
risch	Expression too large to display	1083
elliptic	Expression too large to display	1083

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^3+a)/a/(x*(b*x^3+a))^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/
b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^
```

```
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 24, normalized size = 0.05

$$\frac{2\,\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{b\,x^4+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^4)^(1/2)),x)
[Out] int(1/(x*(a*x + b*x^4)^(1/2)), x)
```

3.103 $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=174

$$\frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3}-\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2}+\frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a}$$

```
[Out] -63/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)+63/64*b^
4*(b*x^(1/2)+a*x)^(1/2)/a^5+21/40*b^2*x*(b*x^(1/2)+a*x)^(1/2)/a^3-9/20*b*x^
(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/2)+a*x)^(1/2)/a-21/32*b^3*x
^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4
```

Rubi [A]
time = 0.11, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 684, 654, 634, 212}

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}}+\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3}-\frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2}+\frac{2x^2\sqrt{ax+b\sqrt{x}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] +
a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*S
qrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*
b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(9b)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a}$$

$$= -\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{(63b^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right.}{40a^2}$$

$$= \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(21b^3)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{}}}{5a}$$

$$= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}}}{20a^2}$$

$$= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}}}{20a^2}$$

$$= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}}}{20a^2}$$

Mathematica [A]
time = 0.22, size = 113, normalized size = 0.65

$$\frac{\sqrt{b\sqrt{x}+ax}\left(315b^4-210ab^3\sqrt{x}+168a^2b^2x-144a^3bx^{3/2}+128a^4x^2\right)}{320a^5}+\frac{63b^5\log\left(b+2a\sqrt{x}-2\sqrt{a}\sqrt{b\sqrt{x}+ax}\right)}{128a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(315*b^4 - 210*a*b^3*Sqrt[x] + 168*a^2*b^2*x - 144*a
^3*b*x^(3/2) + 128*a^4*x^2))/(320*a^5) + (63*b^5*Log[b + 2*a*Sqrt[x] - 2*Sq
rt[a]*Sqrt[b*Sqrt[x] + a*x]])/(128*a^(11/2))
```

Maple [A]
time = 0.39, size = 223, normalized size = 1.28

method	result
derivativedivides	$\frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{4a} - \frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b\left(\frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{3b\left(\right.}{}\right.}{}\right.}{}\right.}{5a}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(544\sqrt{x}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{7}{2}}b-256x\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}+1300\sqrt{x}\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}b^{3}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(b*x^(1/2)+a*x)^(1/2)*(544*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*b-2
56*x*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)+1300*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/
2)*b^3-880*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^2+650*(b*x^(1/2)+a*x)^(1/2)*a^(3
/2)*b^4+640*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/
```

```
a^(1/2))*a*b^5-1280*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*b^4-325*ln(1/2*(2
*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^5)/(x^(1/2)*(a*x
^(1/2)+b))^(1/2)/a^(13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(a*x + b*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(x**2/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A]
time = 0.82, size = 111, normalized size = 0.64

$$\frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a}-\frac{9b}{a^2}\right)+\frac{21b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)\sqrt{x}+\frac{315b^4}{a^5}\right)+\frac{63b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{128a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a - 9*b/a^2) + 21*b
^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(abs(
-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/2))^(1/2),x)
[Out] int(x^2/(a*x + b*x^(1/2))^(1/2), x)
```

3.104 $\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=116

$$\frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3}-\frac{5b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{6a^2}+\frac{2x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}$$

```
[Out] -5/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)+5/4*b^2*(b*
x^(1/2)+a*x)^(1/2)/a^3+2/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/6*b*x^(1/2)*(b*x^(1/
2)+a*x)^(1/2)/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2043, 684, 654, 634, 212}

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}}+\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{5b\sqrt{x}\,\sqrt{ax+b\sqrt{x}}}{6a^2}+\frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt
[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right.}{4a^2}$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{8}$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{1-a}\right.}{}$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{b}}\right.}{4a^{7/2}}$$

Mathematica [A]

time = 0.17, size = 95, normalized size = 0.82

$$\frac{\sqrt{b\sqrt{x}+ax}\,(15b^2-10ab\sqrt{x}+8a^2x)}{12a^3}+\frac{5b^3\log\left(a^3b+2a^4\sqrt{x}-2a^{7/2}\sqrt{b\sqrt{x}+ax}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(15*b^2 - 10*a*b*Sqrt[x] + 8*a^2*x))/(12*a^3) + (5*b
^3*Log[a^3*b + 2*a^4*Sqrt[x] - 2*a^(7/2)*Sqrt[b*Sqrt[x] + a*x]])/(8*a^(7/2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(84) = 168$.
time = 0.38, size = 181, normalized size = 1.56

method	result
derivativedivides	$\frac{2x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right.}{2a^{\frac{3}{2}}}\right.}{4a}\right.}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(16\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{5}{2}}-36\sqrt{b\sqrt{x}+ax}\,\sqrt{x}\,a^{\frac{5}{2}}b-18\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b^2+48\sqrt{\sqrt{x}\right.}{24a^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(b*x^(1/2)+a*x)^(1/2)/a^(9/2)*(16*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)-36*(b*
x^(1/2)+a*x)^(1/2)*x^(1/2)*a^(5/2)*b-18*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^2+4
8*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*b^2-24*a*ln(1/2*(2*a*x^(1/2)+2*(x^(
1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^3+9*ln(1/2*(2*a*x^(1/2)+2*(
b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^3)/(x^(1/2)*(a*x^(1/2)+b))^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A]
time = 1.15, size = 83, normalized size = 0.72

$$\frac{1}{12}\sqrt{ax + b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a} - \frac{5b}{a^2}\right) + \frac{15b^2}{a^3}\right) + \frac{5b^3 \log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3)
 + 5/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b
))/a^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a\,x + b\,\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/2))^(1/2), x)
```

3.105 $\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

```
[Out] -2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)+2*(b*x^(1/2)+a*
x)^(1/2)/a
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2035, 2038, 634, 212}

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x]
 + a*x]])/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx &= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx}{2a}\\
&= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{(2b)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}\\
&= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 65, normalized size = 1.16

$$\frac{2\sqrt{b\sqrt{x}+ax}}{a} + \frac{b\log\left(ab+2a^2\sqrt{x}-2a^{3/2}\sqrt{b\sqrt{x}+ax}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a + (b*Log[a*b + 2*a^2*Sqrt[x] - 2*a^(3/2)*Sqrt[b
*Sqrt[x] + a*x]])/a^(3/2)
```

Maple [**A**]
time = 0.38, size = 83, normalized size = 1.48

method	result

derivativedivides	$\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(b\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}\right)}{\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^(1/2)+a*x)^(1/2)*(b*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/
2)*a^(1/2)+b)/a^(1/2))-2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2))/(x^(1/2)*(a
*x^(1/2)+b))^(1/2)/a^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A]
time = 1.20, size = 54, normalized size = 0.96

$$\frac{b\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{3}{2}}}+\frac{2\sqrt{ax+b\sqrt{x}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(3/2
) + 2*sqrt(a*x + b*sqrt(x))/a
```

Mupad [B]
time = 5.24, size = 72, normalized size = 1.29

$$\frac{4\,x\left(\frac{3\sqrt{b}\,\sqrt{b+a\,\sqrt{x}}}{2\,a\,\sqrt{x}}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1/4}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/4}}\right)\sqrt{\frac{a\,\sqrt{x}}{b}+1}}{3\,\sqrt{a\,x+b\,\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*x*((3*b^(1/2)*(b + a*x^(1/2))^(1/2))/(2*a*x^(1/2)) + (b^(3/2)*asin((a^(1
/2)*x^(1/4)*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/4)))*((a*x^(1/2))/b + 1)^(1/2)
)/(3*(a*x + b*x^(1/2))^(1/2))
```

3.106 $\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=25

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

```
[Out] -4*(b*x^(1/2)+a*x)^(1/2)/b/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

Mathematica [A]
time = 0.07, size = 25, normalized size = 1.00

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.39, size = 160, normalized size = 6.40

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(4\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}\sqrt{a}-2\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}x-2\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}x-\ln\left(\frac{2a}{\ }\right.\right.}{\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^(1/2)+a*x)^(1/2)*(4*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)-2*(b*x^(1/2)+a*x)^(
1/2)*a^(3/2)*x-2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*x-ln(1/2*(2*a*x^(1/2
)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b*x+ln(1/2*(2*a*x^(1/2)+2*(
x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a*b*x)/(x^(1/2)*(a*x^(1/2)
+b))^(1/2)/b^2/x/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)
```

Fricas [A]
time = 6.34, size = 19, normalized size = 0.76

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)
```

Giac [A]
time = 1.31, size = 25, normalized size = 1.00

$$\frac{4}{\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/2))^(1/2)), x)
```

3.107 $\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=84

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}$$

```
[Out] -4/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)+16/15*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x-32
/15*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 2039}

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) + (16*a*Sqrt[b*Sqrt[x] + a*x])/(15
*b^2*x) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(15*b^3*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}} - \frac{(4a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}} + \frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x} + \frac{(8a^2)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{15b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}} + \frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 0.57

$$-\frac{4\sqrt{b\sqrt{x}+ax}\,\left(3b^2-4ab\sqrt{x}+8a^2x\right)}{15b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/2
))
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.38, size = 218, normalized size = 2.60

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}\right)}{5b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\,\left(60\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{5}{2}}a^{\frac{5}{2}}-30\sqrt{b\sqrt{x}+ax}\,x^{\frac{7}{2}}a^{\frac{7}{2}}-30x^{\frac{7}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{7}{2}}-1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(b*x^(1/2)+a*x)^(1/2)*(60*(b*x^(1/2)+a*x)^(3/2)*x^(5/2)*a^(5/2)-30*(b
*x^(1/2)+a*x)^(1/2)*x^(7/2)*a^(7/2)-30*x^(7/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2
```

```
)*a^(7/2)-15*x^(7/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)
/a^(1/2))*a^3*b+15*x^(7/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1
/2)*a^(1/2)+b)/a^(1/2))*a^3*b+12*(b*x^(1/2)+a*x)^(3/2)*x^(3/2)*a^(1/2)*b^2-
28*a^(3/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^2)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^4/x
^(7/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)
```

Fricas [A]
time = 4.04, size = 42, normalized size = 0.50

$$\frac{4\left(4\,abx-\left(8\,a^2x+3\,b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{15\,b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] 4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)
```

Giac [A]
time = 2.37, size = 84, normalized size = 1.00

$$\frac{4\left(20\,a\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+15\,\sqrt{a}\,b\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+3\,b^2\right)}{15\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x)))^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)), x)
```

3.108 $\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=142

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}}+\frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{315b^5\sqrt{x}}$$

```
[Out] -4/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)+32/63*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2-
64/105*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)+256/315*a^3*(b*x^(1/2)+a*x)^(1
/2)/b^4/x-512/315*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(63
*b^2*x^2) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(105*b^3*x^(3/2)) + (256*a^3*Sqr
t[b*Sqrt[x] + a*x])/(315*b^4*x) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(315*b^5*
Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} - \frac{(8a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} + \frac{(16a^2)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} - \frac{(64a^3)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{}}}}{105b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+a}}{315b^4x}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+a}}{315b^4x}$$

Mathematica [A]
time = 0.12, size = 72, normalized size = 0.51

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(35b^4-40ab^3\sqrt{x}+48a^2b^2x-64a^3bx^{3/2}+128a^4x^2\right)}{315b^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a^
3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.38, size = 262, normalized size = 1.85

method	result

derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{16a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\cdots}{\cdots}\right)}{5b}\right)}{7b}\right)}{9b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(1260\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{9}{2}}-630\sqrt{b\sqrt{x}+ax}\,x^{\frac{11}{2}}a^{\frac{11}{2}}+315x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\cdots}}{\cdots}\right)\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(b*x^(1/2)+a*x)^(1/2)*(1260*(b*x^(1/2)+a*x)^(3/2)*x^(9/2)*a^(9/2)-63
0*(b*x^(1/2)+a*x)^(1/2)*x^(11/2)*a^(11/2)+315*x^(11/2)*ln(1/2*(2*a*x^(1/2)+
2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^5*b-315*x^(11/2)*ln(1
/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^5*b-630*x^(11
/2)*a^(11/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)+492*(b*x^(1/2)+a*x)^(3/2)*x^(7/2
)*a^(5/2)*b^2+140*(b*x^(1/2)+a*x)^(3/2)*x^(5/2)*a^(1/2)*b^4-748*a^(7/2)*(b*
x^(1/2)+a*x)^(3/2)*b*x^4-300*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*b^3*x^3)/(x^(1/2
)*(a*x^(1/2)+b))^(1/2)/b^6/a^(1/2)/x^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)
```

Fricas [A]
time = 3.52, size = 64, normalized size = 0.45

$$\frac{4\left(64\,a^{3}bx^{2}+40\,ab^{3}x-\left(128\,a^{4}x^{2}+48\,a^{2}b^{2}x+35\,b^{4}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{315\,b^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*sq
rt(x))*sqrt(a*x + b*sqrt(x))/(b^5*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)
```

Giac [A]
time = 2.85, size = 146, normalized size = 1.03

$$\frac{4\left(1008\,a^2\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+1680\,a^{\frac{3}{2}}b\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+1080\,ab^2\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+315\,\sqrt{a}\,b^3\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+35\,b^4\right)}{315\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(x)
 - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a\,x+b\,\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)), x)
```

3.109 $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=200

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}+\frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{1001b^5x^{3/2}}+$$

```
[Out] -4/13*(b*x^(1/2)+a*x)^(1/2)/b/x^(7/2)+48/143*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^
3-160/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)+1280/3003*a^3*(b*x^(1/2)+a*
x)^(1/2)/b^4/x^2-512/1001*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)+2048/3003*a
^5*(b*x^(1/2)+a*x)^(1/2)/b^6/x-4096/3003*a^6*(b*x^(1/2)+a*x)^(1/2)/b^7/x^(1
/2)
```

Rubi [A]
time = 0.20, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}+\frac{48a\sqrt{ax+b\sqrt{x}}}{143b^2x^3}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) + (48*a*Sqrt[b*Sqrt[x] + a*x])/(1
43*b^2*x^3) - (160*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^3*x^(5/2)) + (1280*a^3
*Sqrt[b*Sqrt[x] + a*x])/(3003*b^4*x^2) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(1
001*b^5*x^(3/2)) + (2048*a^5*Sqrt[b*Sqrt[x] + a*x])/(3003*b^6*x) - (4096*a^
6*Sqrt[b*Sqrt[x] + a*x])/(3003*b^7*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} - \frac{(12a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} + \frac{(120a^2)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{143b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} - \frac{(320a^3)\int \frac{}{x^{5/2}\sqrt{b}}}{429b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+}}{3003b^4x^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+}}{3003b^4x^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+}}{3003b^4x^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+}}{3003b^4x^2}$$

Mathematica [A]
time = 0.13, size = 96, normalized size = 0.48

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(231b^6-252ab^5\sqrt{x}+280a^2b^4x-320a^3b^3x^{3/2}+384a^4b^2x^2-512a^5bx^{5/2}+1024a^6x^3\right)}{3003b^7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 32
0*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(3
003*b^7*x^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.39, size = 306, normalized size = 1.53

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{13b\,x^{\frac{7}{2}}}-\frac{24a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{11b\,x^{3}}-\frac{10a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{13b\cdots}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(12012\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{13}{2}}a^{\frac{13}{2}}-6006\sqrt{b\sqrt{x}+ax}\,x^{\frac{15}{2}}a^{\frac{15}{2}}-6006x^{\frac{15}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}\cdots\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3003*(b*x^(1/2)+a*x)^(1/2)*(12012*(b*x^(1/2)+a*x)^(3/2)*x^(13/2)*a^(13/2
)-6006*(b*x^(1/2)+a*x)^(1/2)*x^(15/2)*a^(15/2)-6006*x^(15/2)*(x^(1/2)*(a*x^
(1/2)+b))^(1/2)*a^(15/2)-3003*x^(15/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x
)^(1/2)*a^(1/2)+b)/a^(1/2))*a^7*b+3003*x^(15/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1
/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^7*b+5868*(b*x^(1/2)+a*x)^(3/
2)*x^(11/2)*a^(9/2)*b^2+3052*(b*x^(1/2)+a*x)^(3/2)*x^(9/2)*a^(5/2)*b^4-7916
*a^(11/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^6+924*(b*x^(1/2)+a*x)^(3/2)*x^(7/2)*a^(
1/2)*b^6-4332*(b*x^(1/2)+a*x)^(3/2)*x^5*a^(7/2)*b^3-1932*(b*x^(1/2)+a*x)^(3
/2)*a^(3/2)*b^5*x^4)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^8/x^(15/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)
```

Fricas [A]
time = 3.84, size = 86, normalized size = 0.43

$$\frac{4\left(512\,a^5bx^3+320\,a^3b^3x^2+252\,ab^5x-\left(1024\,a^6x^3+384\,a^4b^2x^2+280\,a^2b^4x+231\,b^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3003\,b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 384
*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^7
*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)
```

Giac [A]
time = 1.15, size = 208, normalized size = 1.04

$$\frac{4\left(27456\,a^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^6+72072\,a^{\frac{5}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+80080\,a^2b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+48048\,a^{\frac{3}{2}}b^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+16380\,ab^4\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+3003\,\sqrt{a}\,b^5\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+231\,b^6\right)}{3003\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^6 + 72072*a^(5/
2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 48048*a^(3/2)*b^3*(sqrt(a)*sqrt(x) - sq
rt(a*x + b*sqrt(x)))^3 + 16380*a*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x
)))^2 + 3003*sqrt(a)*b^5*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 231*b^
6)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^13
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)), x)
```

3.110 $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\frac{99bx^{3/2}\sqrt{b\sqrt{x}+}}{20a^3}$$

```
[Out] -693/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-4*x^3/a
/(b*x^(1/2)+a*x)^(1/2)+693/64*b^4*(b*x^(1/2)+a*x)^(1/2)/a^6+231/40*b^2*x*(b
*x^(1/2)+a*x)^(1/2)/a^4-99/20*b*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3+22/5*x^2*
(b*x^(1/2)+a*x)^(1/2)/a^2-231/32*b^3*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^5
```

Rubi [A]
time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 682, 684, 654, 634, 212}

$$-\frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}+\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}-\frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3}+\frac{22x^2\sqrt{ax+b\sqrt{x}}}{5a^2}-\frac{4x^3}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^3)/(a*Sqrt[b*Sqrt[x] + a*x]) + (693*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^
6) - (231*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^5) + (231*b^2*x*Sqrt[b*S
qrt[x] + a*x])/(40*a^4) - (99*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^3) + (
22*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a^2) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/
Sqrt[b*Sqrt[x] + a*x]])/(64*a^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\text{Subst}\left(\int \frac{x^7}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{(99b)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a^2}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} + \frac{(693b^2)\,\text{Subst}\Big(}{}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}}}{5a^2}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b}}{20}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b}}{40a}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b}}{40a}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b}}{40a}$$

Mathematica [A]
time = 0.32, size = 137, normalized size = 0.70

$$\frac{\sqrt{b\sqrt{x}+ax}\left(3465b^5+1155ab^4\sqrt{x}-462a^2b^3x+264a^3b^2x^{3/2}-176a^4bx^2+128a^5x^{5/2}\right)}{320a^6\left(b+a\sqrt{x}\right)} + \frac{693b^5\log\left(b+2a\sqrt{x}-2\sqrt{a}\sqrt{b\sqrt{x}+ax}\right)}{128a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(3465*b^5 + 1155*a*b^4*Sqrt[x] - 462*a^2*b^3*x + 264
*a^3*b^2*x^(3/2) - 176*a^4*b*x^2 + 128*a^5*x^(5/2)))/(320*a^6*(b + a*Sqrt[x
])) + (693*b^5*Log[b + 2*a*Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/(128
*a^(13/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(147) = 294$.
time = 0.39, size = 549, normalized size = 2.79

method	result

		$11b\left(\frac{x^{\frac{5}{2}}}{4a\sqrt{b\sqrt{x}+ax}} - 9b\left(\frac{x^{2}}{3a\sqrt{b\sqrt{x}+ax}} - 7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}} - 5b\left(\frac{}{a\sqrt{(}}\right.\right.\right.\right.$

default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(352\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}a^{\frac{11}{2}}b-256\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{2}a^{\frac{13}{2}}-528\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x\,a^{\frac{9}{2}}b^{2}+4060\sqrt{b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(b*x^(1/2)+a*x)^(1/2)/a^(15/2)*(352*(b*x^(1/2)+a*x)^(3/2)*x^(3/2)*a^
(11/2)*b-256*(b*x^(1/2)+a*x)^(3/2)*x^2*a^(13/2)-528*(b*x^(1/2)+a*x)^(3/2)*x
*a^(9/2)*b^2+4060*(b*x^(1/2)+a*x)^(1/2)*x^(3/2)*a^(9/2)*b^3-3136*(b*x^(1/2)
+a*x)^(3/2)*x^(1/2)*a^(7/2)*b^3+10150*(b*x^(1/2)+a*x)^(1/2)*x*a^(7/2)*b^4-8
960*x*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(7/2)*b^4+4480*x*a^3*ln(1/2*(2*a*x^(1
/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^5-2000*(b*x^(1/2)
+a*x)^(3/2)*a^(5/2)*b^4+8120*(b*x^(1/2)+a*x)^(1/2)*x^(1/2)*a^(5/2)*b^5-1015
*x*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^3*b^5-
17920*x^(1/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(5/2)*b^5+8960*x^(1/2)*a^2*ln
(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^6+2
560*(x^(1/2)*(a*x^(1/2)+b))^(3/2)*a^(5/2)*b^4+2030*(b*x^(1/2)+a*x)^(1/2)*a^
(3/2)*b^6-2030*x^(1/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+
b)/a^(1/2))*a^2*b^6-8960*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*b^6+4480*a*l
n(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^7-
1015*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^7)
/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/(a*x^(1/2)+b)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [A]
time = 0.80, size = 150, normalized size = 0.76

$$\frac{1}{320}\sqrt{ax + b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a^2} - \frac{19b}{a^3}\right) + \frac{71b^2}{a^4}\right)\sqrt{x} - \frac{515b^3}{a^5}\right)\sqrt{x} + \frac{2185b^4}{a^6}\right) + \frac{693b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{128a^{\frac{13}{2}}} + \frac{4b^6}{\left(a\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) + \sqrt{a}b\right)a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a^2 - 19*b/a^3) + 7
1*b^2/a^4)*sqrt(x) - 515*b^3/a^5)*sqrt(x) + 2185*b^4/a^6) + 693/128*b^5*log
(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(13/2) +
4*b^6/((a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + sqrt(a)*b)*a^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(a\,x + b\,\sqrt{x}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(1/2))^(3/2), x)
```

3.111 $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}}+\frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4}-\frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3}+\frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2}-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{9/2}}$$

```
[Out] -35/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-4*x^2/a/(b
*x^(1/2)+a*x)^(1/2)+35/4*b^2*(b*x^(1/2)+a*x)^(1/2)/a^4+14/3*x*(b*x^(1/2)+a*
x)^(1/2)/a^2-35/6*b*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 682, 684, 654, 634, 212}

$$-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}+\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4}-\frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^2)/(a*Sqrt[b*Sqrt[x] + a*x]) + (35*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^4)
 - (35*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^3) + (14*x*Sqrt[b*Sqrt[x] + a*
x])/(3*a^2) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*
a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\text{Subst}\left(\int \frac{x^5}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} + \frac{(35b^2)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+}}{3a^2}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+}}{3a^2}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+}}{3a^2}$$

Mathematica [A]
time = 0.33, size = 119, normalized size = 0.86

$$\frac{\sqrt{b\sqrt{x}+ax}\left(105b^3+35ab^2\sqrt{x}-14a^2bx+8a^3x^{3/2}\right)}{12a^4\left(b+a\sqrt{x}\right)} + \frac{35b^3\log\left(a^4b+2a^5\sqrt{x}-2a^{9/2}\sqrt{b\sqrt{x}+ax}\right)}{8a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(105*b^3 + 35*a*b^2*Sqrt[x] - 14*a^2*b*x + 8*a^3*x^(
3/2)))/(12*a^4*(b + a*Sqrt[x])) + (35*b^3*Log[a^4*b + 2*a^5*Sqrt[x] - 2*a^(
9/2)*Sqrt[b*Sqrt[x] + a*x]])/(8*a^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(103) = 206$.

time = 0.39, size = 503, normalized size = 3.62

method	result
derivativedivides	$\frac{2x^{2}}{3a\sqrt{b\sqrt{x}+ax}}-\frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{}{a\sqrt{b}}\right.}{}\right.}{}\right.}{}\right.}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(16x\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}-60\sqrt{b\sqrt{x}+ax}\,x^{\frac{3}{2}}a^{\frac{9}{2}}b+32\sqrt{x}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{7}{2}}b-150\sqrt{b\sqrt{x}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(b*x^(1/2)+a*x)^(1/2)/a^(11/2)*(16*x*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)-60*
(b*x^(1/2)+a*x)^(1/2)*x^(3/2)*a^(9/2)*b+32*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*a^
(7/2)*b-150*(b*x^(1/2)+a*x)^(1/2)*x*a^(7/2)*b^2+240*x*(x^(1/2)*(a*x^(1/2)+b
))^(1/2)*a^(7/2)*b^2-120*x*a^3*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b)
)^(1/2)*a^(1/2)+b)/a^(1/2))*b^3+16*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^2-120*x^
(1/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*b^3+15*x*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/
2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^3*b^3+480*x^(1/2)*(x^(1/2)*(a*x^(1/2)+b
))^(1/2)*a^(5/2)*b^3-240*x^(1/2)*a^2*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1
/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^4-96*(x^(1/2)*(a*x^(1/2)+b))^(3/2)*a^(5
/2)*b^2-30*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^4+30*x^(1/2)*ln(1/2*(2*a*x^(1/2)
+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^2*b^4+240*(x^(1/2)*(a*x^(1/2
)+b))^(1/2)*a^(3/2)*b^4-120*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(
```

```
1/2)*a^(1/2)+b)/a^(1/2))*a*b^5+15*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/
2)*a^(1/2)+b)/a^(1/2))*a*b^5)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/(a*x^(1/2)+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [A]
time = 0.70, size = 122, normalized size = 0.88

$$\frac{1}{12}\sqrt{ax + b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a^2} - \frac{11b}{a^3}\right) + \frac{57b^2}{a^4}\right) + \frac{35b^3\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{8a^{\frac{9}{2}}} + \frac{4b^4}{\left(a\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) + \sqrt{a}b\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a^2 - 11*b/a^3) + 57*b^2/a
^4) + 35/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

```
 - b))/a^(9/2) + 4*b^4/((a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + sqrt
(a)*b)*a^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a\,x+b\,\sqrt{x}\,\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x^2/(a*x + b*x^(1/2))^(3/2), x)
```

3.112 $\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}$$

```
[Out] -6*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-4*x/a/(b*x^(1/2
)+a*x)^(1/2)+6*(b*x^(1/2)+a*x)^(1/2)/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2043, 682, 654, 634, 212}

$$-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}+\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x)/(a*Sqrt[b*Sqrt[x] + a*x]) + (6*Sqrt[b*Sqrt[x] + a*x])/a^2 - (6*b*Arc
Tanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\text{Subst}\left(\int \frac{x^3}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(3b)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(6b)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.23, size = 90, normalized size = 1.17

$$\frac{2\left(3b+a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{a^2\left(b+a\sqrt{x}\right)}+\frac{3b\log\left(a^2b+2a^3\sqrt{x}-2a^{5/2}\sqrt{b\sqrt{x}+ax}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (2*(3*b + a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(a^2*(b + a*Sqrt[x])) + (3*b*Lo
g[a^2*b + 2*a^3*Sqrt[x] - 2*a^(5/2)*Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(59) = 118.
time = 0.38, size = 237, normalized size = 3.08

method	result
derivativedivides	$\frac{2x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+a\sqrt{\ldots}}{\sqrt{a}}\right)\ldots}{\ldots}\right)}{a}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(3x\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\sqrt{a}+b}{2\sqrt{a}}\right)a^2b-6xa^{\frac{5}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^(1/2)+a*x)^(1/2)/a^(5/2)*(3*x*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/
2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^2*b-6*x*a^(5/2)*(x^(1/2)*(a*x^(1/2)+b))^
(1/2)+6*x^(1/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)
+b)/a^(1/2))*a*b^2-12*x^(1/2)*a^(3/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*b+4*a^(
3/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)+3*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1
/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^3-6*a^(1/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/
2)*b^2)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/(a*x^(1/2)+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [**A**]
time = 0.62, size = 94, normalized size = 1.22

$$\frac{3\,b\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{5}{2}}}+\frac{4\,b^2}{\left(a\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+\sqrt{a}\,b\right)a^2}+\frac{2\,\sqrt{ax+b\sqrt{x}}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3*b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5
/2) + 4*b^2/((a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + sqrt(a)*b)*a^2)
 + 2*sqrt(a*x + b*sqrt(x))/a^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/2))^(3/2), x)
```

3.113 $\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

```
[Out] 4*x^(1/2)/b/(b*x^(1/2)+a*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2025}

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sqrt[x] + a*x)^(-3/2),x]
```

```
[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

Mathematica [A]
time = 0.12, size = 31, normalized size = 1.24

$$\frac{4\sqrt{b\sqrt{x}+ax}}{b\left(b+a\sqrt{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]
[Out] (4*Sqrt[b*Sqrt[x] + a*x])/(b*(b + a*Sqrt[x]))
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.38, size = 404, normalized size = 16.16

method	result
derivativedivides	$-\frac{2}{a\sqrt{b\sqrt{x}+ax}}+\frac{2b+4a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(2\sqrt{b\sqrt{x}+ax}\,x\,a^{\frac{5}{2}}+2x\,a^{\frac{5}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}+x\ln\left(\frac{2a\sqrt{x}+2\sqrt{b\sqrt{x}+ax}}{2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] (b*x^(1/2)+a*x)^(1/2)*(2*(b*x^(1/2)+a*x)^(1/2)*x*a^(5/2)+2*x*a^(5/2)*(x^(1/
2)*(a*x^(1/2)+b))^(1/2)+x*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/
2)+b)/a^(1/2))*a^2*b-x*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*
a^(1/2)+b)/a^(1/2))*a^2*b+4*(b*x^(1/2)+a*x)^(1/2)*x^(1/2)*a^(3/2)*b+4*x^(1/
2)*a^(3/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*b+2*x^(1/2)*ln(1/2*(2*a*x^(1/2)+2*
(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^2-2*x^(1/2)*ln(1/2*(2*a*x^(1/
2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^2-4*a^(3/2)*(x^(
1/2)*(a*x^(1/2)+b))^(3/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)*b^2+2*a^(1/2)*(x^
(1/2)*(a*x^(1/2)+b))^(1/2)*b^2+ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*
a^(1/2)+b)/a^(1/2))*b^3-ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)
*a^(1/2)+b)/a^(1/2))*b^3)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^2/(a*x^(1/2)+b)^2
/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*sqrt(x))^(-3/2), x)
```

Fricas [A]
time = 6.19, size = 36, normalized size = 1.44

$$\frac{4\sqrt{ax+b\sqrt{x}}\,\left(a\sqrt{x}-b\right)}{a^2bx-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a^2*b*x - b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*sqrt(x))**(-3/2), x)
```

Giac [A]
time = 0.59, size = 34, normalized size = 1.36

$$\frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) + b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))
```

Mupad [B]
time = 5.43, size = 40, normalized size = 1.60

$$-\frac{4\,x\,\left(\frac{b}{a\,\sqrt{x}}+1\right)}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}\,\left(\sqrt{\frac{b}{a\,\sqrt{x}}+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] -(4*x*(b/(a*x^(1/2)) + 1))/((a*x + b*x^(1/2))^(3/2)*((b/(a*x^(1/2)) + 1)^(1
/2) + 1))
```

3.114 $\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}-\frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x}+\frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}$$

[Out] 4/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)-16/3*(b*x^(1/2)+a*x)^(1/2)/b^2/x+32/3*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}+\frac{4\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}-\frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x}-\frac{(8a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b^2}\\
&= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}-\frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x}+\frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 55, normalized size = 0.70

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(b^2-4ab\sqrt{x}-8a^2x\right)}{3b^3\left(b+a\sqrt{x}\right)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(b^2 - 4*a*b*Sqrt[x] - 8*a^2*x))/(3*b^3*(b + a*Sq
rt[x])*x)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.46, size = 524, normalized size = 6.63

method	result
derivativedivides	$-\frac{4}{3b\sqrt{x}\sqrt{b\sqrt{x}+ax}}+\frac{16a\left(b+2a\sqrt{x}\right)}{3b^3\sqrt{b\sqrt{x}+ax}}$

default	$\frac{\sqrt{b\sqrt{x}+ax}\,\left(24\left(b\sqrt{x}+ax\right)^{\frac{3}{2}} x^{\frac{5}{2}} a^{\frac{7}{2}}-6\sqrt{b\sqrt{x}+ax}\, x^{\frac{7}{2}} a^{\frac{9}{2}}-6x^{\frac{7}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\, a^{\frac{9}{2}}-3x^{\frac{7}{2}}\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^(1/2)+a*x)^(1/2)*(24*(b*x^(1/2)+a*x)^(3/2)*x^(5/2)*a^(7/2)-6*(b*x^
(1/2)+a*x)^(1/2)*x^(7/2)*a^(9/2)-6*x^(7/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^
(9/2)-3*x^(7/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1
/2))*a^4*b+3*x^(7/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^
(1/2)+b)/a^(1/2))*a^4*b+44*(b*x^(1/2)+a*x)^(3/2)*x^2*a^(5/2)*b-12*(b*x^(1/2
)+a*x)^(1/2)*x^3*a^(7/2)*b-12*x^3*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(7/2)*b-6
*x^3*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^3*b^
2+6*x^3*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1
/2))*a^3*b^2-12*x^(5/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)*a^(7/2)+16*(b*x^(1/2)
+a*x)^(3/2)*x^(3/2)*a^(3/2)*b^2-6*(b*x^(1/2)+a*x)^(1/2)*x^(5/2)*a^(5/2)*b^2
-6*x^(5/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(5/2)*b^2-3*x^(5/2)*ln(1/2*(2*a*
x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^2*b^3+3*x^(5/2)*ln(1/
2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^2*b^3-
4*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*b^3*x)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^4/a^
(1/2)/(a*x^(1/2)+b)^2/x^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

Fricas [A]
time = 2.94, size = 63, normalized size = 0.80

$$-\frac{4\left(4\,a^2bx-b^3-\left(8\,a^3x-5\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3\left(a^2b^3x^2-b^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/
(a^2*b^3*x^2 - b^5*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/2))^(3/2)), x)
```

3.115 $\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}-\frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{35b^5\sqrt{x}}$$

```
[Out] 4/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-32/7*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2+192/35
*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)-256/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x
+512/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2
) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^(3/2)) - (256*a^2*Sqrt[b*Sqrt[x
] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}+\frac{8\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}-\frac{(48a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b^2}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}+\frac{(192a^2)\int \frac{}{x^{3/2}}}{3}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}-\frac{256a^2\sqrt{b\sqrt{x}}}{35b^4x}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}-\frac{256a^2\sqrt{b\sqrt{x}}}{35b^4x}$$

Mathematica [A]
time = 0.17, size = 81, normalized size = 0.59

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^4-8ab^3\sqrt{x}+16a^2b^2x-64a^3bx^{3/2}-128a^4x^2\right)}{35b^5\left(b+a\sqrt{x}\right)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^4 - 8*a*b^3*Sqrt[x] + 16*a^2*b^2*x - 64*a^3*
b*x^(3/2) - 128*a^4*x^2))/(35*b^5*(b + a*Sqrt[x])*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.38, size = 570, normalized size = 4.16

method	result
derivativedivides	$-\frac{4}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{16a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a\left(b+2a\sqrt{x}\right)}{3b^{3}\sqrt{b\sqrt{x}+a\cdots}}\cdots\right)}{5b}\cdots\right)}{7b}\cdots$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(560\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{11}{2}}-210\sqrt{b\sqrt{x}+ax}\,x^{\frac{11}{2}}a^{\frac{13}{2}}+105x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\cdots}}{2\sqrt{\cdots}}\right)\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(b*x^(1/2)+a*x)^(1/2)*(560*(b*x^(1/2)+a*x)^(3/2)*x^(9/2)*a^(11/2)-210*
(b*x^(1/2)+a*x)^(1/2)*x^(11/2)*a^(13/2)+105*x^(11/2)*ln(1/2*(2*a*x^(1/2)+2*
(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^6*b-210*x^(11/2)*a^(13/
2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)-105*x^(11/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1
/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^6*b+256*(b*x^(1/2)+a*x)^(3/2)*x^(7/2)*
a^(7/2)*b^2+932*(b*x^(1/2)+a*x)^(3/2)*x^4*a^(9/2)*b-420*(b*x^(1/2)+a*x)^(1/
2)*x^5*a^(11/2)*b+210*x^5*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/
2)*a^(1/2)+b)/a^(1/2))*a^5*b^2-420*x^5*a^(11/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/
2)*b-210*x^5*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2)
)*a^5*b^2-140*x^(9/2)*a^(11/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)-64*(b*x^(1/2)+
a*x)^(3/2)*x^3*a^(5/2)*b^3-210*(b*x^(1/2)+a*x)^(1/2)*x^(9/2)*a^(9/2)*b^2+10
5*x^(9/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^
(1/2))*a^4*b^3-210*x^(9/2)*a^(9/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*b^2-105*x^
(9/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^4*b
^3+32*(b*x^(1/2)+a*x)^(3/2)*x^(5/2)*a^(3/2)*b^4-20*(b*x^(1/2)+a*x)^(3/2)*a^
(1/2)*b^5*x^2)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^6/a^(1/2)/(a*x^(1/2)+b)^2/x^
(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

Fricas [A]
time = 2.41, size = 87, normalized size = 0.64

$$-\frac{4\left(64\,a^4bx^2-24\,a^2b^3x-5\,b^5-\left(128\,a^5x^2-80\,a^3b^2x-13\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\left(a^2b^5x^3-b^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] -4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x -
13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^2\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)), x)
```

3.116 $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{77b^5x^{3/2}}-\cdots$$

```
[Out] 4/b/x^(5/2)/(b*x^(1/2)+a*x)^(1/2)-48/11*(b*x^(1/2)+a*x)^(1/2)/b^2/x^3+160/3
3*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)-1280/231*a^2*(b*x^(1/2)+a*x)^(1/2)/b^
4/x^2+512/77*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)-2048/231*a^4*(b*x^(1/2)+
a*x)^(1/2)/b^6/x+4096/231*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-\frac{48\sqrt{ax+b\sqrt{x}}}{11b^2x^3}+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^
3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^(5/2)) - (1280*a^2*Sqrt[b*Sqrt
[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^(3/2)
) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x]
 + a*x])/(231*b^7*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}+\frac{12\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}-\frac{(120a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}+\frac{(320a^2)\int \frac{}{x^{5/2}}}{3}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}}}{231b^4x^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}}}{231b^4x^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}}}{231b^4x^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}}}{231b^4x^2}$$

Mathematica [A]

time = 0.19, size = 105, normalized size = 0.54

$$-\frac{4\sqrt{b\sqrt{x}+ax}\,\left(21b^6-28ab^5\sqrt{x}+40a^2b^4x-64a^3b^3x^{3/2}+128a^4b^2x^2-512a^5bx^{5/2}-1024a^6x^3\right)}{231b^7\left(b+a\sqrt{x}\right)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(21*b^6 - 28*a*b^5*Sqrt[x] + 40*a^2*b^4*x - 64*a^
3*b^3*x^(3/2) + 128*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) - 1024*a^6*x^3))/(231*b
^7*(b + a*Sqrt[x])*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.40, size = 614, normalized size = 3.15

method	result
derivativedivides	$-\frac{4}{11b\,x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{24a\left(-\frac{2}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{10a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}\ldots}}\ldots\right.}{\ldots}\right.}{\ldots}\right.}{11b}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\,\left(-2310\sqrt{b\sqrt{x}+ax}\,x^{\frac{13}{2}}a^{\frac{13}{2}}b^{2}-2310x^{\frac{13}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{13}{2}}b^{2}-2310\sqrt{b\sqrt{x}\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/231*(b*x^(1/2)+a*x)^(1/2)*(-2310*(b*x^(1/2)+a*x)^(1/2)*x^(13/2)*a^(13/2)*
b^2-2310*x^(13/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(13/2)*b^2-2310*(b*x^(1/2
)+a*x)^(1/2)*x^(15/2)*a^(17/2)-2310*x^(15/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*
a^(17/2)-1155*x^(15/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+
b)/a^(1/2))*a^8*b+1155*x^(15/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b
))^(1/2)*a^(1/2)+b)/a^(1/2))*a^8*b-2310*x^7*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2
)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^7*b^2+2310*x^7*ln(1/2*(2*a*x^(1/2)+2*(x^
(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^7*b^2-924*x^(13/2)*(x^(1/2
)*(a*x^(1/2)+b))^(3/2)*a^(15/2)+256*(b*x^(1/2)+a*x)^(3/2)*x^(9/2)*a^(7/2)*b
^4-1155*x^(13/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(
1/2))*a^6*b^3+1155*x^(13/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(
1/2)*a^(1/2)+b)/a^(1/2))*a^6*b^3+2048*(b*x^(1/2)+a*x)^(3/2)*x^(11/2)*a^(11/
2)*b^2+8716*(b*x^(1/2)+a*x)^(3/2)*x^6*a^(13/2)*b-4620*(b*x^(1/2)+a*x)^(1/2)
*x^7*a^(15/2)*b-4620*x^7*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(15/2)*b-512*(b*x^
(1/2)+a*x)^(3/2)*x^5*a^(9/2)*b^3+5544*(b*x^(1/2)+a*x)^(3/2)*x^(13/2)*a^(15/
2)-84*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*b^7*x^3-160*(b*x^(1/2)+a*x)^(3/2)*x^4*a
^(5/2)*b^5+112*(b*x^(1/2)+a*x)^(3/2)*x^(7/2)*a^(3/2)*b^6)/(x^(1/2)*(a*x^(1/
2)+b))^(1/2)/b^8/x^(13/2)/a^(1/2)/(a*x^(1/2)+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

Fricas [A]
time = 4.43, size = 109, normalized size = 0.56

$$-\frac{4\left(512\,a^6bx^3-192\,a^4b^3x^2-68\,a^2b^5x-21\,b^7-\left(1024\,a^7x^3-640\,a^5b^2x^2-104\,a^3b^4x-49\,ab^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{231\left(a^2b^7x^4-b^9x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^7
*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*sq
rt(x))/(a^2*b^7*x^4 - b^9*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)), x)
```

3.117 $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=204

$$-\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{}}}{30a}$$

```
[Out] 231/256*b^6*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-231/256
*b^5*(b*x^(1/2)+a*x)^(1/2)/a^6-77/160*b^3*x*(b*x^(1/2)+a*x)^(1/2)/a^4+33/80
*b^2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3-11/30*b*x^2*(b*x^(1/2)+a*x)^(1/2)/a^
2+1/3*x^(5/2)*(b*x^(1/2)+a*x)^(1/2)/a+77/128*b^4*x^(1/2)*(b*x^(1/2)+a*x)^(1
/2)/a^5
```

Rubi [A]
time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2043, 684, 654, 634, 212}

$$\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}}-\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3}-\frac{11bx^2\sqrt{ax+b\sqrt{x}}}{30a^2}+\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-231*b^5*Sqrt[b*Sqrt[x] + a*x])/(256*a^6) + (77*b^4*Sqrt[x]*Sqrt[b*Sqrt[x]
 + a*x])/(128*a^5) - (77*b^3*x*Sqrt[b*Sqrt[x] + a*x])/(160*a^4) + (33*b^2*x
^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(80*a^3) - (11*b*x^2*Sqrt[b*Sqrt[x] + a*x])/(
30*a^2) + (x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(3*a) + (231*b^6*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(256*a^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(11b)\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{6a}$$

$$= -\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} + \frac{(33b^2)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{}\right.}{20a^2}$$

$$= \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(231b^3)\,\text{Subs}}{}$$

$$= -\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{}}}{3}$$

$$= \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{}}{}$$

$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}}{}$$

$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}}{}$$

$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}}{}$$

<u>Mathematica</u> [**A**]
time = 0.24, size = 126, normalized size = 0.62

$$\frac{\sqrt{b\sqrt{x}+ax}\,\left(-3465b^5+2310ab^4\sqrt{x}-1848a^2b^3x+1584a^3b^2x^{3/2}-1408a^4bx^2+1280a^5x^{5/2}\right)}{3840a^6} - \frac{231b^6\log\left(b+2a\sqrt{x}-2\sqrt{a}\sqrt{b\sqrt{x}+ax}\right)}{512a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-3465*b^5 + 2310*a*b^4*Sqrt[x] - 1848*a^2*b^3*x + 1
584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1280*a^5*x^(5/2)))/(3840*a^6) - (231
*b^6*Log[b + 2*a*Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/(512*a^(13/2))
```

Maple [A]
time = 0.39, size = 245, normalized size = 1.20

method	result

		$11b\left(\frac{x^2\sqrt{b\sqrt{x}+ax}}{5a}-\frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{4a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{}}{}\right.}{}\right.}{}\right.}{}\right.$

default	$\frac{\sqrt{b\sqrt{x}+ax}\left(2560x^{\frac{3}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{11}{2}}+8544\sqrt{x}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{7}{2}}b^{2}-5376\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}bx+16860\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7680*(b*x^(1/2)+a*x)^(1/2)*(2560*x^(3/2)*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)+8
544*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*b^2-5376*(b*x^(1/2)+a*x)^(3/2)*a^
(9/2)*b*x+16860*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*b^4-12240*(b*x^(1/2)+
a*x)^(3/2)*a^(5/2)*b^3+8430*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^5-15360*(x^(1/2
)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*b^5+7680*a*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a
*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^6-4215*ln(1/2*(2*a*x^(1/2)+2*(b*x^
(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^6)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/a
^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A]
time = 0.55, size = 125, normalized size = 0.61

$$\frac{1}{3840}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\left(8\sqrt{x}\left(\frac{10\sqrt{x}}{a}-\frac{11b}{a^2}\right)+\frac{99b^2}{a^3}\right)\sqrt{x}-\frac{231b^3}{a^4}\right)\sqrt{x}+\frac{1155b^4}{a^5}\right)\sqrt{x}-\frac{3465b^5}{a^6}\right)-\frac{231b^6\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{512a^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2)
+ 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 3465
*b^5/a^6) - 231/512*b^6*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*
sqrt(x))) - b))/a^(13/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{5/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.118 $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=146

$$-\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{9/2}}$$

```
[Out] 35/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-35/32*b^3*
(b*x^(1/2)+a*x)^(1/2)/a^4-7/12*b*x*(b*x^(1/2)+a*x)^(1/2)/a^2+1/2*x^(3/2)*(b
*x^(1/2)+a*x)^(1/2)/a+35/48*b^2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2043, 684, 654, 634, 212}

$$\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}}-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3}-\frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2}+\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^3*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (35*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] +
 a*x])/(48*a^3) - (7*b*x*Sqrt[b*Sqrt[x] + a*x])/(12*a^2) + (x^(3/2)*Sqrt[b*
Sqrt[x] + a*x])/(2*a) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/(32*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{(7b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= -\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{(35b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{24a^2}$$

$$= \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{(35b^3)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+}}{2a}$$

$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+}}{2a}$$

$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+}}{2a}$$

Mathematica [A]
time = 0.20, size = 108, normalized size = 0.74

$$\frac{\sqrt{b\sqrt{x}+ax}\,\left(-105b^3+70ab^2\sqrt{x}-56a^2bx+48a^3x^{3/2}\right)}{96a^4} - \frac{35b^4\log\left(a^4b+2a^5\sqrt{x}-2a^{9/2}\sqrt{b\sqrt{x}+ax}\right)}{64a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-105*b^3 + 70*a*b^2*Sqrt[x] - 56*a^2*b*x + 48*a^3*x
^(3/2)))/(96*a^4) - (35*b^4*Log[a^4*b + 2*a^5*Sqrt[x] - 2*a^(9/2)*Sqrt[b*Sq
rt[x] + a*x]])/(64*a^(9/2))
```

Maple [A]
time = 0.39, size = 203, normalized size = 1.39

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{2a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{b}{2}\right.}{}\right.}{}\right.}{6a}\right.}{4a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(96\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}+348\sqrt{x}\,\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}b^{2}-208\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{5}{2}}b+174\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(b*x^(1/2)+a*x)^(1/2)*(96*(b*x^(1/2)+a*x)^(3/2)*x^(1/2)*a^(7/2)+348*x
^(1/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*b^2-208*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*
b+174*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^3-384*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a
^(3/2)*b^3+192*a*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2
)+b)/a^(1/2))*b^4-87*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)
/a^(1/2))*a*b^4)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/a^(11/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A]
time = 0.69, size = 97, normalized size = 0.66

$$\frac{1}{96}\sqrt{ax + b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{6\sqrt{x}}{a} - \frac{7b}{a^2}\right) + \frac{35b^2}{a^3}\right)\sqrt{x} - \frac{105b^3}{a^4}\right) - \frac{35b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{64a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/a
^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x)
- sqrt(a*x + b*sqrt(x))) - b))/a^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a\,x + b\,\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.119 $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=87

$$-\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2}+\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}$$

```
[Out] 3/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-3/2*b*(b*x^(
1/2)+a*x)^(1/2)/a^2+x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2043, 684, 654, 634, 212}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\,\sqrt{ax+b\sqrt{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-3*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a +
(3*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a} - \frac{(3b)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{2a}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 0.99

$$\frac{\left(-3b+2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{3b^2\log\left(a^2b+2a^3\sqrt{x}-2a^{5/2}\sqrt{b\sqrt{x}+ax}\right)}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((-3*b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) - (3*b^2*Log[a^2*b + 2
*a^3*Sqrt[x] - 2*a^(5/2)*Sqrt[b*Sqrt[x] + a*x]])/(4*a^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(63) = 126$.
time = 0.38, size = 160, normalized size = 1.84

method	result
derivativedivides	$\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{2a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(4\sqrt{x}\,\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}+2\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b-8\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}b+4a\right.}{4\sqrt{\sqrt{x}\,\left(a\sqrt{x}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^(1/2)+a*x)^(1/2)*(4*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)+2*(b*x^(
1/2)+a*x)^(1/2)*a^(3/2)*b-8*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*b+4*a*ln(
1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^2-b^
2*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a)/(x^(1/
2)*(a*x^(1/2)+b))^(1/2)/a^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{ax + b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A]
time = 0.59, size = 69, normalized size = 0.79

$$\frac{1}{2}\sqrt{ax + b\sqrt{x}}\left(\frac{2\sqrt{x}}{a} - \frac{3b}{a^2}\right) - \frac{3b^2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(-2*sqrt
(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{a\,x + b\,\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.120 $\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=34

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}$$

```
[Out] 4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2038, 634, 212}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = 2\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$
$$= 4\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)$$
$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 1.18

$$-\frac{2\log\left(b+2a\sqrt{x}-2\sqrt{a}\sqrt{b\sqrt{x}+ax}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-2*Log[b + 2*a*Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(24) = 48$.
time = 0.39, size = 133, normalized size = 3.91

method	result
derivativedivides	$\frac{2\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{\sqrt{a}}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(2\sqrt{b\sqrt{x}+ax}\sqrt{a}+b\ln\left(\frac{2a\sqrt{x}+2\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\left(a\sqrt{x}\right.}\right.}{\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^(1/2)+a*x)^(1/2)*(2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*a*x^(1/2
)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))-2*(x^(1/2)*(a*x^(1/2)+b))^(1/
```

```
2)*a^(1/2)+b*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)
/a^(1/2)))/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b/a^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(a*x + b*sqrt(x))), x)
```

Giac [**A**]
time = 0.63, size = 37, normalized size = 1.09

$$-\frac{2\,\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/sqrt(
a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.121 $\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=54

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}$$

```
[Out] -4/3*(b*x^(1/2)+a*x)^(1/2)/b/x+8/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqr
t[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx} - \frac{(2a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx} + \frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}$$

Mathematica [A]
time = 0.09, size = 35, normalized size = 0.65

$$-\frac{4\left(b-2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*(b - 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.52, size = 194, normalized size = 3.59

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(6x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}+6x^{\frac{5}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{5}{2}}-3x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}}}{2}\right.\right.}{3\sqrt{\sqrt{x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^(1/2)+a*x)^(1/2)*(6*x^(5/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)+6*x^(5/
2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(5/2)-3*x^(5/2)*ln(1/2*(2*a*x^(1/2)+2*(x
^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^2*b+3*x^(5/2)*ln(1/2*(2*a
*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^2*b-12*x^(3/2)*(b*x^
(1/2)+a*x)^(3/2)*a^(3/2)+4*(b*x^(1/2)+a*x)^(3/2)*b*a^(1/2)*x)/(x^(1/2)*(a*x
^(1/2)+b))^(1/2)/b^3/a^(1/2)/x^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)
```

Fricas [A]
time = 3.74, size = 29, normalized size = 0.54

$$\frac{4\sqrt{ax+b\sqrt{x}}\,\left(2\,a\sqrt{x}-b\right)}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{3}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)
```

Giac [A]
time = 0.60, size = 53, normalized size = 0.98

$$\frac{4\left(3\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)}{3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqrt
(x) - sqrt(a*x + b*sqrt(x)))^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{3/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.122 $\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=112

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}+\frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x}+\frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}$$

```
[Out] -4/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2+24/35*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)-
32/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x+64/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^4/x
^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2041, 2039}

$$\frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}}-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) + (24*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^2
*x^(3/2)) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x) + (64*a^3*Sqrt[b*Sqrt
[x] + a*x])/(35*b^4*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} - \frac{(6a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} + \frac{(24a^2)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} - \frac{(16a^3)\int \frac{1}{x\sqrt{b\sqrt{x}}}}{35b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} + \frac{64a^3\sqrt{b\sqrt{x}+a}}{35b^4\sqrt{x}}$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 0.53

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^3-6ab^2\sqrt{x}+8a^2bx-16a^3x^{3/2}\right)}{35b^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^3 - 6*a*b^2*Sqrt[x] + 8*a^2*b*x - 16*a^3*x^(
3/2)))/(35*b^4*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.39, size = 240, normalized size = 2.14

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{7b\,x^2}-\frac{12a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}\right)}{5b}\right)}{7b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(70x^{\frac{9}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}+70x^{\frac{9}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{9}{2}}-140x^{\frac{7}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{7}{2}}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(b*x^(1/2)+a*x)^(1/2)*(70*x^(9/2)*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)+70*x^
(9/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(9/2)-140*x^(7/2)*(b*x^(1/2)+a*x)^(3/
2)*a^(7/2)+35*x^(9/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b
)/a^(1/2))*a^4*b-35*x^(9/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(
1/2)*a^(1/2)+b)/a^(1/2))*a^4*b-44*x^(5/2)*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*b^2
+76*a^(5/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^3+20*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*b^
3*x^2)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^5/x^(9/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)
```

Fricas [A]
time = 3.59, size = 50, normalized size = 0.45

$$-\frac{4\left(8\,a^{2}bx+5\,b^{3}-2\left(8\,a^{3}x+3\,ab^{2}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\,b^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/35*(8*a^2*b*x + 5*b^3 - 2*(8*a^3*x + 3*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt
(x))/(b^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{5}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)
```

Giac [A]
time = 0.54, size = 115, normalized size = 1.03

$$\frac{4\left(70\,a^{\frac{3}{2}}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}+84\,ab\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{2}+35\,\sqrt{a}\,b^{2}\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+5\,b^{3}\right)}{35\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x) -
sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.123 $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=170

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}+\frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}+$$

```
[Out] -4/11*(b*x^(1/2)+a*x)^(1/2)/b/x^3+40/99*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2)
-320/693*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2+128/231*a^3*(b*x^(1/2)+a*x)^(1/2
)/b^4/x^(3/2)-512/693*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x+1024/693*a^5*(b*x^(1/
2)+a*x)^(1/2)/b^6/x^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2041, 2039}

$$\frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) + (40*a*Sqrt[b*Sqrt[x] + a*x])/(99*b^
2*x^(5/2)) - (320*a^2*Sqrt[b*Sqrt[x] + a*x])/(693*b^3*x^2) + (128*a^3*Sqrt[
b*Sqrt[x] + a*x])/(231*b^4*x^(3/2)) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(693*
b^5*x) + (1024*a^5*Sqrt[b*Sqrt[x] + a*x])/(693*b^6*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} - \frac{(10a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} + \frac{(80a^2)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{99b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} - \frac{(160a^3)\int \frac{1}{x^2\sqrt{b}}}{231}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+}}{231b^4x^{3/2}}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+}}{231b^4x^{3/2}}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+}}{231b^4x^{3/2}}$$

Mathematica [A]
time = 0.11, size = 83, normalized size = 0.49

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(63b^5-70ab^4\sqrt{x}+80a^2b^3x-96a^3b^2x^{3/2}+128a^4bx^2-256a^5x^{5/2}\right)}{693b^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(63*b^5 - 70*a*b^4*Sqrt[x] + 80*a^2*b^3*x - 96*a^
3*b^2*x^(3/2) + 128*a^4*b*x^2 - 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.41, size = 284, normalized size = 1.67

method	result

derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{11b\,x^{3}}-\frac{20a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(\cdots\right.}{\cdots}\right)}{\cdots}\right)}{9b\cdots}\right)}{11b\cdots}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(1386x^{\frac{13}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{13}{2}}+1386x^{\frac{13}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{13}{2}}-2772x^{\frac{11}{2}}\left(b\sqrt{x}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/693*(b*x^(1/2)+a*x)^(1/2)*(1386*x^(13/2)*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)+
1386*x^(13/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(13/2)-2772*x^(11/2)*(b*x^(1/
2)+a*x)^(3/2)*a^(11/2)-693*x^(13/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/
2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^6*b+693*x^(13/2)*ln(1/2*(2*a*x^(1/2)+2*(
b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^6*b-1236*x^(9/2)*(b*x^(1/2)+a*x)
^(3/2)*a^(7/2)*b^2+1748*a^(9/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^5-532*x^(7/2)*(b*
x^(1/2)+a*x)^(3/2)*a^(3/2)*b^4+852*x^4*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^3+25
2*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*b^5*x^3)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^7/
a^(1/2)/x^(13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)
```

Fricas [A]
time = 2.97, size = 72, normalized size = 0.42

$$-\frac{4 \left(128 \, a^{4} b x^{2} + 80 \, a^{2} b^{3} x + 63 \, b^{5} - 2 \left(128 \, a^{5} x^{2} + 48 \, a^{3} b^{2} x + 35 \, a b^{4}\right) \sqrt{x}\right) \sqrt{a x + b \sqrt{x}}}{693 \, b^{6} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^2
*x + 35*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^6*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}} \sqrt{a x + b \sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(a*x + b*sqrt(x))), x)
```

Giac [A]
time = 0.54, size = 177, normalized size = 1.04

$$\frac{4 \left(3696 \, a^{\frac{5}{2}} \left(\sqrt{a} \sqrt{x} - \sqrt{a x + b \sqrt{x}}\right)^{5} + 7920 \, a^{2} b \left(\sqrt{a} \sqrt{x} - \sqrt{a x + b \sqrt{x}}\right)^{4} + 6930 \, a^{\frac{3}{2}} b^{2} \left(\sqrt{a} \sqrt{x} - \sqrt{a x + b \sqrt{x}}\right)^{3} + 3080 \, a b^{3} \left(\sqrt{a} \sqrt{x} - \sqrt{a x + b \sqrt{x}}\right)^{2} + 693 \, \sqrt{a} b^{4} \left(\sqrt{a} \sqrt{x} - \sqrt{a x + b \sqrt{x}}\right) + 63 \, b^{5}\right)}{693 \left(\sqrt{a} \sqrt{x} - \sqrt{a x + b \sqrt{x}}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x
+ b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
 + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2} \sqrt{a\, x + b \sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.124 $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4}-\frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3}+\frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2}$$

[Out] 315/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)-4*x^(5/2)/a/(b*x^(1/2)+a*x)^(1/2)-315/32*b^3*(b*x^(1/2)+a*x)^(1/2)/a^5-21/4*b*x*(b*x^(1/2)+a*x)^(1/2)/a^3+9/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+105/16*b^2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4

Rubi [A]
time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2043, 682, 684, 654, 634, 212}

$$\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-\frac{4x^{5/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(5/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (315*b^3*Sqrt[b*Sqrt[x] + a*x])/(32*a^5) + (105*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(16*a^4) - (21*b*x*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) + (9*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (315*b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(32*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\text{Subst}\left(\int \frac{x^6}{(bx+ax^2)^{3/2}}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{18\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{(63b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{4a^2}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{(105b^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}}}{2a^2}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{}}}{4a^3}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{}}}{4a^3}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{}}}{4a^3}$$

Mathematica [A]
time = 0.29, size = 124, normalized size = 0.73

$$\frac{\sqrt{b\sqrt{x}+ax}\,\left(-315b^4-105ab^3\sqrt{x}+42a^2b^2x-24a^3bx^{3/2}+16a^4x^2\right)}{32a^5\left(b+a\sqrt{x}\right)} - \frac{315b^4\log\left(b+2a\sqrt{x}-2\sqrt{a}\sqrt{b\sqrt{x}+ax}\right)}{64a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

[Out] (Sqrt[b*Sqrt[x] + a*x]*(-315*b^4 - 105*a*b^3*Sqrt[x] + 42*a^2*b^2*x - 24*a^3*b*x^(3/2) + 16*a^4*x^2))/(32*a^5*(b + a*Sqrt[x])) - (315*b^4*Log[b + 2*a*Sqrt[x] - 2*Sqrt[a]*Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(125) = 250$.
time = 0.39, size = 527, normalized size = 3.08

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{2a\sqrt{b\sqrt{x}+ax}} - \frac{9b\left(\frac{x^{2}}{3a\sqrt{b\sqrt{x}+ax}} - \frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}} - \frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}} - \frac{3b\left(-\frac{\cdots}{a\sqrt{b\cdots}}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

default	$\frac{\sqrt{b\sqrt{x}+ax}\left(32x^{\frac{3}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{11}{2}}+276x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}b^{2}-48\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}bx+690x\sqrt{b\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(b*x^(1/2)+a*x)^(1/2)/a^(13/2)*(32*x^(3/2)*(b*x^(1/2)+a*x)^(3/2)*a^(11
/2)+276*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*b^2-48*(b*x^(1/2)+a*x)^(3/2)*
a^(9/2)*b*x+690*x*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*b^3-768*x*a^(7/2)*(x^(1/2)*
(a*x^(1/2)+b))^(1/2)*b^3+384*x*a^3*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2
)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^4-192*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*a^(7/
2)*b^2-69*x*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))
*a^3*b^4+552*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*b^4-1536*x^(1/2)*a^(5/2)
*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*b^4+768*x^(1/2)*a^2*ln(1/2*(2*a*x^(1/2)+2*(x
^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^5-112*(b*x^(1/2)+a*x)^(3/
2)*a^(5/2)*b^3+256*a^(5/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)*b^3-138*x^(1/2)*ln
(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^2*b^5+138*(
b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^5-768*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*
b^5+384*a*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^
(1/2))*b^6-69*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2
))*a*b^6)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/(a*x^(1/2)+b)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [A]
time = 0.64, size = 136, normalized size = 0.80

$$\frac{1}{32}\sqrt{ax + b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{2\sqrt{x}}{a^2} - \frac{5b}{a^3}\right) + \frac{41b^2}{a^4}\right)\sqrt{x} - \frac{187b^3}{a^5}\right) - \frac{315b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{64a^{\frac{11}{2}}} - \frac{4b^5}{\left(a\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) + \sqrt{a}b\right)a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(2*sqrt(x)/a^2 - 5*b/a^3) + 41*b^2
/a^4)*sqrt(x) - 187*b^3/a^5) - 315/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(
x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2) - 4*b^5/((a*(sqrt(a)*sqrt(x) - s
qrt(a*x + b*sqrt(x))) + sqrt(a)*b)*a^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.125 $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}$$

```
[Out] 15/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)-4*x^(3/2)/a
/(b*x^(1/2)+a*x)^(1/2)-15/2*b*(b*x^(1/2)+a*x)^(1/2)/a^3+5*x^(1/2)*(b*x^(1/2
)+a*x)^(1/2)/a^2
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2043, 682, 684, 654, 634, 212}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}}-\frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3}+\frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(3/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (15*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^
3) + (5*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a^2 + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\text{Subst}\left(\int \frac{x^4}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{10\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} - \frac{(15b)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right.}{2a^2}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{\left(15b^2\right)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{15b^2\tanh^{-1}\left(\right.}{2a^{7/}}$$

<u>Mathematica</u> [A]
time = 0.29, size = 106, normalized size = 0.94

$$\frac{\sqrt{b\sqrt{x}+ax}\left(-15b^2-5ab\sqrt{x}+2a^2x\right)}{2a^3\left(b+a\sqrt{x}\right)} - \frac{15b^2\log\left(a^3b+2a^4\sqrt{x}-2a^{7/2}\sqrt{b\sqrt{x}+ax}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-15*b^2 - 5*a*b*Sqrt[x] + 2*a^2*x))/(2*a^3*(b + a*S
qrt[x])) - (15*b^2*Log[a^3*b + 2*a^4*Sqrt[x] - 2*a^(7/2)*Sqrt[b*Sqrt[x] + a
*x]])/(4*a^(7/2))
```

<u>Maple</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(83) = 166$.
time = 0.39, size = 440, normalized size = 3.89

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{a\sqrt{b\sqrt{x}+ax}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{}}{ba\sqrt{b\sqrt{x}}}\right.}{2a}\right.}{2a}\right.}{2a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(4x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}+10x\sqrt{b\sqrt{x}+ax}\,a^{\frac{7}{2}}b-32x\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{7}{2}}b+16\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^(1/2)+a*x)^(1/2)/a^(9/2)*(4*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)+
10*x*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*b-32*x*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(
7/2)*b+16*x*a^3*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)
+b)/a^(1/2))*b^2-x*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a
^(1/2))*a^3*b^2+8*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*b^2-64*x^(1/2)*(x^(
1/2)*(a*x^(1/2)+b))^(1/2)*a^(5/2)*b^2+32*x^(1/2)*a^2*ln(1/2*(2*a*x^(1/2)+2*
(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^3+16*(x^(1/2)*(a*x^(1/2
)+b))^(3/2)*a^(5/2)*b-2*x^(1/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)
*a^(1/2)+b)/a^(1/2))*a^2*b^3+2*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^3-32*(x^(1/2
)*(a*x^(1/2)+b))^(1/2)*a^(3/2)*b^3+16*a*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x
^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*b^4-ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a
*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^4)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/(a*x^(1/
2)+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [**A**]
time = 0.73, size = 108, normalized size = 0.96

$$\frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a^2}-\frac{7b}{a^3}\right)-\frac{15b^2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{4a^{\frac{7}{2}}}-\frac{4b^3}{\left(a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+\sqrt{a}\,b\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a^2 - 7*b/a^3) - 15/4*b^2*log(abs(-2*s
qrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(7/2) - 4*b^3/((a*
(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + sqrt(a)*b)*a^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.126 $\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

```
[Out] 4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)-4*x^(1/2)/a/(b*x^(
1/2)+a*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2043, 666, 634, 212}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[
b*Sqrt[x] + a*x]])/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[
```

```
e^2*((p + 2)/(c*(p + 1))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\text{Subst}\left(\int \frac{x^2}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\ &= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}\\ &= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 76, normalized size = 1.27

$$-\frac{4\sqrt{b\sqrt{x}+ax}}{a\left(b+a\sqrt{x}\right)}-\frac{2\log\left(ab+2a^2\sqrt{x}-2a^{3/2}\sqrt{b\sqrt{x}+ax}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(a*(b + a*Sqrt[x])) - (2*Log[a*b + 2*a^2*Sqrt[x]
 - 2*a^(3/2)*Sqrt[b*Sqrt[x] + a*x]])/a^(3/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(44) = 88$.
time = 0.43, size = 240, normalized size = 4.00

method	result
derivativedivides	$-\frac{2\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{a}+\frac{2\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right.}{a^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{b\sqrt{x}+ax}\left(2x\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{5}{2}}-x\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^(1/2)+a*x)^(1/2)/a^(3/2)*(2*x*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(5/2)
-x*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*
a^2*b+4*x^(1/2)*a^(3/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*b-2*x^(1/2)*ln(1/2*(2
*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^2-2*a^(3
/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)+2*a^(1/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*b
^2-ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*
b^3)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b/(a*x^(1/2)+b)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[1]%%%},[2,2]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,3]
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.127 $\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}$$

```
[Out] -4*(b+2*a*x^(1/2))/b^2/(b*x^(1/2)+a*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2038, 627}

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 46, normalized size = 1.53

$$-\frac{4\left(b+2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{b^2\left(b+a\sqrt{x}\right)\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
[Out] (-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^2*(b + a*Sqrt[x])*Sqrt[x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(24) = 48.
time = 0.42, size = 111, normalized size = 3.70

method	result
derivativedivides	$-\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}$
default	$-\frac{4\sqrt{b\sqrt{x}+ax}\left(x\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^2+2\sqrt{x}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}ab-\left(\sqrt{x}\left(a\sqrt{x}+b\right)\right)^{\frac{3}{2}}a^2x+\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}\right.}{\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b^3\left(a\sqrt{x}+b\right)^2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -4*(b*x^(1/2)+a*x)^(1/2)*(x*(b*x^(1/2)+a*x)^(3/2)*a^2+2*x^(1/2)*(b*x^(1/2)+
a*x)^(3/2)*a*b-(x^(1/2)*(a*x^(1/2)+b))^(3/2)*a^2*x+(b*x^(1/2)+a*x)^(3/2)*b^
2)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^3/(a*x^(1/2)+b)^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.
time = 3.33, size = 54, normalized size = 1.80

$$\frac{4\left(abx-\left(2a^2x-b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{a^2b^2x^2-b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b^
4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)
```

Giac [A]
time = 0.55, size = 26, normalized size = 0.87

$$-\frac{4\left(\frac{2\,a\sqrt{x}}{b^2}+\frac{1}{b}\right)}{\sqrt{ax+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x}\,\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.128 $\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}$$

```
[Out] 4/b/x/(b*x^(1/2)+a*x)^(1/2)-24/5*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)+32/5*a*(
b*x^(1/2)+a*x)^(1/2)/b^3/x-64/5*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2040, 2041, 2039}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x}-\frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2))
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(
5*b^4*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx\sqrt{b\sqrt{x}+ax}}+\frac{6\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}-\frac{(24a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b^2}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x}+\frac{(16a^2)\int \frac{1}{x\sqrt{b\sqrt{x}}}}{5b^3}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}$$

Mathematica [A]
time = 0.17, size = 70, normalized size = 0.65

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(b^3-2ab^2\sqrt{x}+8a^2bx+16a^3x^{3/2}\right)}{5b^4\left(b+a\sqrt{x}\right)x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3/
2)))/(5*b^4*(b + a*Sqrt[x])*x^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.39, size = 548, normalized size = 5.12

method	result

derivativedivides	$-\frac{4}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{12a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a\left(b+2a\sqrt{x}\right)}{3b^3\sqrt{b\sqrt{x}+ax}}\right)}{5b}$
default	$\frac{2\sqrt{b\sqrt{x}+ax}\left(10x^{\frac{9}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{11}{2}}+10x^{\frac{9}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{11}{2}}-30x^{\frac{7}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*x^(1/2)+a*x)^(1/2)*(10*x^(9/2)*(b*x^(1/2)+a*x)^(1/2)*a^(11/2)+10*x^(
9/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(11/2)-30*x^(7/2)*(b*x^(1/2)+a*x)^(3/2
)*a^(9/2)+10*x^(7/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)*a^(9/2)+10*x^(7/2)*(b*x^
(1/2)+a*x)^(1/2)*a^(7/2)*b^2+10*x^(7/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(7/
2)*b^2-5*x^(9/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2
)+b)/a^(1/2))*a^5*b+5*x^(9/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a
^(1/2)+b)/a^(1/2))*a^5*b-16*x^(5/2)*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^2+20*x^
4*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*b+20*x^4*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(9
/2)*b-5*x^(7/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)
+b)/a^(1/2))*a^3*b^3+5*x^(7/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*
a^(1/2)+b)/a^(1/2))*a^3*b^3-52*x^3*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*b-2*x^(3/2
)*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*b^4+4*x^2*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*b^3
-10*x^4*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1
/2))*a^4*b^2+10*x^4*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/
a^(1/2))*a^4*b^2)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^5/(a*x^(1/2)+b)^2/a^(1/2)
/x^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

Fricas [A]
time = 2.80, size = 79, normalized size = 0.74

$$\frac{4\left(8a^3bx^2-3ab^3x-\left(16a^4x^2-10a^2b^2x-b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{5\left(a^2b^4x^3-b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*s
qrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} \left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.129 $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=165

$$\frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}-\frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{63b^5x}-10$$

```
[Out] 4/b/x^2/(b*x^(1/2)+a*x)^(1/2)-40/9*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2)+320/63
*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2-128/21*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(3/
2)+512/63*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x-1024/63*a^4*(b*x^(1/2)+a*x)^(1/2)
/b^6/x^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2040, 2041, 2039}

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x}-\frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}}+\frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2}-\frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) - (40*Sqrt[b*Sqrt[x] + a*x])/(9*b^2*x^(5/2)
) + (320*a*Sqrt[b*Sqrt[x] + a*x])/(63*b^3*x^2) - (128*a^2*Sqrt[b*Sqrt[x] +
a*x])/(21*b^4*x^(3/2)) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(63*b^5*x) - (1024
*a^4*Sqrt[b*Sqrt[x] + a*x])/(63*b^6*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} + \frac{10\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} - \frac{(80a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b^2}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} + \frac{(160a^2)\int \frac{}{x^2\sqrt{b}}}{21b}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+}}{21b^4x^{3/2}}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+}}{21b^4x^{3/2}}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+}}{21b^4x^{3/2}}$$

Mathematica [A]
time = 0.19, size = 96, normalized size = 0.58

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(7b^5-10ab^4\sqrt{x}+16a^2b^3x-32a^3b^2x^{3/2}+128a^4bx^2+256a^5x^{5/2}\right)}{63b^6\left(b+a\sqrt{x}\right)x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^3
*b^2*x^(3/2) + 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*(b + a*Sqrt[x])*x^
(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.40, size = 592, normalized size = 3.59

method	result
derivativedivides	$-\frac{4}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{20a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{}}}\right.}{}\right)}{7b}\right)}{9b}$
default	$\frac{4\sqrt{b\sqrt{x}+ax}\left(126x^{\frac{13}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{15}{2}}+126x^{\frac{13}{2}}a^{\frac{15}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}-315x^{\frac{11}{2}}\left(b\sqrt{x}+ax\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/63*(b*x^(1/2)+a*x)^(1/2)*(126*x^(13/2)*(b*x^(1/2)+a*x)^(1/2)*a^(15/2)+126
*x^(13/2)*a^(15/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)-315*x^(11/2)*(b*x^(1/2)+a*
x)^(3/2)*a^(13/2)+63*x^(11/2)*a^(13/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)+126*x^
(11/2)*(b*x^(1/2)+a*x)^(1/2)*a^(11/2)*b^2+126*x^(11/2)*a^(11/2)*(x^(1/2)*(a
*x^(1/2)+b))^(1/2)*b^2-128*x^(9/2)*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*b^2+63*x^(
13/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^7*b
-63*x^(13/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)
/a^(1/2))*a^7*b+252*x^6*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*b+252*x^6*a^(13/2)*(
x^(1/2)*(a*x^(1/2)+b))^(1/2)*b+63*x^(11/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)
+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^5*b^3-63*x^(11/2)*ln(1/2*(2*a*x^(1/2)+2*(
x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^5*b^3-508*x^5*(b*x^(1/2)
+a*x)^(3/2)*a^(11/2)*b-16*x^(7/2)*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^4+32*x^4*
(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*b^3-7*x^(5/2)*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*b
^6+126*x^6*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*
a^6*b^2-126*x^6*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)
```

```
+b)/a^(1/2))*a^6*b^2+10*x^3*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*b^5)/(x^(1/2)*(a*
x^(1/2)+b))^(1/2)/b^7/x^(11/2)/a^(1/2)/(a*x^(1/2)+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

Fricas [A]
time = 2.40, size = 101, normalized size = 0.61

$$\frac{4\left(128\,a^5bx^3-48\,a^3b^3x^2-17\,ab^5x-\left(256\,a^6x^3-160\,a^4b^2x^2-26\,a^2b^4x-7\,b^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{63\left(a^2b^6x^4-b^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] 4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4*
b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x^4
 - b^8*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{5}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,\left(a\,x+b\,\sqrt{x}\,\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.130 $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}}+\frac{2560a^3\sqrt{b\sqrt{x}+ax}}{429b^5x^2}-\cdots$$

```
[Out] 4/b/x^3/(b*x^(1/2)+a*x)^(1/2)-56/13*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(7/2)+672/1
43*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^3-2240/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x
^(5/2)+2560/429*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^2-1024/143*a^4*(b*x^(1/2)+a
*x)^(1/2)/b^6/x^(3/2)+4096/429*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x-8192/429*a^6
*(b*x^(1/2)+a*x)^(1/2)/b^8/x^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2040, 2041, 2039}

$$-\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x}-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}}+\frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2}-\frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}}+\frac{672a\sqrt{ax+b\sqrt{x}}}{143b^3x^3}-\frac{56\sqrt{ax+b\sqrt{x}}}{13b^2x^{7/2}}+\frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2
)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x]
 + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2)
 - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sq
rt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[
x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
```

```
x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
- j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}+\frac{14\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}-\frac{(168a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b^2}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}+\frac{(1680a^2)\int \frac{}{x^3\sqrt{}}}{143}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}\cdots}}{429b^4x^{5/2}}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}\cdots}}{429b^4x^{5/2}}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}\cdots}}{429b^4x^{5/2}}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}\cdots}}{429b^4x^{5/2}}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}\cdots}}{429b^4x^{5/2}}$$

Mathematica [A]
time = 0.21, size = 120, normalized size = 0.54

$$-\frac{4\sqrt{b\sqrt{x}+ax}\left(33b^7-42ab^6\sqrt{x}+56a^2b^5x-80a^3b^4x^{3/2}+128a^4b^3x^2-256a^5b^2x^{5/2}+1024a^6bx^3+2048a^7x^{7/2}\right)}{429b^8\left(b+a\sqrt{x}\right)x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a^
3*b^4*x^(3/2) + 128*a^4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 + 20
48*a^7*x^(7/2)))/(429*b^8*(b + a*Sqrt[x])*x^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.38, size = 636, normalized size = 2.85

method	result

derivativedivides	$-\frac{4}{13b\,x^{3}\sqrt{b\sqrt{x}+ax}}-\frac{28a\left(-\frac{2}{11b\,x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{12a\left(-\frac{2}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{10a\left(-\frac{}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{}}}\right.}{}\right.}{}\right.}{}$

default	$\frac{2\sqrt{b\sqrt{x}+ax}\left(2574x^{\frac{17}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{19}{2}}+2574x^{\frac{17}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{19}{2}}-6006x^{\frac{15}{2}}\left(b\sqrt{x}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/429*(b*x^(1/2)+a*x)^(1/2)*(2574*x^(17/2)*(b*x^(1/2)+a*x)^(1/2)*a^(19/2)+2
574*x^(17/2)*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(19/2)-6006*x^(15/2)*(b*x^(1/2
)+a*x)^(3/2)*a^(17/2)+858*x^(15/2)*(x^(1/2)*(a*x^(1/2)+b))^(3/2)*a^(17/2)+2
574*x^(15/2)*(b*x^(1/2)+a*x)^(1/2)*a^(15/2)*b^2+2574*x^(15/2)*(x^(1/2)*(a*x
^(1/2)+b))^(1/2)*a^(15/2)*b^2-2048*x^(13/2)*(b*x^(1/2)+a*x)^(3/2)*a^(13/2)*
b^2+1287*x^(17/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^
(1/2))*a^9*b-1287*x^(17/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*(a*x^(1/2)+b))^(1
/2)*a^(1/2)+b)/a^(1/2))*a^9*b-256*x^(11/2)*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*b^
4+5148*x^8*(b*x^(1/2)+a*x)^(1/2)*a^(17/2)*b+5148*x^8*(x^(1/2)*(a*x^(1/2)+b)
)^(1/2)*a^(17/2)*b+1287*x^(15/2)*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2
)*a^(1/2)+b)/a^(1/2))*a^7*b^3-1287*x^(15/2)*ln(1/2*(2*a*x^(1/2)+2*(x^(1/2)*
(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^7*b^3-9244*x^7*(b*x^(1/2)+a*x)^(
3/2)*a^(15/2)*b-112*x^(9/2)*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^6+512*x^6*(b*x^
(1/2)+a*x)^(3/2)*a^(11/2)*b^3+160*x^5*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*b^5-66*
x^(7/2)*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*b^8+2574*x^8*ln(1/2*(2*a*x^(1/2)+2*(b
*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a^8*b^2-2574*x^8*ln(1/2*(2*a*x^(1/2
)+2*(x^(1/2)*(a*x^(1/2)+b))^(1/2)*a^(1/2)+b)/a^(1/2))*a^8*b^2+84*x^4*(b*x^(
1/2)+a*x)^(3/2)*a^(3/2)*b^7)/(x^(1/2)*(a*x^(1/2)+b))^(1/2)/b^9/x^(15/2)/(a*
x^(1/2)+b)^2/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

Fricas [A]
time = 3.53, size = 123, normalized size = 0.55

$$\frac{4\left(1024\,a^7bx^4-384\,a^5b^3x^3-136\,a^3b^5x^2-75\,ab^7x-\left(2048\,a^8x^4-1280\,a^6b^2x^3-208\,a^4b^4x^2-98\,a^2b^6x-33\,b^8\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{429\left(a^2b^8x^5-b^{10}x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (2
048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)*s
qrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}} \left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(7/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{7/2}\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.131 $\int x^3 \sqrt{b\sqrt[3]{x} + ax} \, dx$

Optimal. Leaf size=301

$$-\frac{884b^6\sqrt{b\sqrt[3]{x} + ax}}{14421a^6} + \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x} + ax}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x} + ax}}{216315a^4} + \frac{476b^3x^2\sqrt{b\sqrt[3]{x} + ax}}{19665a^3} - \frac{28b^2x}{}$$

```
[Out] -884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+884/24035*b^5*x^(2/3)*(b*x^(1/3)+a
*x)^(1/2)/a^5-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+476/19665*b
^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^3-28/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/
a^2+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+
442/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(25/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2046, 2049, 2036, 335, 226}

$$\frac{442b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14421a^{25/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{884b^6\sqrt{ax+b\sqrt[3]{x}}}{14421a^6} + \frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^4} + \frac{476b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^3} - \frac{28b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^2} + \frac{4bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{207a} + \frac{2}{9}x^4\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-884*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^6) + (884*b^5*x^(2/3)*Sqrt[b*x^(1
/3) + a*x])/(24035*a^5) - (6188*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*
a^4) + (476*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^3) - (28*b^2*x^(8/3)*Sq
rt[b*x^(1/3) + a*x])/(1311*a^2) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207
*a) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/9 + (442*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^
(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elliptic
F[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(25/4)*Sqrt[b*x^(1/3)
 + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\text{Subst}\left(\int x^{11}\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)$$
$$= \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{9}(2b)\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)$$
$$= \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}-\frac{(14b^2)\,\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{\right.}}{69a}$$
$$= -\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{(238b^3)\,\text{S}}{}$$
$$= \frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{}$$
$$= -\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+$$
$$= \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.13, size = 155, normalized size = 0.51

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(-9945b^6+3978ab^5x^{2/3}-3094a^2b^4x^{4/3}+2618a^3b^3x^2-2310a^4b^2x^{8/3}+2090a^5bx^{10/3}+24035a^6x^4\right)+9945b^6\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{216315a^6\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-9945*b^6 + 3978*a*b^5*x
^(2/3) - 3094*a^2*b^4*x^(4/3) + 2618*a^3*b^3*x^2 - 2310*a^4*b^2*x^(8/3) + 2
090*a^5*b*x^(10/3) + 24035*a^6*x^4) + 9945*b^6*Hypergeometric2F1[-1/2, 1/4,
 5/4, -((a*x^(2/3))/b)]))/(216315*a^6*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A]
time = 0.35, size = 264, normalized size = 0.88

method	result
derivativedivides	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-6188$
default	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-6188$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a-28/1
311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+476/19665*b^3*x^2*(b*x^(1/3)+a*x)
^(1/2)/a^3-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+884/24035*b^5*
x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+4
42/14421*b^7/a^7*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b
)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{ax + b\sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*x + b*x**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\, \sqrt{a\,x + b\,x^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^(1/3))^(1/2), x)
```

3.132 $\int x^2 \sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal. Leaf size=411

$$\frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}$$

```
[Out] 44/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(9/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/
3)+a*x)^(1/2)-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+220/4641*b^3*x*(
b*x^(1/3)+a*x)^(1/2)/a^3-60/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+4/11
9*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)-44/221*b^
(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^
(1/2))^2)^(1/2)/a^(19/4)/(b*x^(1/3)+a*x)^(1/2)+22/221*b^(21/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(
1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(
19/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\frac{22b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{44b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{9/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{44b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{663a^4}+\frac{220b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^2}+\frac{4bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{119a}+\frac{2}{7}x^3\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (44*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(9/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*S
qrt[b*x^(1/3) + a*x]) - (44*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^4) +
(220*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^3) - (60*b^2*x^(5/3)*Sqrt[b*x^(1/
3) + a*x])/(1547*a^2) + (4*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a) + (2*x^
3*Sqrt[b*x^(1/3) + a*x])/7 - (44*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x]) + (22
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
```

```
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\text{Subst}\left(\int x^8\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{7}(2b)\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} - \frac{(30b^2)\text{ Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{\right.}}{119a}$$

$$= -\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} + \frac{(330b^3)\text{ S}}{}$$

$$= \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} + \frac{2}{7}x^3\sqrt{b}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx}{}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx}{}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx}{}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx}{}$$

$$= \frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{}}}{4641}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.11, size = 136, normalized size = 0.33

$$\frac{2\sqrt[3]{x}\,\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(-385b^4+110ab^3x^{2/3}-90a^2b^2x^{4/3}+78a^3bx^2+663a^4x^{8/3}\right)+385b^4\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{4641a^4\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-385*b^4 + 110*a
*b^3*x^(2/3) - 90*a^2*b^2*x^(4/3) + 78*a^3*b*x^2 + 663*a^4*x^(8/3)) + 385*b
^4*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(4641*a^4*Sqrt[1 +
 (a*x^(2/3))/b])
```

Maple [A]
time = 0.37, size = 273, normalized size = 0.66

method	result
derivativedivides	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^2}+\frac{220b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^3}-\frac{44b^4x^{\frac{1}{3}}}{}$
default	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^2}+\frac{220b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^3}-\frac{44b^4x^{\frac{1}{3}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^3*(b*x^(1/3)+a*x)^(1/2)+4/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a-60/15
47*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+220/4641*b^3*x*(b*x^(1/3)+a*x)^(1/
2)/a^3-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+22/221*b^5/a^5*(-a*b)^(
```

```
1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)
+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b
)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{ax + b\sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x**2*sqrt(a*x + b*x**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a\,x+b\,x^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x^2*(a*x + b*x^(1/3))^(1/2), x)
```

3.133 $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal. Leaf size=213

$$\frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2}+\frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}-\frac{6b^{15/4}\left(\sqrt{b}+\sqrt{a}\right.}{}$$

```
[Out] 12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-36/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2
)/a^2+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/5*x^2*(b*x^(1/3)+a*x)^(1/2)-
6/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(
1/2)+b^(1/2))^2)^(1/2)/a^(13/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2043, 2046, 2049, 2036, 335, 226}

$$-\frac{6b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{13/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{12b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^2}+\frac{4bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{55a}+\frac{2}{5}x^2\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (12*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^3) - (36*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a^2) + (4*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a) + (2*x^2*Sqrt[
b*x^(1/3) + a*x])/5 - (6*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x
^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(77*a^(13/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\text{Subst}\left(\int x^5\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{5}(2b)\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(18b^2)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right.}{55a}$$

$$= -\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{(18b^3)\,\text{Sub}}{}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 118, normalized size = 0.55

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(45b^3-18ab^2x^{2/3}+14a^2bx^{4/3}+77a^3x^2\right)-45b^3\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{385a^3\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(45*b^3 - 18*a*b^2*x^(2/3
) + 14*a^2*b*x^(4/3) + 77*a^3*x^2) - 45*b^3*Hypergeometric2F1[-1/2, 1/4, 5/
4, -((a*x^(2/3))/b)]))/(385*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A]
time = 0.34, size = 198, normalized size = 0.93

method	result
derivativedivides	$\frac{2x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^{2}x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^{2}}+\frac{12b^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^{3}}-\frac{6b^{4}\sqrt{-a}}{}$
default	$\frac{2x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^{2}x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^{2}}+\frac{12b^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^{3}}-\frac{6b^{4}\sqrt{-a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^2*(b*x^(1/3)+a*x)^(1/2)+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a-36/385
*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-
6/77*b^4/a^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{ax+b\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*x + b*x**(1/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x*(a*x + b*x^(1/3))^(1/2), x)
```

3.134 $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal. Leaf size=323

$$-\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}{}$$

```
[Out] -4/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(3/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)
+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/3*x*(b*x^(1/3)+a*x)^(1
/2)+4/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/co
s(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)
/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a
^(1/2)+b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)-2/5*b^(9/4)*x^(1/6)*
(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*
(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)
/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2029, 2043, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{2b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{4b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{3/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{15a}+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-4*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(3/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a) + (2*x*Sqr
t[b*x^(1/3) + a*x])/3 + (4*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*
x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[b*x^(1/3) + a*x]) - (2*b^(9/4)*(S
qrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^
2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{9}(2b)\int \frac{\sqrt[3]{x}}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$

$$= \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{3}(2b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{5a}$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(2b^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax}}\right.}{5a\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(4b^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax}}\right.}{5a\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(4b^{5/2}\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+c}}\right.}{5a^{3/2}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 94, normalized size = 0.29

$$\frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)\sqrt{1+\frac{ax^{2/3}}{b}}-b\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{3a\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(1/3) + a*x],x]

[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))*Sqrt[1 + (a*x^(2/3))/b] - b*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(3*a*Sqrt[1 + (a*x^(2/3))/b])

Maple [A]
time = 0.35, size = 207, normalized size = 0.64

method	result
derivativedivides	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)}{\sqrt{-ab}}}}{}$
default	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)}{\sqrt{-ab}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3*x*(b*x^(1/3)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a-2/5*b^2/a^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2

```
)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + b\sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(sqrt(a*x + b*x**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

Mupad [B]
time = 5.18, size = 40, normalized size = 0.12

$$\frac{6\,x\,\sqrt{a\,x+b\,x^{1/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{7\,\sqrt{\frac{a\,x^{2/3}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2),x)
[Out] (6*x*(a*x + b*x^(1/3))^(1/2)*hypergeom([-1/2, 7/4], 11/4, -(a*x^(2/3))/b))/
(7*((a*x^(2/3))/b + 1)^(1/2))
```

3.135 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$

Optimal. Leaf size=123

$$2\sqrt{b\sqrt[3]{x}+ax}+\frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)+2*b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x
^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 2046, 2036, 335, 226}

$$\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}}+2\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x,x]
```

```
[Out] 2*Sqrt[b*x^(1/3) + a*x] + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/
4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx &= 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + (2b)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{\left(2b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{\left(4b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 54, normalized size = 0.44

$$\frac{6\sqrt{b\sqrt[3]{x}+ax}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x,x]
```

```
[Out] (6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((a*x^(2/3))/b)
])/Sqrt[1 + (a*x^(2/3))/b]
```

Maple [A]
time = 0.36, size = 132, normalized size = 1.07

method	result
derivativedivides	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\;\mathrm{EllipticF}}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\;\mathrm{EllipticF}}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)+2*b/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^
(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(
-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x + b\,x^{1/3}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x, x)
```

3.136 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

Optimal. Leaf size=325

$$\frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}-\frac{12a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{\cdots}{\left(\sqrt{b}\cdots\right.}}}{5b\cdots}$$

```
[Out] 12/5*a^(3/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a
*x)^(1/2)-6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3
)-12/5*a^(5/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)+6/5*a^(5/4)*x^(1/6)*(
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6
)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(
x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/
b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$\frac{6a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{12a^{3/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{5b\sqrt[3]{x}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{5x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (12*a^(3/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(5*x) - (12*a*Sqrt[b*x^(1/3)
+ a*x])/(5*b*x^(1/3)) - (12*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a
*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4
)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (6*a^(5/4)*(
Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))
^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} + \frac{1}{5}(6a)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{(6a^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(6a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\right.}{5b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(12a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax}}\right.}{5b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(12a^{3/2}\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+a}}\right.}{5\sqrt{b}\,\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{12a^{5/4}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 59, normalized size = 0.18

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}\; {}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}\; x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^(2/3))
/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x)
```

Maple [A]
time = 0.34, size = 213, normalized size = 0.66

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\cdots\right.}{\sqrt{-ab}\cdots}}\cdots}{\cdots}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\cdots\right.}{\sqrt{-ab}\cdots}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*(b+a*x^(2/3))*a/b/(x^(1/3)*(b+a*x^(2/3)))
^(1/2)+6/5/b*a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/
2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^
```

```
(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1
/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*Ellipt
icF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**2,x)
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^2,x)
[Out] int((a*x + b*x^(1/3))^(1/2)/x^2, x)
```

3.137 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}+\frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}+\frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}}{77b^{9/4}\sqrt{b\sqrt[3]{x}+a}}$$

```
[Out] -6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20/
77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^(11/4)*x^(1/6)*(cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
)*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^
(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b
*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2045, 2050, 2036, 335, 226}

$$\frac{10a^{11/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{11x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(11*x^2) - (12*a*Sqrt[b*x^(1/3) + a*x])/(77*b*x^
(4/3)) + (20*a^2*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) + (10*a^(11/4)*(Sq
rt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2
]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} + \frac{1}{11}(6a)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} - \frac{(30a^2)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{(10a^3)\,\text{Subst}\left(\int \frac{}{\sqrt{bx}} \right.}{77}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(10a^3\sqrt{b+ax^{2/3}}\,\sqrt[6]{}\right.}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(20a^3\sqrt{b+ax^{2/3}}\,\sqrt[6]{}\right.}{7}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 59, normalized size = 0.31

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}\;{}_2F_1\left(-\frac{11}{4},-\frac{1}{2};-\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{11\sqrt{1+\frac{ax^{2/3}}{b}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((a*x^(2/3)
)/b)])/(11*Sqrt[1 + (a*x^(2/3))/b]*x^2)
```

Maple [A]
time = 0.36, size = 179, normalized size = 0.95

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^2}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^2x^{\frac{2}{3}}}+\frac{10a^2\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^2}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^2x^{\frac{2}{3}}}+\frac{10a^2\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20/
77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^2/b^2*(-a*b)^(1/2)*((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*
EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x^3, x)
```

3.138 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

Optimal. Leaf size=413

$$-\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}}+\frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}}-\frac{308a}{}$$

```
[Out] -308/1105*a^(9/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^
(1/3)+a*x)^(1/2)-6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1
/2)/b/x^(7/3)+44/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*
x^(1/3)+a*x)^(1/2)/b^3/x+308/1105*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+308
/1105*a^(17/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)-154/1105*a^(17/4)*x^
(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)
*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(
1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)
^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{154a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{308a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{308a^{9/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{308a^4\sqrt{ax+b\sqrt[3]{x}}}{1105b^4\sqrt[3]{x}}-\frac{308a^3\sqrt{ax+b\sqrt[3]{x}}}{3315b^3x}+\frac{44a^2\sqrt{ax+b\sqrt[3]{x}}}{663b^2x^{5/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{221bx^{7/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{17x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-308*a^(9/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(17*x^3) - (12*a*Sqrt[
b*x^(1/3) + a*x])/(221*b*x^(7/3)) + (44*a^2*Sqrt[b*x^(1/3) + a*x])/(663*b^2
*x^(5/3)) - (308*a^3*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (308*a^4*Sqrt[b*
x^(1/3) + a*x])/(1105*b^4*x^(1/3)) + (308*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a
*x]) - (154*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} + \frac{1}{17}(6a)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} - \frac{(66a^2)\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{221b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} + \frac{(154a^3)\,\text{Subst}\left(\int \frac{}{x^3\sqrt{}}\right.}{66}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}$$

$$= -\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 59, normalized size = 0.14

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}\ {}_2F_1\left(-\frac{17}{4},-\frac{1}{2};-\frac{13}{4};-\frac{ax^{2/3}}{b}\right)}{17\sqrt{1+\frac{ax^{2/3}}{b}}\ x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-17/4, -1/2, -13/4, -((a*x^(2/3
))/b)])/(17*Sqrt[1 + (a*x^(2/3))/b]*x^3)
```

Maple [A]
time = 0.35, size = 281, normalized size = 0.68

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{17x^3}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{221b\,x^{\frac{7}{3}}}+\frac{44a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^2x^{\frac{5}{3}}}-\frac{308a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^3x}+\frac{308}{1105b^4\sqrt{x}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{17x^3}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{221b\,x^{\frac{7}{3}}}+\frac{44a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^2x^{\frac{5}{3}}}-\frac{308a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^3x}+\frac{308}{1105b^4\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+44
/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*x^(1/3)+a*x)^(1/
2)/b^3/x+308/1105*(b+a*x^(2/3))*a^4/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-154/1
```

```
105*a^4/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*
(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/
2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*
(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF
(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x^4, x)
```

3.139 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

Optimal. Leaf size=276

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}}+\frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}}-\frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}+\frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}-\frac{26}{}$$

```
[Out] -6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+6
8/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x)^
(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/33649*
a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^(23/4)*x^(1/6)*(cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*
a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)
/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2045, 2050, 2036, 335, 226}

$$-\frac{1326a^{23/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{33649b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}}+\frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{884a^3\sqrt{ax+b\sqrt[3]{x}}}{24035b^3x^2}+\frac{68a^2\sqrt{ax+b\sqrt[3]{x}}}{2185b^2x^{8/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{437bx^{10/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{23x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(23*x^4) - (12*a*Sqrt[b*x^(1/3) + a*x])/(437*b*x
^(10/3)) + (68*a^2*Sqrt[b*x^(1/3) + a*x])/(2185*b^2*x^(8/3)) - (884*a^3*Sqr
t[b*x^(1/3) + a*x])/(24035*b^3*x^2) + (7956*a^4*Sqrt[b*x^(1/3) + a*x])/(168
245*b^4*x^(4/3)) - (2652*a^5*Sqrt[b*x^(1/3) + a*x])/(33649*b^5*x^(2/3)) - (
1326*a^(23/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + S
qrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1
/2])/(33649*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} + \frac{1}{23}(6a)\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} - \frac{(102a^2)\,\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{437b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} + \frac{(442a^3)\,\text{Subst}\left(\int \frac{}{x^6\sqrt{}}\right.}{21}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 59, normalized size = 0.21

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}\ {}_2F_1\left(-\frac{23}{4},-\frac{1}{2};-\frac{19}{4};-\frac{ax^{2/3}}{b}\right)}{23\sqrt{1+\frac{ax^{2/3}}{b}}\ x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-23/4, -1/2, -19/4, -((a*x^(2/3
))/b)])/(23*Sqrt[1 + (a*x^(2/3))/b]*x^4)
```

Maple [A]
time = 0.40, size = 245, normalized size = 0.89

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^4}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^2x^{\frac{8}{3}}}-\frac{884a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^3x^2}+\frac{7956a^4\sqrt{b}}{1682}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^4}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^2x^{\frac{8}{3}}}-\frac{884a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^3x^2}+\frac{7956a^4\sqrt{b}}{1682}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+6
8/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x)^
(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/33649*
a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^5/b^5*(-a*b)^(1/2)*((x^(
1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))
*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2
)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x + b\,x^{1/3}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x^5, x)
```

3.140 $\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx$

Optimal. Leaf size=298

$$\frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^8}{}$$

```
[Out] 2/9*x^3*(b*x^(1/3)+a*x)^(3/2)+1768/100947*b^6*(b*x^(1/3)+a*x)^(1/2)/a^5-176
8/168245*b^5*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+1768/216315*b^4*x^(4/3)*(b*x
^(1/3)+a*x)^(1/2)/a^3-136/19665*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+8/1311*b^
2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a+4/69*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)-884
/100947*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6
)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*
a^(1/2)+b^(1/2))^2)^(1/2)/a^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2046, 2049, 2036, 335, 226}

$$-\frac{884b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{100947a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{1768b^6\sqrt{ax+b\sqrt[3]{x}}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^3}-\frac{136b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{9}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (1768*b^6*Sqrt[b*x^(1/3) + a*x])/(100947*a^5) - (1768*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(168245*a^4) + (1768*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(2163
15*a^3) - (136*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^2) + (8*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/69 +
 (2*x^3*(b*x^(1/3) + a*x)^(3/2))/9 - (884*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*a^(21/4)*Sqrt[b*x^(1/3) +
 a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}dx &= 3\text{Subst}\left(\int x^8\left(bx+ax^3\right)^{3/2}dx,x,\sqrt[3]{x}\right)\\
&= \frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{3}(2b)\text{Subst}\left(\int x^9\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{69}\left(4b^2\right)\text{Subst}\left(\int\frac{x^{10}}{\sqrt{bx+ax^3}}\right.\\
&= \frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(68b^3\right)}{}\\
&= -\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}\\
&= \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}\\
&= -\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.13, size = 142, normalized size = 0.48

$$
\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(3315b^4-7293ab^3x^{2/3}+12155a^2b^2x^{4/3}-17765a^3bx^2+24035a^4x^{8/3}\right)-3315b^6\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{216315a^5\sqrt{1+\frac{ax^{2/3}}{b}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]*(3315*b
^4 - 7293*a*b^3*x^(2/3) + 12155*a^2*b^2*x^(4/3) - 17765*a^3*b*x^2 + 24035*a
^4*x^(8/3)) - 3315*b^6*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)])
)/(216315*a^5*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A]
time = 0.38, size = 196, normalized size = 0.66

method	result
default	$-\frac{2\left(-216755x^{\frac{11}{3}}a^6b^2-380380x^{\frac{13}{3}}a^7b+616a^5b^3x^3+1768x^{\frac{5}{3}}a^3b^5-952x^{\frac{7}{3}}a^4b^4-168245a^8x^5+6630b^7\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}}{\sqrt{}}}\right.}{1514205a^6}$
derivativedivides	$\frac{2a\,x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{58b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207}+\frac{8b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a}-\frac{136b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^2}+\frac{176}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1514205*(-216755*x^(11/3)*a^6*b^2-380380*x^(13/3)*a^7*b+616*a^5*b^3*x^3+
1768*x^(5/3)*a^3*b^5-952*x^(7/3)*a^4*b^4-168245*a^8*x^5+6630*b^7*(-a*b)^(1/
2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3
)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-5304*a^2*b^6*x-13260*x^(1/
3)*a*b^7)/a^6/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x^3 + b*x^(7/3))*sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + b*x**(1/3))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(1/3))^(3/2), x)
```

3.141 $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=408

$$-\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}}}{1547a}$$

```
[Out] 2/7*x^2*(b*x^(1/3)+a*x)^(3/2)-88/1105*b^5*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x^
(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+88/3315*b^4*x^(1/3)*(b*x^(1/3)
+a*x)^(1/2)/a^3-88/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^2+24/1547*b^2*x^(5/3)
*(b*x^(1/3)+a*x)^(1/2)/a+12/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)+88/1105*b^(
21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))
,1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(
1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)-44/1105*b^(21/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(
1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(
15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.77, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2043, 2046, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{88b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{88b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{3315a^3}-\frac{88b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{7}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-88*b^5*(b + a*x^(2/3))*x^(1/3))/(1105*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))
*Sqrt[b*x^(1/3) + a*x]) + (88*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(3315*a^3)
 - (88*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^2) + (24*b^2*x^(5/3)*Sqrt[b*x^(
1/3) + a*x])/(1547*a) + (12*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/119 + (2*x^2*(
b*x^(1/3) + a*x)^(3/2))/7 + (88*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(
b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a
^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
```

```
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx = 3\text{Subst}\left(\int x^5\left(bx+ax^3\right)^{3/2}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{7}(6b)\text{Subst}\left(\int x^6\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{119}\left(12b^2\right)\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\right.$$

$$= \frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{(132b^3}{}$$

$$= -\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2$$

$$= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx$$

$$= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx$$

$$= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx$$

$$= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx$$

$$= -\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{}}{4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 123, normalized size = 0.30

$$\frac{2\sqrt[3]{x}\,\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(77b^2-143abx^{2/3}+221a^2x^{4/3}\right)-77b^4\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{1547a^3\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]
*(77*b^2 - 143*a*b*x^(2/3) + 221*a^2*x^(4/3)) - 77*b^4*Hypergeometric2F1[-3
/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(1547*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A]
time = 0.36, size = 261, normalized size = 0.64

method	result
default	$\frac{622x^{\frac{8}{3}}a^4b^2}{1547}+\frac{80x^{\frac{10}{3}}a^5b}{119}-\frac{16a^3b^3x^2}{4641}-\frac{88b^6\sqrt{\frac{ax^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(ax^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticE}\Big(\sqrt{}}{1105}$
derivativedivides	$\frac{2ax^3\sqrt{bx^{\frac{1}{3}}+ax}}{7}+\frac{46bx^{\frac{7}{3}}\sqrt{bx^{\frac{1}{3}}+ax}}{119}+\frac{24b^2x^{\frac{5}{3}}\sqrt{bx^{\frac{1}{3}}+ax}}{1547a}-\frac{88b^3x\sqrt{bx^{\frac{1}{3}}+ax}}{4641a^2}+\frac{88b^4x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23205/a^4*(4665*x^(8/3)*a^4*b^2+7800*x^(10/3)*a^5*b-40*a^3*b^3*x^2-924*b^
6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3)
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*b^6*((a*x^(1/3)+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))+3315*a^6*x^4+308*x^(2/3)*a*b^5+88*x^(4/3)*a^2*b^
4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a*x^2 + b*x^(4/3))*sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x*(a*x + b*x**(1/3))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x*(a*x + b*x^(1/3))^(3/2), x)
```

3.142 $\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=208

$$-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{4b^{15/4}\left(\sqrt{b}+\sqrt{a}\right.}{}$$

```
[Out] 2/5*x*(b*x^(1/3)+a*x)^(3/2)-8/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^2+24/385*b^2*x
^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a+12/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)+4/77*b
^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
)),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b
^(1/2))^2)^(1/2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2029, 2043, 2046, 2049, 2036, 335, 226}

$$\frac{4b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-8*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^2) + (24*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a) + (12*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/55 + (2*x*(b*x^(1/3) +
 a*x)^(3/2))/5 + (4*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(77*a^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}dx=\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(2b)\int \sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,dx$$

$$=\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(6b)\text{Subst}\left(\int x^3\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)$$

$$=\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{55}\left(12b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}dx,x\right.$$

$$=\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(12b^3\right)\text{Subs}}{}$$

$$=-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}\right.$$

$$=-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}\right.$$

$$=-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}\right.$$

$$=-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 106, normalized size = 0.51

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-\left(\left(5b-11ax^{2/3}\right)\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\right)+5b^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{55a^2\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-((5*b - 11*a*x^(2/3))*(b + a*x^(2/3))^2*Sqrt[1 +
 (a*x^(2/3))/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)
]))/(55*a^2*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A]
time = 0.35, size = 163, normalized size = 0.78

method	result
default	$\frac{\frac{262x^{\frac{5}{3}}a^{3}b^{2}}{385}+\frac{56a^{4}bx^{\frac{7}{3}}}{55}+\frac{4b^{4}\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{77}}{a^{3}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$\frac{2a\,x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{34b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55}+\frac{24b^{2}x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a}-\frac{8b^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^{2}}+\frac{4b^{4}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/385*(131*x^(5/3)*a^3*b^2+196*a^4*b*x^(7/3)+10*b^4*(-a*b)^(1/2)*((a*x^(1/3
)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-8*a^2*b^3*x+77*a^5*x^3-20*a*b^4*x^(1/3)
)/a^3/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral((a*x + b*x**(1/3))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

Mupad [B]
time = 5.19, size = 40, normalized size = 0.19

$$\frac{2\,x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{9}{4};\ \frac{13}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{3\,\left(\frac{a\,x^{2/3}}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2),x)
[Out] (2*x*(a*x + b*x^(1/3))^(3/2)*hypergeom([-3/2, 9/4], 13/4, -(a*x^(2/3))/b))/
(3*((a*x^(2/3))/b + 1)^(3/2))
```

3.143 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=319

$$\frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{8b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}{}$$

```
[Out] 2/3*(b*x^(1/3)+a*x)^(3/2)+8/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(1/2)/(x^(1/3)*a^
(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4/5*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)-8/
5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)
+b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^(1/3)+a*x)^(1/2)+4/5*b^(9/4)*x^(1/6)*(cos(2
*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/
3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/
4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2046, 2029, 2057, 335, 311, 226, 1210}

$$\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{8b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5\sqrt{a}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4}{5}b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{3}\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (8*b^2*(b + a*x^(2/3))*x^(1/3))/(5*Sqrt[a]*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/5 + (2*(b*x^(1/3)
+ a*x)^(3/2))/3 - (8*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (4*b^(9/4)*(Sqrt[b]
 + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(
1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + (2b)\text{Subst}\left(\int \sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{1}{5}\left(4b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{\right.$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(4b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{}}{\sqrt{b+}}\right.}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(8b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{}{\sqrt{b+}}\right.}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(8b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{5\sqrt{a}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 60, normalized size = 0.19

$$\frac{2b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((a*x
^(2/3))/b)])/Sqrt[1 + (a*x^(2/3))/b]
```

Maple [A]
time = 0.35, size = 228, normalized size = 0.71

method	result
derivativedivides	$\frac{2ax\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{22b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15}+\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)\cdots}{\sqrt{-ab}}}\cdots}{\cdots}$
default	$\frac{8b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{4b^{3}\sqrt{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a*(12*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)
-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*Elliptic
E(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-6*b^3*((a*x^(1
/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+11*a*b^2*x^(2/3)+16*a^2*b*x^(4/3)+5*a
^3*x^2)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x, x)
```

3.144 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=144

$$4a\sqrt{b\sqrt[3]{x}+ax}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}+\frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\ldots}{}\right.\right.}{\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] -2*(b*x^(1/3)+a*x)^(3/2)/x+4*a*(b*x^(1/3)+a*x)^(1/2)+4*a^(3/4)*b^(3/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/
2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(
1/2)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2045, 2046, 2036, 335, 226}

$$\frac{4a^{3/4}b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}+4a\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] 4*a*Sqrt[b*x^(1/3) + a*x] - (2*(b*x^(1/3) + a*x)^(3/2))/x + (4*a^(3/4)*b^(3
/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(
1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/Sqrt[
b*x^(1/3) + a*x]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (6a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (4ab)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(4ab\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+\ldots}}\ldots\right.}{\sqrt{b\sqrt[3]{x}+ax}}$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(8ab\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\ldots\right.}{\sqrt{b\sqrt[3]{x}+ax}}$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/\ldots}}{\left(\sqrt{b}+\sqrt{a}\ldots\right.}}\ldots}{\sqrt{b\sqrt[3]{x}+\ldots}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 60, normalized size = 0.42

$$-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}\;x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^(2/3)
)/b)])/(Sqrt[1 + (a*x^(2/3))/b]*x^(2/3))
```

Maple [**A**]
time = 0.35, size = 130, normalized size = 0.90

method	result

default	$\frac{4x^{\frac{1}{3}}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-}}{x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{x^{\frac{2}{3}}}+2a\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{\sqrt{b\,x^{\frac{1}{3}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/x^(1/3)*(2*x^(1/3)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x
^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*El
lipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(
1/2)*b+x^(4/3)*a^2-b^2)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

3.145 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=350

$$\frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}-\frac{8a^{9/4}\left(\sqrt{b}\right.}{}$$

```
[Out] -2/3*(b*x^(1/3)+a*x)^(3/2)/x^2+8/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3)
*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-4/5*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5*a
^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-8/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ellipti
cE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(
1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+
a*x)^(1/2)+4/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^
(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$\frac{4a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{8a^{5/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{8a^2\sqrt{ax+b\sqrt[3]{x}}}{5b\sqrt[3]{x}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x^2}-\frac{4a\sqrt{ax+b\sqrt[3]{x}}}{5x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (8*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b
*x^(1/3) + a*x]) - (4*a*Sqrt[b*x^(1/3) + a*x])/(5*x) - (8*a^2*Sqrt[b*x^(1/3
) + a*x])/(5*b*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(3*x^2) - (8*a^(9/4)*
(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3)
)^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4
)*Sqrt[b*x^(1/3) + a*x]) + (4*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
 a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1
/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + (2a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{1}{5}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{\ldots}\right.$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{b\ldots}}\ldots\right.}{\ldots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(4a^3\sqrt{b+ax^{2/3}}\,\sqrt[6]{\ldots}\right.}{\ldots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(8a^3\sqrt{b+ax^{2/3}}\,\sqrt[6]{\ldots}\right.}{\ldots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(8a^{5/2}\sqrt{b+ax^{2/3}}\ldots\right.}{\ldots}$$

$$= \frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2(\ldots}{\ldots}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 62, normalized size = 0.18

$$-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\ {}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{3\sqrt{1+\frac{ax^{2/3}}{b}}\ x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^(2/3
))/b)])/(3*Sqrt[1 + (a*x^(2/3))/b]*x^(5/3))
```

Maple [A]
time = 0.34, size = 339, normalized size = 0.97

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{3x^{\frac{5}{3}}}-\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15x}-\frac{8\left(b+a\,x^{\frac{2}{3}}\right)a^{2}}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{4a^{2}\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)}{\sqrt{-ab}}}}{}$
default	$-\frac{2\left(-12a^{2}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(-12*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/
3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(8/3
)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))+6*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+12*(b*x^(1/3)+a*x)^(1/2)*x^(10/3
)*a^3+12*(b*x^(1/3)+a*x)^(1/2)*x^(8/3)*a^2*b+16*x^2*(x^(1/3)*(b+a*x^(2/3)))
```

```
^(1/2)*a*b^2+11*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b+5*x^(4/3)*(x^(1
/3)*(b+a*x^(2/3)))^(1/2)*b^3)/b/x^3/(b+a*x^(2/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**3,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

3.146 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=213

$$-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{4a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}{}$$

```
[Out] -2/5*(b*x^(1/3)+a*x)^(3/2)/x^3-12/55*a*(b*x^(1/3)+a*x)^(1/2)/x^2-24/385*a^2
*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+8/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)
+4/77*a^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2045, 2050, 2036, 335, 226}

$$\frac{4a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{5x^3}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{55x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-12*a*Sqrt[b*x^(1/3) + a*x])/(55*x^2) - (24*a^2*Sqrt[b*x^(1/3) + a*x])/(38
5*b*x^(4/3)) + (8*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) - (2*(b*x^(1/
3) + a*x)^(3/2))/(5*x^3) + (4*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{1}{5}(6a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{1}{55}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx,\right.$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}-\frac{\left(12a^3\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/}}{5x^3}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/}}{5x^3}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/}}{5x^3}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/}}{5x^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 62, normalized size = 0.29

$$-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\;{}_2F_1\left(-\frac{15}{4},-\frac{3}{2};-\frac{11}{4};-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}\;x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((a*x^(2
/3))/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x^(8/3))
```

Maple [A]
time = 0.34, size = 168, normalized size = 0.79

method	result
default	$\dfrac{\dfrac{4a^{3}\sqrt{-ab}\,\sqrt{\dfrac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\dfrac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\dfrac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\dfrac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\dfrac{\sqrt{2}}{2}\right.}{77}}{b^{2}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{14}{3}}}$
derivativedivides	$-\dfrac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x^{\frac{8}{3}}}-\dfrac{34a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55x^{2}}-\dfrac{24a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b\,x^{\frac{4}{3}}}+\dfrac{8a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{2}x^{\frac{2}{3}}}+\dfrac{4a^{3}\sqrt{-ab}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/385*(10*a^3*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1
/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^
(14/3)-131*x^(11/3)*a^2*b^2+8*x^(13/3)*a^3*b-196*a*b^3*x^3+20*a^4*x^5-77*x^
(7/3)*b^4)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(14/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**4,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^4,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

3.147 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=438

$$-\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}-88$$

```
[Out] -2/7*(b*x^(1/3)+a*x)^(3/2)/x^4-88/1105*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(
x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-12/119*a*(b*x^(1/3)+a*x)^(1/
2)/x^3-24/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3)+a
*x)^(1/2)/b^2/x^(5/3)-88/3315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*a^5*(
b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+88/1105*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(
1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Elli
pticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+
b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1
/3)+a*x)^(1/2)-44/1105*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(
2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{44a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{88a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{88a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88a^5\sqrt{ax+b\sqrt[3]{x}}}{1105b^4\sqrt[3]{x}}-\frac{88a^4\sqrt{ax+b\sqrt[3]{x}}}{3315b^3x}+\frac{88a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^2x^{5/3}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{1547bx^{7/3}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{7x^4}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{119x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-88*a^(11/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (12*a*Sqrt[b*x^(1/3) + a*x])/(119*x^3) - (24*a^2
*Sqrt[b*x^(1/3) + a*x])/(1547*b*x^(7/3)) + (88*a^3*Sqrt[b*x^(1/3) + a*x])/(
4641*b^2*x^(5/3)) - (88*a^4*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (88*a^5*S
qrt[b*x^(1/3) + a*x])/(1105*b^4*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(7*x
^4) + (88*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b
] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4
)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44*a^(21/4)*(Sqrt[b] + S
qrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)
*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{13}}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4}+\frac{1}{7}(6a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4}+\frac{1}{119}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx\right.$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4}-\frac{\left(132a^3\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3}}{7x^4}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}-\frac{88a^4\sqrt{b\sqrt[3]{x}+}}{3315b^3x}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}-\frac{88a^4\sqrt{b\sqrt[3]{x}+}}{3315b^3x}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}-\frac{88a^4\sqrt{b\sqrt[3]{x}+}}{3315b^3x}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}-\frac{88a^4\sqrt{b\sqrt[3]{x}+}}{3315b^3x}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}-\frac{88a^4\sqrt{b\sqrt[3]{x}+}}{3315b^3x}$$

$$= -\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+}}{1547bx^{7/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 62, normalized size = 0.14

$$-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\ {}_2F_1\left(-\frac{21}{4},-\frac{3}{2};-\frac{17}{4};-\frac{ax^{2/3}}{b}\right)}{7\sqrt{1+\frac{ax^{2/3}}{b}}\ x^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-21/4, -3/2, -17/4, -((a*x^(2
/3))/b)])/(7*Sqrt[1 + (a*x^(2/3))/b]*x^(11/3))
```

Maple [A]
time = 0.35, size = 411, normalized size = 0.94

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{7x^{\frac{11}{3}}}-\frac{46a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119x^{3}}-\frac{24a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b\,x^{\frac{7}{3}}}+\frac{88a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^{2}x^{\frac{5}{3}}}-\frac{88a^{4}\sqrt{b\,x^{\frac{1}{3}}}}{3315b^{3}}$
default	$-\frac{88a^{5}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\ \mathrm{EllipticE}\left(\sqrt{\frac{a\,x}{}}\right.}{1105}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/23205*(-924*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^
(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(
20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2))+462*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a
*b)^(1/2))^(1/2)*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+924*(b*x^(1/3)+a*x)^(1/2)
*x^(22/3)*a^6+924*(b*x^(1/3)+a*x)^(1/2)*x^(20/3)*a^5*b-88*x^6*(x^(1/3)*(b+a
*x^(2/3)))^(1/2)*a^4*b^2-308*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*b-4
```

```
665*x^(14/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4+40*x^(16/3)*(x^(1/3)*(b+
a*x^(2/3)))^(1/2)*a^3*b^3-7800*x^4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^5-3315
*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^6)/b^4/x^7/(b+a*x^(2/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**5,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^5,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

3.148 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=301

$$-\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}}+\frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}}-\frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2}+\frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}$$

```
[Out] -2/9*(b*x^(1/3)+a*x)^(3/2)/x^5-4/69*a*(b*x^(1/3)+a*x)^(1/2)/x^4-8/1311*a^2*
(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+136/19665*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(8/3)-1768/216315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+1768/168245*a^5*(b*x^(1
/3)+a*x)^(1/2)/b^4/x^(4/3)-1768/100947*a^6*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3
)-884/100947*a^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2045, 2050, 2036, 335, 226}

$$-\frac{884a^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{100947b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}}+\frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{1768a^4\sqrt{ax+b\sqrt[3]{x}}}{216315b^3x^2}+\frac{136a^3\sqrt{ax+b\sqrt[3]{x}}}{19665b^2x^{8/3}}-\frac{8a^2\sqrt{ax+b\sqrt[3]{x}}}{1311bx^{10/3}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^5}-\frac{4a\sqrt{ax+b\sqrt[3]{x}}}{69x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-4*a*Sqrt[b*x^(1/3) + a*x])/(69*x^4) - (8*a^2*Sqrt[b*x^(1/3) + a*x])/(1311
*b*x^(10/3)) + (136*a^3*Sqrt[b*x^(1/3) + a*x])/(19665*b^2*x^(8/3)) - (1768*
a^4*Sqrt[b*x^(1/3) + a*x])/(216315*b^3*x^2) + (1768*a^5*Sqrt[b*x^(1/3) + a*
x])/(168245*b^4*x^(4/3)) - (1768*a^6*Sqrt[b*x^(1/3) + a*x])/(100947*b^5*x^(
2/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(9*x^5) - (884*a^(27/4)*(Sqrt[b] + Sqrt
[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*El
lipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*b^(21/4)*Sqrt[b*
x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{1}{3}(2a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{1}{69}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x\right.$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{\left(68a^3\right)\text{Subst}\left(\int \frac{}{x^8}\right.}{}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/}}{9x^5}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}}}{216315b^3x^2}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}}}{216315b^3x^2}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}}}{216315b^3x^2}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}}}{216315b^3x^2}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}}}{216315b^3x^2}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}}}{216315b^3x^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 62, normalized size = 0.21

$$-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\;{}_2F_1\left(-\frac{27}{4},-\frac{3}{2};-\frac{23}{4};-\frac{ax^{2/3}}{b}\right)}{9\sqrt{1+\frac{ax^{2/3}}{b}}\;x^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-27/4, -3/2, -23/4, -((a*x^(2
/3))/b)])/(9*Sqrt[1 + (a*x^(2/3))/b]*x^(14/3))
```

Maple [A]
time = 0.34, size = 201, normalized size = 0.67

method	result
default	$-\frac{2\left(6630a^6\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-a}}{\sqrt{-ab}}}\right.\right.}{1514205b^5}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{9x^{\frac{14}{3}}}-\frac{58a\sqrt{b\,x^{\frac{1}{3}}+ax}}{207x^4}-\frac{8a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311b\,x^{\frac{10}{3}}}+\frac{136a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665b^2x^{\frac{8}{3}}}-\frac{1768a^4\sqrt{b\,x}}{216315}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/1514205*(6630*a^6*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*x^(26/3)-1768*x^(23/3)*a^5*b^2+5304*x^(25/3)*a^6*b+952*a^4*b^3*x^7+216
755*x^(17/3)*a^2*b^5-616*x^(19/3)*a^3*b^4+380380*a*b^6*x^5+13260*a^7*x^9+16
8245*x^(13/3)*b^7)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(26/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**6,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^6,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

3.149 $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=304

$$\frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5}-\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}+350$$

```
[Out] 11050/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^7-2210/4807*b^5*x^(2/3)*(b*x^(1/3)+
a*x)^(1/2)/a^6+15470/43263*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-1190/3933*
b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^4+350/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2
)/a^3-50/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(1/3)+a*x)^(
1/2)/a-5525/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(29/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 2049, 2036, 335, 226}

$$-\frac{5525b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14421a^{29/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{11050b^6\sqrt{ax+b\sqrt[3]{x}}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{43263a^5}-\frac{1190b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{3933a^4}+\frac{350b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^3}-\frac{50bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{207a^2}+\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (11050*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^7) - (2210*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(4807*a^6) + (15470*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(43263
*a^5) - (1190*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(3933*a^4) + (350*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a^3) - (50*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(
207*a^2) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/(9*a) - (5525*b^(27/4)*(Sqrt[b] +
Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6
)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(29/4)*Sqrt
[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{x^{14}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{(25b)\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{9a}$$

$$= -\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} + \frac{(175b^2)\,\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx, x,\right.}{69a^2}$$

$$= \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{(2975b^3)\,\text{Su}}{}$$

$$= -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4}{}$$

$$= \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50}{}$$

$$= -\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} +$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{11}{}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{11}{}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{11}{}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{11}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.08, size = 161, normalized size = 0.53

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(16575b^7+6630ab^6x^{2/3}-2210a^2b^5x^{4/3}+1190a^3b^4x^2-770a^4b^3x^{8/3}+550a^5b^2x^{10/3}-418a^6bx^4+4807a^7x^{14/3}-16575b^7\sqrt{1+\frac{ax^{2/3}}{b}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{43263a^7\left(b+ax^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(16575*b^7 + 6630*a*b^6*x^(2/3) - 2210*a^2*b^5*x^(
4/3) + 1190*a^3*b^4*x^2 - 770*a^4*b^3*x^(8/3) + 550*a^5*b^2*x^(10/3) - 418*
a^6*b*x^4 + 4807*a^7*x^(14/3) - 16575*b^7*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(43263*a^7*(b + a*x^(2/3)))
```

Maple [A]
time = 0.35, size = 196, normalized size = 0.64

method	result
default	$-\frac{-1100x^{\frac{11}{3}}a^6b^2+836x^{\frac{13}{3}}a^7b+1540a^5b^3x^3+4420x^{\frac{5}{3}}a^3b^5-2380x^{\frac{7}{3}}a^4b^4-9614a^8x^5+16575b^7\sqrt{-ab}\ \sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-}}{\sqrt{-a}}}}{43263\sqrt{x^{\frac{1}{3}}}}$
derivativedivides	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9a}-\frac{50b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a^2}+\frac{350b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^3}-\frac{1190b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{3933a^4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/43263*(-1100*x^(11/3)*a^6*b^2+836*x^(13/3)*a^7*b+1540*a^5*b^3*x^3+4420*x
^(5/3)*a^3*b^5-2380*x^(7/3)*a^4*b^4-9614*a^8*x^5+16575*b^7*(-a*b)^(1/2)*((a
*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-13260*a^2*b^6*x-33150*x^(1/3)*a*
b^7)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^8
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^5 - a*b*x^(13/3) + b^2*x^(11/3))*sqrt(a*x + b*x^(1/3))/(a^3
*x^2 + b^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{ax + b\sqrt[3]{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a*x + b*x**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a\,x + b\,x^{1/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(1/3))^(1/2), x)
```

3.150 $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=414

$$-\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5}-\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4}+\frac{570b^2x^{5/3}\sqrt{}}{15}$$

```
[Out] -418/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(11/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^
(1/3)+a*x)^(1/2)+418/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-2090/4641*b^
3*x*(b*x^(1/3)+a*x)^(1/2)/a^4+570/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^
3-38/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)/
a+418/221*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)-209/221*b^(21/4)
*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*
2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))
^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2043, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{209b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{418b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{418b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{11/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{418b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{663a^5}-\frac{2090b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^4}+\frac{570b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^3}-\frac{38bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{119a^2}+\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-418*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(11/2)*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) + (418*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^5
) - (2090*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^4) + (570*b^2*x^(5/3)*Sqrt[b
*x^(1/3) + a*x])/(1547*a^3) - (38*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^2
) + (2*x^3*Sqrt[b*x^(1/3) + a*x])/(7*a) + (418*b^(21/4)*(Sqrt[b] + Sqrt[a]*
x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Ellipt
icE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3)
 + a*x]) - (209*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(
Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/
b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{(19b)\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7a}$$

$$= -\frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} + \frac{(285b^2)\text{ Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x,\right.}{119a^2}$$

$$= \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{(3135b^3)\text{ S}}{}$$

$$= -\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3}{}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38b}{}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38b}{}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38b}{}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38b}{}$$

$$= -\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{}}{46}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 143, normalized size = 0.35

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(1463b^5\sqrt[3]{x}+418ab^4x-190a^2b^3x^{5/3}+114a^3b^2x^{7/3}-78a^4bx^3+663a^5x^{11/3}-1463b^5\sqrt{1+\frac{ax^{2/3}}{b}}\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{4641a^5\left(b+ax^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(1463*b^5*x^(1/3) + 418*a*b^4*x - 190*a^2*b^3*x^(5
/3) + 114*a^3*b^2*x^(7/3) - 78*a^4*b*x^3 + 663*a^5*x^(11/3) - 1463*b^5*Sqrt
[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3))/
b)]))/(4641*a^5*(b + a*x^(2/3)))
```

Maple [A]
time = 0.35, size = 261, normalized size = 0.63

method	result
default	$-\frac{-228x^{\frac{8}{3}}a^4b^2+156x^{\frac{10}{3}}a^5b+380a^3b^3x^2+8778b^6\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{El}}{}$
derivativedivides	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a}-\frac{38b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^2}+\frac{570b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^3}-\frac{2090b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^4}+\frac{418}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4641/a^6*(-228*x^(8/3)*a^4*b^2+156*x^(10/3)*a^5*b+380*a^3*b^3*x^2+8778*b
^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3
)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-4389*b^6*((a*x^(1/3)+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*
```

```
b)^(1/2))^(1/2),1/2*2^(1/2))-1326*a^6*x^4-2926*x^(2/3)*a*b^5-836*x^(4/3)*a^
2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral((a^2*x^4 - a*b*x^(10/3) + b^2*x^(8/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ax + b\sqrt[3]{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x**3/sqrt(a*x + b*x**(1/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x^3/(a*x + b*x^(1/3))^(1/2), x)
```

3.151 $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=216

$$-\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4}+\frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3}-\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2}+\frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a}+\frac{39b^{15/4}\left(\sqrt{b}+\right.}{}$$

```
[Out] -78/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^4+234/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1
/2)/a^3-26/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/3)+a*x)^(
1/2)/a+39/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(17/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 2049, 2036, 335, 226}

$$\frac{39b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{17/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{78b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^4}+\frac{234b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^3}-\frac{26bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{55a^2}+\frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-78*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^4) + (234*b^2*x^(2/3)*Sqrt[b*x^(1/3)
+ a*x])/(385*a^3) - (26*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a^2) + (2*x^2*
Sqrt[b*x^(1/3) + a*x])/(5*a) + (39*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*a^(17/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{(13b)\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a}$$

$$= -\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{(117b^2)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x,\right.}{55a^2}$$

$$= \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{(117b^3)\text{Su}}{}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\cdot}}{}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\cdot}}{}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\cdot}}{}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\cdot}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 124, normalized size = 0.57

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-195b^4-78ab^3x^{2/3}+26a^2b^2x^{4/3}-14a^3bx^2+77a^4x^{8/3}+195b^4\sqrt{1+\frac{ax^{2/3}}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{385a^4\left(b+ax^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-195*b^4 - 78*a*b^3*x^(2/3) + 26*a^2*b^2*x^(4/3)
- 14*a^3*b*x^2 + 77*a^4*x^(8/3) + 195*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^4*(b + a*x^(2/3)))
```

Maple [A]
time = 0.34, size = 163, normalized size = 0.75

method	result
default	$-\frac{-52x^{\frac{5}{3}}a^3b^2+28a^4bx^{\frac{7}{3}}-195b^4\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{EllipticF}}{385\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^5}$
derivativedivides	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a}-\frac{26b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^2}+\frac{234b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^3}-\frac{78b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^4}+\frac{39b^4\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/385*(-52*x^(5/3)*a^3*b^2+28*a^4*b*x^(7/3)-195*b^4*(-a*b)^(1/2)*((a*x^(1/
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+156*a^2*b^3*x-154*a^5*x^3+390*a*b^4*x^
(1/3))/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^3 - a*b*x^(7/3) + b^2*x^(5/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a\,x + b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/3))^(1/2), x)
```

3.152 $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=326

$$\frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2}+\frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a}-\frac{14b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}{}$$

```
[Out] 14/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(5/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)
+a*x)^(1/2)-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(1/3)+a*x)
^(1/2)/a-14/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)+7/5*b^(9/4)*x
^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^
(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2
)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2043, 2049, 2057, 335, 311, 226, 1210}

$$\frac{7b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{14b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{14b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{5/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{14b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{15a^2}+\frac{2x\sqrt{ax+b\sqrt[3]{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (14*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(5/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (14*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^2) + (2*x*
Sqrt[b*x^(1/3) + a*x])/(3*a) - (14*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[
(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x]) + (7*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(11/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} - \frac{(7b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{3a}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{(7b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right.}{5a^2}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(7b^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{}}{\sqrt{b+}}\right.}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^{5/2}\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{5a^{5/2}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 106, normalized size = 0.33

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-7b^2\sqrt[3]{x}-2abx+5a^2x^{5/3}+7b^2\sqrt{1+\frac{ax^{2/3}}{b}}\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{15a^2\left(b+ax^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^(1/3) + a*x],x]
```

[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-7*b^2*x^(1/3) - 2*a*b*x + 5*a^2*x^(5/3) + 7*b^2*Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(15*a^2*(b + a*x^(2/3)))

Maple [A]
time = 0.35, size = 228, normalized size = 0.70

method	result
derivativedivides	$\frac{2x\sqrt{b x^{\frac{1}{3}}+ax}}{3a}-\frac{14b x^{\frac{1}{3}}\sqrt{b x^{\frac{1}{3}}+ax}}{15a^{2}}+\frac{7b^{2}\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)}{\sqrt{-ab}}}}{}$
default	$-\frac{-42b^{3}\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/15/a^3*(-42*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+21*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+14*a*b^2*x^(2/3)+4*a^2*b*x^(4/3)-10*a^3*x^2)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax + b\sqrt[3]{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x/sqrt(a*x + b*x**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a\,x + b\,x^{1/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x/(a*x + b*x^(1/3))^(1/2), x)
```

3.153 $\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=126

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)/a-b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x
^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2035, 2038, 2036, 335, 226}

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b\int \frac{1}{x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{3a}$$

$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{a}$$

$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{\left(b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}$$

$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{\left(2b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}$$

$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\ldots\right.\right.}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 80, normalized size = 0.63

$$\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(b+ax^{2/3}-b\sqrt{1+\frac{ax^{2/3}}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{a\left(b+ax^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(b + a*x^(2/3) - b*Sqrt[1 + (a*x^(2/3))/b]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a*(b + a*x^(2/3)))
```

Maple [A]
time = 0.38, size = 127, normalized size = 1.01

method	result
default	$\frac{-b\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+\cdots}{\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{2}}$
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{a}-\frac{b\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\cdots\right)}{a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/
3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2*a*b*x^(1/3
)+2*a^2*x)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^3 + b^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt[3]{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [B]
time = 5.27, size = 40, normalized size = 0.32

$$\frac{2\,x\,\sqrt{\frac{b}{a\,x^{2/3}}+1}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ -\frac{b}{a\,x^{2/3}}\right)}{\sqrt{a\,x+b\,x^{1/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] (2*x*(b/(a*x^(2/3)) + 1)^(1/2)*hypergeom([-3/4, 1/2], 1/4, -b/(a*x^(2/3))))
/(a*x + b*x^(1/3))^(1/2)
```

3.154 $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=294

$$\frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}}-\frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,E}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 6*(b+a*x^(2/3))*x^(1/3)*a^(1/2)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)
^(1/2)-6*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-6*a^(1/4)*x^(1/6)*(cos(2*arctan(a^
(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ell
ipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)
+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1
/3)+a*x)^(1/2)+3*a^(1/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(
x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2043, 2050, 2057, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{a}\,\sqrt[6]{x}\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{6\sqrt[4]{a}\,\sqrt[6]{x}\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{6\sqrt{a}\,\sqrt[3]{x}\left(ax^{2/3}+b\right)}{b\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (6*Sqrt[a]*(b + a*x^(2/3))*x^(1/3))/(b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x
^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) - (6*a^(1/4)*(Sqrt[b
] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^
(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*
x^(1/3) + a*x]) + (3*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{(3a)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(3a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6\sqrt{a}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} -$$

$$= \frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} - \frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 54, normalized size = 0.18

$$-\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^(2/3))
/b)])/Sqrt[b*x^(1/3) + a*x]
```

Maple [A]
time = 0.38, size = 253, normalized size = 0.86

method	result
derivativedivides	$-\frac{6\left(b+a\,x^{\frac{2}{3}}\right)}{b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\right.}{}$
default	$-\frac{3\left(-2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3*(-2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)
^(1/2))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*b+(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a
*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*b+2*(b*x^(1/3)+a*x)^(1/2)*x^(2/3)*a+2*(b*x^(1/3)+a*x)^(1/2)*b)/x
^(1/3)/(b+a*x^(2/3))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^4 + b^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*x**(1/3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{a\,x + b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/3))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/3))^(1/2)), x)
```

3.155 $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=163

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}+\frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{7b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] -6/7*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+10/7*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/
3)+5/7*a^(7/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 2050, 2036, 335, 226}

$$\frac{5a^{7/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{10a\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{2/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) + (10*a*Sqrt[b*x^(1/3) + a*x])/(7*
b^2*x^(2/3)) + (5*a^(7/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(7*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} - \frac{(15a)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{(5a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{\left(5a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{b}\cdots}\right.}{7b^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{\left(10a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+\cdots}}\right.}{7b^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{\cdots}\right.}}}{7b^{9/4}\sqrt{b\sqrt[3]{\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 59, normalized size = 0.36

$$-\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\;{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{ax^{2/3}}{b}\right)}{7x\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((a*x^(2/3)
)/b)])/(7*x*Sqrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.35, size = 142, normalized size = 0.87

method	result

default	$\frac{5a\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{7b^2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{4}{3}}}$
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b\,x^{\frac{4}{3}}}+\frac{10a\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b^2x^{\frac{2}{3}}}+\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{7b^2\sqrt{b\,x^{\frac{1}{3}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(5*a*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*
x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*E
llipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(4/3)
+4*a*b*x+10*x^(5/3)*a^2-6*b^2*x^(1/3))/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^
(4/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^5 + b^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(1/3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/3))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/3))^(1/2)), x)
```

3.156 $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=388

$$-\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}}+\frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}}-\frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}+\frac{154a^3}{}$$

```
[Out] -154/65*a^(7/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1
/3)+a*x)^(1/2)-6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*x)
^(1/2)/b^2/x^(5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*a^3*(b*x^
(1/3)+a*x)^(1/2)/b^4/x^(1/3)+154/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE
(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/
2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a
*x)^(1/2)-77/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(
x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2043, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{77a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{154a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{154a^{7/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{65b^4\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{154a^3\sqrt{ax+b\sqrt[3]{x}}}{65b^4\sqrt[3]{x}}-\frac{154a^2\sqrt{ax+b\sqrt[3]{x}}}{195b^3x}+\frac{22a\sqrt{ax+b\sqrt[3]{x}}}{39b^2x^{5/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-154*a^(7/2)*(b + a*x^(2/3))*x^(1/3))/(65*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) + (22*a*S
qrt[b*x^(1/3) + a*x])/(39*b^2*x^(5/3)) - (154*a^2*Sqrt[b*x^(1/3) + a*x])/(1
95*b^3*x) + (154*a^3*Sqrt[b*x^(1/3) + a*x])/(65*b^4*x^(1/3)) + (154*a^(13/4
)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/
3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(
15/4)*Sqrt[b*x^(1/3) + a*x]) - (77*a^(13/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} - \frac{(33a)\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{13b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} + \frac{(77a^2)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right.}{39b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} - \frac{(77a^3)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+}}{65b^4\sqrt[3]{x}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+}}{65b^4\sqrt[3]{x}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+}}{65b^4\sqrt[3]{x}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+}}{65b^4\sqrt[3]{x}}$$

$$= -\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 59, normalized size = 0.15

$$-\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\ {}_2F_1\left(-\frac{13}{4},\frac{1}{2};-\frac{9}{4};-\frac{ax^{2/3}}{b}\right)}{13x^2\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-13/4, 1/2, -9/4, -((a*x^(2/3
))/b)])/(13*x^2*Sqrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.35, size = 363, normalized size = 0.94

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{13b\,x^{\frac{7}{3}}}+\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{39b^2x^{\frac{5}{3}}}-\frac{154a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{195b^3x}+\frac{154\left(b+a\,x^{\frac{2}{3}}\right)a^3}{65b^4\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-$ $77a^3\sqrt{}$
default	$\frac{-462a^3b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/195*(-462*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1
/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(10
/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))+231*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b
)^(1/2))^(1/2)*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*(b*x^(1/3)+a*x)^(1/2)*x
^(10/3)*a^3*b-44*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^2-154*x^(10/3)
*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b+20*x^2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a
```

```
*b^3+462*(b*x^(1/3)+a*x)^(1/2)*x^4*a^4-90*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(
1/2)*b^4)/x^(11/3)/(b+a*x^(2/3))/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^6 + b^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(1/3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/3))^(1/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(1/3))^(1/2)), x)
```

3.157 $\int \frac{1}{x^4 \sqrt{b\sqrt[3]{x} + ax}} \, dx$

Optimal. Leaf size=251

$$-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}}+\frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}}-\frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2}+\frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}}-\frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}}$$

```
[Out] -6/19*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+34/95*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(8/3)-442/1045*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+3978/7315*a^3*(b*x^(1/3)+a
*x)^(1/2)/b^4/x^(4/3)-1326/1463*a^4*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-663/1
463*a^(19/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1
/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 2050, 2036, 335, 226}

$$-\frac{663a^{19/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1463b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{1326a^4\sqrt{ax+b\sqrt[3]{x}}}{1463b^5x^{2/3}}+\frac{3978a^3\sqrt{ax+b\sqrt[3]{x}}}{7315b^4x^{4/3}}-\frac{442a^2\sqrt{ax+b\sqrt[3]{x}}}{1045b^3x^2}+\frac{34a\sqrt{ax+b\sqrt[3]{x}}}{95b^2x^{8/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) + (34*a*Sqrt[b*x^(1/3) + a*x])/(
95*b^2*x^(8/3)) - (442*a^2*Sqrt[b*x^(1/3) + a*x])/(1045*b^3*x^2) + (3978*a^
3*Sqrt[b*x^(1/3) + a*x])/(7315*b^4*x^(4/3)) - (1326*a^4*Sqrt[b*x^(1/3) + a*
x])/(1463*b^5*x^(2/3)) - (663*a^(19/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(1463*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} - \frac{(51a)\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{19b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} + \frac{(221a^2)\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{\right.}}{95b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} - \frac{(1989a^3)\text{Subst}\left(\right.}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}\ -}}{7315b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}\ -}}{7315b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}\ -}}{7315b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}\ -}}{7315b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}\ -}}{7315b^4x^{4/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 59, normalized size = 0.24

$$-\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\ {}_2F_1\left(-\frac{19}{4},\frac{1}{2};-\frac{15}{4};-\frac{ax^{2/3}}{b}\right)}{19x^3\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]

[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-19/4, 1/2, -15/4, -((a*x^(2/3))/b)])/(19*x^3*Sqrt[b*x^(1/3) + a*x])

Maple [A]
time = 0.36, size = 179, normalized size = 0.71

method	result
default	$-\frac{3315 a^4 \sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\right.}{7315 b^5 \sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{19 b\,x^{\frac{10}{3}}}+\frac{34 a\sqrt{b\,x^{\frac{1}{3}}+ax}}{95 b^2 x^{\frac{8}{3}}}-\frac{442 a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{1045 b^3 x^2}+\frac{3978 a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7315 b^4 x^{\frac{4}{3}}}-\frac{1326 a^4\sqrt{}}{146}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/7315*(3315*a^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(16/3)+2652*a^4*b*x^5+6630*x^(17/3)*a^5+476*x^(11/3)*a^2*b^3-884*x^(13/3)*a^3*b^2-308*a*b^4*x^3+2310*x^(7/3)*b^5)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(16/3)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^7 + b^3*x^5), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*x**(1/3))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(1/3))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(1/3))^(1/2)), x)
```

3.158 $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=437

$$-\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}}}{4641a^5}$$

[Out] -3*x^4/a/(b*x^(1/3)+a*x)^(1/2)-4807/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(13/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4807/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^6-24035/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+6555/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-437/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+23/7*x^3*(b*x^(1/3)+a*x)^(1/2)/a^2+4807/221*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(27/4)/(b*x^(1/3)+a*x)^(1/2)-4807/442*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(27/4)/(b*x^(1/3)+a*x)^(1/2)

Rubi [A]

time = 0.46, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2047, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{27/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{4807b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{13/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{663a^6}-\frac{24035b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^5}+\frac{6555b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^4}-\frac{437bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{119a^3}+\frac{23x^3\sqrt{ax+b\sqrt[3]{x}}}{7a^2}-\frac{3x^4}{a\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(b*x^(1/3) + a*x)^(3/2),x]

[Out] (-4807*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(13/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (3*x^4)/(a*Sqrt[b*x^(1/3) + a*x]) + (4807*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^6) - (24035*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^5) + (6555*b^2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(1547*a^4) - (437*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^3) + (23*x^3*Sqrt[b*x^(1/3) + a*x])/(7*a^2) + (4807*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]

```
*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*a^(27/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{x^{14}}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\text{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} - \frac{(437b)\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right.}{14a^2}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} + \frac{(6555b^2)\,\text{Sub}}{}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{}}{}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx}{}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\,\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b}{}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\,\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b}{}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\,\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b}{}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\,\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b}{}$$

$$= -\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\,\sqrt{b\sqrt[3]{}}}{663a^6}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 131, normalized size = 0.30

$$\frac{2x^{2/3}\left(-33649b^5+4807ab^4x^{2/3}-2185a^2b^3x^{4/3}+1311a^3b^2x^2-897a^4bx^{8/3}+663a^5x^{10/3}+33649b^5\sqrt{1+\frac{ax^{2/3}}{b}}\ {}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{4641a^6\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(-33649*b^5 + 4807*a*b^4*x^(2/3) - 2185*a^2*b^3*x^(4/3) + 1311*a
^3*b^2*x^2 - 897*a^4*b*x^(8/3) + 663*a^5*x^(10/3) + 33649*b^5*Sqrt[1 + (a*x
^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(4641*a^6*S
qrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.35, size = 384, normalized size = 0.88

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}b^5}{a^6\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a^2}-\frac{80b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^3}+\frac{1914b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^4}-$ 10
default	$5244x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^4b^2-3588x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^5b-8740x^2\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^3b^3-20189$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9282/a^7*(5244*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b^2-3588*x^(10/3
)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*b-8740*x^2*(x^(1/3)*(b+a*x^(2/3)))^(1/2
)*a^3*b^3-201894*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x
^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*(x
^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))+100947*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
```

```
/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/
2))^(1/2)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2652*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^6*x^
4+27846*(b*x^(1/3)+a*x)^(1/2)*x^(2/3)*a*b^5+39452*x^(2/3)*(x^(1/3)*(b+a*x^(
2/3)))^(1/2)*a*b^5+19228*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4)/x^(
1/3)/(b+a*x^(2/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a^4*x^6 + 3*a^2*b^2*x^(14/3) - 2*a*b^3*x^4 - (2*a^3*b*x^5 - b^4*x
^3)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x**4/(a*x + b*x**(1/3))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

3.159 $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}+\frac{17x^2\sqrt{b\sqrt[3]{x}}}{5a^2}$$

```
[Out] -3*x^3/a/(b*x^(1/3)+a*x)^(1/2)-663/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+1989/38
5*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-221/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1
/2)/a^3+17/5*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+663/154*b^(15/4)*x^(1/6)*(cos(2*
arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1
/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3
)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(21/
4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2047, 2049, 2036, 335, 226}

$$\frac{663b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4}-\frac{221bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{55a^3}+\frac{17x^2\sqrt{ax+b\sqrt[3]{x}}}{5a^2}-\frac{3x^3}{a\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x^3)/(a*Sqrt[b*x^(1/3) + a*x]) - (663*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^
5) + (1989*b^2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(385*a^4) - (221*b*x^(4/3)*Sq
rt[b*x^(1/3) + a*x])/(55*a^3) + (17*x^2*Sqrt[b*x^(1/3) + a*x])/(5*a^2) + (6
63*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqr
t[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2
])/(154*a^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{x^{11}}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{51\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} - \frac{(221b)\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10a^2}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} + \frac{(1989b^2)\text{Sub}}{}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{}}{}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{}}{5}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{}}{5}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{}}{5}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{}}{5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 124, normalized size = 0.52

$$\frac{\sqrt{b\sqrt[3]{x}+ax}\left(-3315b^4-1326ab^3x^{2/3}+442a^2b^2x^{4/3}-238a^3bx^2+154a^4x^{8/3}+3315b^4\sqrt{1+\frac{ax^{2/3}}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{385a^5\left(b+ax^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*x^(1/3) + a*x]*(-3315*b^4 - 1326*a*b^3*x^(2/3) + 442*a^2*b^2*x^(4/3
) - 238*a^3*b*x^2 + 154*a^4*x^(8/3) + 3315*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^5*(b + a*x^(2/3)))
```

Maple [A]
time = 0.35, size = 260, normalized size = 1.09

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}b^4}{a^5\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a^2}-\frac{56b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^3}+\frac{834b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^4}-$
default	$-\frac{-884x^{\frac{5}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^3b^2+476x^{\frac{7}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^4b-3315\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/770*(-884*x^(5/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b^2+476*x^(7/3)*(x^(
1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b-3315*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(
1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^4+2652*x*(x^(1/3)*(b+a
*x^(2/3)))^(1/2)*a^2*b^3-308*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*x^3+2310*(b*
x^(1/3)+a*x)^(1/2)*x^(1/3)*a*b^4+4320*x^(1/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)
*a*b^4)/x^(1/3)/(b+a*x^(2/3))/a^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 + 3*a^2*b^2*x^(11/3) - 2*a*b^3*x^3 - (2*a^3*b*x^4 - b^4*x
^2)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*x**(1/3))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

3.160 $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3}+\frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2}-\frac{77b^{9/4}}{}$$

```
[Out] -3*x^2/a/(b*x^(1/3)+a*x)^(1/2)+77/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x^(1
/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-77/15*b*x^(1/3)*(b*x^(1/3)+a*x)^
(1/2)/a^3+11/3*x*(b*x^(1/3)+a*x)^(1/2)/a^2-77/5*b^(9/4)*x^(1/6)*(cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^
(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(
b*x^(1/3)+a*x)^(1/2)+77/10*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*
x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$,
Rules used = {2043, 2047, 2049, 2057, 335, 311, 226, 1210}

$$\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{77b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{77b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{15a^3}+\frac{11x\sqrt{ax+b\sqrt[3]{x}}}{3a^2}-\frac{3x^2}{a\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (77*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (3*x^2)/(a*Sqrt[b*x^(1/3) + a*x]) - (77*b*x^(1/3)*Sqr
t[b*x^(1/3) + a*x])/(15*a^3) + (11*x*Sqrt[b*x^(1/3) + a*x])/(3*a^2) - (77*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) + (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*Arc
Tan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(10*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
```

```
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{x^8}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{33\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} - \frac{(77b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{6a^2}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\sqrt{b+ax}\right.}{}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\sqrt{b+ax}\right.}{}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^{5/2}\sqrt{b+a\right.}{}$$

$$= \frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 94, normalized size = 0.27

$$\frac{2x^{2/3}\left(77b^2-11abx^{2/3}+5a^2x^{4/3}-77b^2\sqrt{1+\frac{ax^{2/3}}{b}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{15a^3\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(77*b^2 - 11*a*b*x^(2/3) + 5*a^2*x^(4/3) - 77*b^2*Sqrt[1 + (a*x^
(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(15*a^3*Sqrt
[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.35, size = 312, normalized size = 0.89

method	result
derivativedivides	$-\frac{3x^{\frac{2}{3}}b^2}{a^3\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3a^2}-\frac{32b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a^3}+\frac{77b^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-a}\cdots}{a}\right.\cdots}{\sqrt{-ab}\cdots}}\cdots}{\cdots}$
default	$-\frac{-462b^3\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\sqrt{\frac{a\cdots}{\cdots}}\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/a^4*(-462*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^
(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*(x^
(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))+231*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^
(1/2)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2))+90*(b*x^(1/3)+a*x)^(1/2)*x^(2/3)*a*b^2+64*x^(
2/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^2+44*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))
^(1/2)*a^2*b-20*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*x^2)/x^(1/3)/(b+a*x^(2/3)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 + 3*a^2*b^2*x^(8/3) - 2*a*b^3*x^2 - (2*a^3*b*x^3 - b^4*x)
*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*x**(1/3))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

3.161 $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right.}{2a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] -3*x/a/(b*x^(1/3)+a*x)^(1/2)+5*(b*x^(1/3)+a*x)^(1/2)/a^2-5/2*b^(3/4)*x^(1/6
)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(
1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2)
)*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/
2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2043, 2047, 2049, 2036, 335, 226}

$$-\frac{5b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2}-\frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x)/(a*Sqrt[b*x^(1/3) + a*x]) + (5*Sqrt[b*x^(1/3) + a*x])/a^2 - (5*b^(3/
4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1
/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(
9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{x^5}{(bx+ax^3)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{15\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{(5b)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a^2}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{\left(5b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+a}\ldots}\right.}{2a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{\left(5b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\ldots\right.}{a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\ldots}}\ldots}{2a^{9/4}\sqrt{b\sqrt[3]{x}\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 82, normalized size = 0.55

$$\frac{\sqrt{b\sqrt[3]{x}+ax}\left(5b+2ax^{2/3}-5b\sqrt{1+\frac{ax^{2/3}}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{a^2\left(b+ax^{2/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*x^(1/3) + a*x]*(5*b + 2*a*x^(2/3) - 5*b*Sqrt[1 + (a*x^(2/3))/b]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a^2*(b + a*x^(2/3)))
```

Maple [A]
time = 0.36, size = 184, normalized size = 1.23

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}}b}{a^2\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{a^2}-\frac{5b\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)}{\sqrt{-ab}}}}{2a^3\sqrt{b}}$
default	$\frac{-5\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\right.}{2x^{\frac{1}{3}}\left(b-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-5*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^
(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*b+6*(b*x^(1/3)+a*x)^(1/2)*x^(1/3)*a*b+4*x^(1/3)*(x^(1
/3)*(b+a*x^(2/3)))^(1/2)*a*b+4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*x)/x^(1/3)
/(b+a*x^(2/3))/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x/(a*x + b*x**(1/3))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x/(a*x + b*x^(1/3))^(3/2), x)
```

3.162 $\quad \int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}\,b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}+\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}}{a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 3*x^(2/3)/b/(b*x^(1/3)+a*x)^(1/2)-3*(b+a*x^(2/3))*x^(1/3)/b/a^(1/2)/(x^(1/3
)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+3*x^(1/6)*(cos(2*arctan(a^(1/4)*x^
(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(s
in(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2)
)*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^(1
/3)+a*x)^(1/2)-3/2*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2031, 2043, 2057, 335, 311, 226, 1210}

$$-\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{3\sqrt[3]{x}\left(ax^{2/3}+b\right)}{\sqrt{a}\,b\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(-3/2),x]
```

```
[Out] (-3*(b + a*x^(2/3))*x^(1/3))/(Sqrt[a]*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*
x^(1/3) + a*x]) + (3*x^(2/3))/(b*Sqrt[b*x^(1/3) + a*x]) + (3*(Sqrt[b] + Sqr
t[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*E
llipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[b
*x^(1/3) + a*x]) - (3*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqr
t[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\int \frac{1}{\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{2b}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{3\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{2b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} + \frac{\left(3\sqrt{b \cdots\right.}{\cdots}$$

$$= -\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}\,b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} + \frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 62, normalized size = 0.21

$$\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{2/3}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(-3/2),x]
```

```
[Out] (2*Sqrt[1 + (a*x^(2/3))/b]*x^(2/3)*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^
(2/3))/b)])/(b*Sqrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.35, size = 242, normalized size = 0.82

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}}{b\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}} - \frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(\ldots\right.}{\ldots}$
default	$\frac{-3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{\ldots}}{\sqrt{-\ldots}}}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/a*(-2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a
*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*b+(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/
(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*b+2*(b*x^(1/3)+a*x)^(1/2)*x^(2/3)*a)/x^(1/3)/(b+a*x^(2/3))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^5 + 2*a^3*b^3*x^3 + b^6*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

Mupad [B]
time = 5.35, size = 40, normalized size = 0.14

$$\frac{2\,x\,\left(\frac{a\,x^{2/3}}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{2};\ \frac{7}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] (2*x*((a*x^(2/3))/b + 1)^(3/2)*hypergeom([3/4, 3/2], 7/4, -(a*x^(2/3))/b))/
(a*x + b*x^(1/3))^(3/2)
```

3.163 $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}}{\sqrt[4]{}}\right.\right.}{2b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 3/b/x^(1/3)/(b*x^(1/3)+a*x)^(1/2)-5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)-5/2*a
^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^
(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2048, 2050, 2036, 335, 226}

$$-\frac{5a^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}}+\frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) - (5*Sqrt[b*x^(1/3) + a*x])/(b^2*x^(2/3
)) - (5*a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] +
 Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)],
 1/2])/(2*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{1}{x\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} + \frac{15\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{(5a)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b^2}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b}}\right.}{2b^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+a}}\right.}{b^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/}}{\left(\sqrt{b}+\sqrt{a}\right.}}}{2b^{9/4}\sqrt{b\sqrt[3]{}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 62, normalized size = 0.39

$$-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((a*x^(2/3))
/b)])/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.36, size = 180, normalized size = 1.14

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}a}{b^2\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{b^2x^{\frac{2}{3}}}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right.}{\sqrt{-ab}}}}{2b^2\sqrt{}}$
default	$-\frac{5x^{\frac{2}{3}}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^2x\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(5*x^(2/3)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3
)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*Ellipti
cF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(x^(1/3)*(b+a
*x^(2/3)))^(1/2)*(-a*b)^(1/2)+6*(b*x^(1/3)+a*x)^(1/2)*x*a+4*x^(1/3)*(x^(1/3
)*(b+a*x^(2/3)))^(1/2)*b+4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*x)/b^2/x/(b+a*x^
(2/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^6 + 2*a^3*b^3*x^4 + b^6*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(1/3))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/3))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/3))^(3/2)), x)
```

3.164 $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=383

$$\frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}+\frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}}+\frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x}-\frac{77a^2\sqrt{b}}{5b}$$

```
[Out] 3/b/x^(4/3)/(b*x^(1/3)+a*x)^(1/2)+77/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x
^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-11/3*(b*x^(1/3)+a*x)^(1/2)/b^
2/x^(5/3)+77/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x-77/5*a^2*(b*x^(1/3)+a*x)^(1/2
)/b^4/x^(1/3)-77/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)+77/10*a^
(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))
,1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(
1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2048, 2050, 2057, 335, 311, 226, 1210}

$$\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{77a^{5/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^2\sqrt{ax+b\sqrt[3]{x}}}{5b^4\sqrt[3]{x}}+\frac{77a\sqrt{ax+b\sqrt[3]{x}}}{15b^3x}-\frac{11\sqrt{ax+b\sqrt[3]{x}}}{3b^2x^{5/3}}+\frac{3}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(4/3)*Sqrt[b*x^(1/3) + a*x]) + (77*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/
(5*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (11*Sqrt[b*x^(1
/3) + a*x])/(3*b^2*x^(5/3)) + (77*a*Sqrt[b*x^(1/3) + a*x])/(15*b^3*x) - (77
*a^2*Sqrt[b*x^(1/3) + a*x])/(5*b^4*x^(1/3)) - (77*a^(9/4)*(Sqrt[b] + Sqrt[a
]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elli
pticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(15/4)*Sqrt[b*x^(1/3)
 + a*x]) + (77*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sq
rt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^
(1/4)], 1/2])/(10*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{1}{x^4 \left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{33\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} - \frac{(77a)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right.}{6b^2}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} + \frac{(77a^2)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 64, normalized size = 0.17

$$-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\;{}_2F_1\left(-\frac{9}{4},\frac{3}{2};-\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{3bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((a*x^(2/3)
)/b)])/(3*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.36, size = 339, normalized size = 0.89

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{3b^{2}x^{\frac{5}{3}}}+\frac{32a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15b^{3}x}-\frac{62\left(b+a\,x^{\frac{2}{3}}\right)a^{2}}{5b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{3x^{\frac{2}{3}}a^{3}}{b^{4}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\ldots$ 77a
default	$-\frac{-462a^{2}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(-462*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1
/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(8/
3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))+231*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)
^(1/2))^(1/2)*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*(b*x^(1/3)+a*x)^(1/2)*x^(
10/3)*a^3+372*(b*x^(1/3)+a*x)^(1/2)*x^(8/3)*a^2*b-44*x^2*(x^(1/3)*(b+a*x^(2
/3)))^(1/2)*a*b^2-64*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b+20*x^(4/3)
*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^3)/x^3/(b+a*x^(2/3))/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^7 + 2*a^3*b^3*x^5 + b^6*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**2*(a*x + b*x**(1/3))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/3))^(3/2)),x)
[Out] int(1/(x^2*(a*x + b*x^(1/3))^(3/2)), x)
```

3.165 $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=246

$$\frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}+$$

```
[Out] 3/b/x^(7/3)/(b*x^(1/3)+a*x)^(1/2)-17/5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)+22
1/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2-1989/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4
/x^(4/3)+663/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)+663/154*a^(15/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/
2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(
1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2048, 2050, 2036, 335, 226}

$$\frac{663a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}}-\frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}}+\frac{221a\sqrt{ax+b\sqrt[3]{x}}}{55b^3x^2}-\frac{17\sqrt{ax+b\sqrt[3]{x}}}{5b^2x^{8/3}}+\frac{3}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(7/3)*Sqrt[b*x^(1/3) + a*x]) - (17*Sqrt[b*x^(1/3) + a*x])/(5*b^2*x^(
8/3)) + (221*a*Sqrt[b*x^(1/3) + a*x])/(55*b^3*x^2) - (1989*a^2*Sqrt[b*x^(1/
3) + a*x])/(385*b^4*x^(4/3)) + (663*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^5*x^(2
/3)) + (663*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(154*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{1}{x^7\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{51\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} - \frac{(221a)\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \right.}{10b^2}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} + \frac{(1989a^2)\,\text{Subst}}{}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}}}{385b^4x^{4/}}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}}}{385b^4x^{4/}}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}}}{385b^4x^{4/}}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}}}{385b^4x^{4/}}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}}}{385b^4x^{4/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 64, normalized size = 0.26

$$-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\ {}_2F_1\left(-\frac{15}{4},\frac{3}{2};-\frac{11}{4};-\frac{ax^{2/3}}{b}\right)}{5bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-15/4, 3/2, -11/4, -((a*x^(2/
3))/b)])/(5*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.35, size = 261, normalized size = 1.06

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5b^{2}x^{\frac{8}{3}}}+\frac{56a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55b^{3}x^{2}}-\frac{834a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b^{4}x^{\frac{4}{3}}}+\frac{432a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{5}x^{\frac{2}{3}}}+\frac{3x}{b^{5}\sqrt{\left(x^{\frac{2}{3}}\right.}}$
default	$\frac{3315\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{14}{3}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/770*(3315*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF((
(a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(14/3)*(x^(1/3)
*(b+a*x^(2/3)))^(1/2)*(-a*b)^(1/2)*a^3-884*x^(11/3)*(x^(1/3)*(b+a*x^(2/3)))
^(1/2)*a^2*b^2+2652*x^(13/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b+476*x^3*(x
^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^3+2310*(b*x^(1/3)+a*x)^(1/2)*x^5*a^4+4320*(
x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*x^5-308*x^(7/3)*(x^(1/3)*(b+a*x^(2/3)))^(1
/2)*b^4)/b^5/x^5/(b+a*x^(2/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^8 + 2*a^3*b^3*x^6 + b^6*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**3*(a*x + b*x**(1/3))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/3))^(3/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(1/3))^(3/2)), x)
```

3.166 $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=471

$$\frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221b^7\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}}+\frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3}-\frac{6555}{}$$

```
[Out] 3/b/x^(10/3)/(b*x^(1/3)+a*x)^(1/2)-4807/221*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/
b^7/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-23/7*(b*x^(1/3)+a*x)^(1
/2)/b^2/x^(11/3)+437/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^3-6555/1547*a^2*(b*x
^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+24035/4641*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(5
/3)-4807/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4807/221*a^5*(b*x^(1/3)+a*x)^(
1/2)/b^7/x^(1/3)+4807/221*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*
x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)-
4807/442*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)
*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2048, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{442b^{27/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221b^{27/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{4807a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221b^7\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807a^5\sqrt{ax+b\sqrt[3]{x}}}{221b^7\sqrt[3]{x}}-\frac{4807a^4\sqrt{ax+b\sqrt[3]{x}}}{663b^6x}+\frac{24035a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^5x^{5/3}}-\frac{6555a^2\sqrt{ax+b\sqrt[3]{x}}}{1547b^4x^{7/3}}+\frac{437a\sqrt{ax+b\sqrt[3]{x}}}{119b^3x^3}-\frac{23\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{11/3}}+\frac{3}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(10/3)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(11/2)*(b + a*x^(2/3))*x^(1/
3))/(221*b^7*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (23*Sqrt[
b*x^(1/3) + a*x])/(7*b^2*x^(11/3)) + (437*a*Sqrt[b*x^(1/3) + a*x])/(119*b^3
*x^3) - (6555*a^2*Sqrt[b*x^(1/3) + a*x])/(1547*b^4*x^(7/3)) + (24035*a^3*Sq
rt[b*x^(1/3) + a*x])/(4641*b^5*x^(5/3)) - (4807*a^4*Sqrt[b*x^(1/3) + a*x])/
(663*b^6*x) + (4807*a^5*Sqrt[b*x^(1/3) + a*x])/(221*b^7*x^(1/3)) + (4807*a^
(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2
21*b^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1
/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[
```

```
2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*b^(27/4)*Sqrt[b*x^(1/3) + a
*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{1}{x^{10}\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\text{Subst}\left(\int \frac{1}{x^{11}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} - \frac{(437a)\text{Subst}\left(\int \frac{1}{x^{9}\sqrt{bx+ax^3}}\,dx, x,\right.}{14b^2}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} + \frac{(6555a^2)\,\text{Subs}}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}}}{1547b^4x}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}}}{1547b^4x}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}}}{1547b^4x}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}}}{1547b^4x}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}}}{1547b^4x}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}}}{1547b^4x}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}}}{1547b^4x}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{20417\left(\ldots\right)} - \frac{23\sqrt{b\sqrt[3]{x}+a}}{7b^2x^{11/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 64, normalized size = 0.14

$$-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,{}_2F_1\left(-\frac{21}{4},\frac{3}{2};-\frac{17}{4};-\frac{ax^{2/3}}{b}\right)}{7bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-21/4, 3/2, -17/4, -((a*x^(2/
3))/b)])/(7*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A]
time = 0.35, size = 411, normalized size = 0.87

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b^2x^{\frac{11}{3}}}+\frac{80a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119b^3x^3}-\frac{1914a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b^4x^{\frac{7}{3}}}+\frac{10112a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^5x^{\frac{5}{3}}}-\frac{2818a^4}{}$
default	$-\frac{201894a^5b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\mathrm{EllipticE}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9282*(201894*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*
x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x
^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2),1/2*2^(1/2))-100947*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*
a/(-a*b)^(1/2))^(1/2)*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*
x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-201894*(b*x^(1/3)+a*
```

```
x)^(1/2)*x^(22/3)*a^6-174048*(b*x^(1/3)+a*x)^(1/2)*x^(20/3)*a^5*b+19228*x^6
*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b^2+39452*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)
))^(1/2)*a^5*b+5244*x^(14/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4-8740*x^(
16/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b^3-3588*x^4*(x^(1/3)*(b+a*x^(2/3))
)^(1/2)*a*b^5+2652*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^6)/x^7/(b+a*x^(
2/3))/b^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^9 + 2*a^3*b^3*x^7 + b^6*x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**4*(a*x + b*x**(1/3))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(1/3))^(3/2)),x)
[Out] int(1/(x^4*(a*x + b*x^(1/3))^(3/2)), x)
```

3.167 $\int x^3 \sqrt{bx^{2/3}+ax}\, dx$

Optimal. Leaf size=371

$$-\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(bx^{2/3}+a\right.}{10140585a^{11}\sqrt[3]{x}}$$

```
[Out] -524288/4345965*b^9*(b*x^(2/3)+a*x)^(3/2)/a^10+8388608/152108775*b^12*(b*x^
(2/3)+a*x)^(3/2)/a^13/x-4194304/50702925*b^11*(b*x^(2/3)+a*x)^(3/2)/a^12/x^
(2/3)+1048576/10140585*b^10*(b*x^(2/3)+a*x)^(3/2)/a^11/x^(1/3)+65536/482885
*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^9-360448/2414425*b^7*x^(2/3)*(b*x^(2/3
)+a*x)^(3/2)/a^8+90112/557175*b^6*x*(b*x^(2/3)+a*x)^(3/2)/a^7-45056/260015*
b^5*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/a^6+2816/15295*b^4*x^(5/3)*(b*x^(2/3)+a*x
)^(3/2)/a^5-1408/7245*b^3*x^2*(b*x^(2/3)+a*x)^(3/2)/a^4+352/1725*b^2*x^(7/3
)*(b*x^(2/3)+a*x)^(3/2)/a^3-16/75*b*x^(8/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/9*x
^3*(b*x^(2/3)+a*x)^(3/2)/a
```

Rubi [A]
time = 0.42, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$\frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^9\left(ax+bx^{2/3}\right)^{3/2}}{4345965a^{10}}+\frac{65536b^8\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{482885a^9}-\frac{360448b^7x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{2414425a^8}+\frac{90112b^6x\left(ax+bx^{2/3}\right)^{3/2}}{557175a^7}-\frac{45056b^5x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{260015a^6}+\frac{2816b^4x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(ax+bx^{2/3}\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{75a^2}+\frac{2x^3\left(ax+bx^{2/3}\right)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-524288*b^9*(b*x^(2/3) + a*x)^(3/2))/(4345965*a^10) + (8388608*b^12*(b*x^(
2/3) + a*x)^(3/2))/(152108775*a^13*x) - (4194304*b^11*(b*x^(2/3) + a*x)^(3/
2))/(50702925*a^12*x^(2/3)) + (1048576*b^10*(b*x^(2/3) + a*x)^(3/2))/(10140
585*a^11*x^(1/3)) + (65536*b^8*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(482885*a^9
) - (360448*b^7*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))/(2414425*a^8) + (90112*b^6
*x*(b*x^(2/3) + a*x)^(3/2))/(557175*a^7) - (45056*b^5*x^(4/3)*(b*x^(2/3) +
a*x)^(3/2))/(260015*a^6) + (2816*b^4*x^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(1529
5*a^5) - (1408*b^3*x^2*(b*x^(2/3) + a*x)^(3/2))/(7245*a^4) + (352*b^2*x^(7/
3)*(b*x^(2/3) + a*x)^(3/2))/(1725*a^3) - (16*b*x^(8/3)*(b*x^(2/3) + a*x)^(3
/2))/(75*a^2) + (2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx = \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a} - \frac{(8b)\int x^{8/3}\sqrt{bx^{2/3}+ax}\,dx}{9a}$$
$$= -\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2} + \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a} + \frac{(176b^2)\int x^{7/3}\sqrt{bx^{2/3}+ax}\,dx}{225a^2}$$
$$= \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3} - \frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2} + \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a} - \frac{(704b \cdots}{}$$
$$= -\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4} + \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3} - \frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2} \cdots$$
$$= \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5} - \frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4} + \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3} \cdots$$
$$= -\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6} + \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5} - \frac{1408b^3x^2\left(bx^{2/3}+ax \cdots\right.}{7245a^4}$$
$$= \frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7} - \frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6} + \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right) \cdots}{15295a^5}$$
$$= -\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8} + \frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7} - \frac{45056b^5x^{4/3}\left(bx^{2/3}+ \cdots\right.}{260015a^6}$$
$$= \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9} - \frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8} + \frac{90112b^6x\left(bx^{2/3}+a \cdots\right.}{557175a^7}$$
$$= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} + \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9} - \frac{360448b^7x^{2/3}\left(bx^{2/3}+ \cdots\right.}{2414425a^8}$$
$$= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} + \frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}} + \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+a \cdots\right.}{482885a^9}$$
$$= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} - \frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}} + \frac{1048576b^{10}\left(bx^{2/3}+ax \cdots\right.}{10140585a^{11}\sqrt[3]{x}}$$
$$= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} + \frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x} - \frac{4194304b^{11}\left(bx^{2/3}+ax \cdots\right.}{50702925a^{12}x^{2/3}}$$

Mathematica [A]
time = 0.11, size = 185, normalized size = 0.50

$$\frac{2\sqrt{bx^{2/3}+ax}\left(4194304b^{13}-2097152ab^{12}\sqrt[3]{x}+1572864a^2b^{11}x^{2/3}-1310720a^3b^{10}x+1146880a^4b^9x^{4/3}-1032192a^5b^8x^{5/3}+946176a^6b^7x^2-878592a^7b^6x^{7/3}+823680a^8b^5x^{8/3}-777920a^9b^4x^3+739024a^{10}b^3x^{10/3}-705432a^{11}b^2x^{11/3}+676039a^{12}bx^4+16900975a^{13}x^{13/3}\right)}{152108775a^{13}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(4194304*b^13 - 2097152*a*b^12*x^(1/3) + 1572864*a
^2*b^11*x^(2/3) - 1310720*a^3*b^10*x + 1146880*a^4*b^9*x^(4/3) - 1032192*a^
5*b^8*x^(5/3) + 946176*a^6*b^7*x^2 - 878592*a^7*b^6*x^(7/3) + 823680*a^8*b^
5*x^(8/3) - 777920*a^9*b^4*x^3 + 739024*a^10*b^3*x^(10/3) - 705432*a^11*b^2
*x^(11/3) + 676039*a^12*b*x^4 + 16900975*a^13*x^(13/3)))/(152108775*a^13*x^
(1/3))
```

Maple [A]
time = 0.38, size = 156, normalized size = 0.42

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(16900975a^{12}x^{4}-16224936a^{11}b\,x^{\frac{11}{3}}+15519504a^{10}b^{2}x^{\frac{10}{3}}-14780480a^{9}b^{3}x^{3}+14002560a^{8}b\right.}{}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(16224936a^{11}b\,x^{\frac{11}{3}}-15519504a^{10}b^{2}x^{\frac{10}{3}}-14002560a^{8}b^{4}x^{\frac{8}{3}}+13178880a^{7}b^{5}x^{\frac{7}{3}}+1135411\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/152108775*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(16224936*a^11*b*x^(11/3)-
15519504*a^10*b^2*x^(10/3)-14002560*a^8*b^4*x^(8/3)+13178880*a^7*b^5*x^(7/3
)+11354112*a^5*b^7*x^(5/3)-10321920*a^4*b^8*x^(4/3)-16900975*a^12*x^4+14780
480*a^9*b^3*x^3-7864320*a^2*b^10*x^(2/3)-12300288*a^6*b^6*x^2+6291456*a*b^1
1*x^(1/3)+9175040*a^3*b^9*x-4194304*b^12)/x^(1/3)/a^13
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(277) = 554.
time = 291.53, size = 1293, normalized size = 3.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/304217550*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(6
4*a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 - 3
93725113600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6 -
114688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*(488
16*a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10 -
7340032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b^8 + 1
2288*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(43743616*a^12
 + 89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 42432*(7217086464
*a^15 + 4969216*a^12 + 165*a^9)*b^4 + 7524608*(20570112*a^15 - 2101*a^12)*b
^3 + 2821728*(7815168*a^15 + 181*a^12)*b^2 + 2028117*(2072576*a^15 - 3*a^12
)*b)*x - 4*(16900975*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^
4 - 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x
^5 + 739024*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*b^7 - 11468
8*a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a^10)*b^3)*x^
4 - 878592*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7*b^10 - 114688
*a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^7)*b^6)*x^3 + 1
146880*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b^13 - 114688*a^4
*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^4)*b^9)*x^2 - 209
7152*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 - 114688*a*b^15 - 23
04*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x + (70368744177664*b^19
 + 26388279066624*b^18 + 824633720832*b^17 - 481036337152*b^16 - 9663676416
*b^15 - 4194304*(262144*a^3 + 27)*b^13 + 3623878656*b^14 + 676039*(16777216
*a^12*b^7 + 6291456*a^12*b^6 + 196608*a^12*b^5 - 114688*a^12*b^4 - 2304*a^1
2*b^3 + 864*a^12*b^2 - (262144*a^15 + 27*a^12)*b)*x^4 - 777920*(16777216*a^
9*b^10 + 6291456*a^9*b^9 + 196608*a^9*b^8 - 114688*a^9*b^7 - 2304*a^9*b^6 +
 864*a^9*b^5 - (262144*a^12 + 27*a^9)*b^4)*x^3 + 946176*(16777216*a^6*b^13
+ 6291456*a^6*b^12 + 196608*a^6*b^11 - 114688*a^6*b^10 - 2304*a^6*b^9 + 864
*a^6*b^8 - (262144*a^9 + 27*a^6)*b^7)*x^2 - 1310720*(16777216*a^3*b^16 + 62
91456*a^3*b^15 + 196608*a^3*b^14 - 114688*a^3*b^13 - 2304*a^3*b^12 + 864*a^
3*b^11 - (262144*a^6 + 27*a^3)*b^10)*x)*x^(2/3) - 24*(29393*(16777216*a^11*
b^8 + 6291456*a^11*b^7 + 196608*a^11*b^6 - 114688*a^11*b^5 - 2304*a^11*b^4
+ 864*a^11*b^3 - (262144*a^14 + 27*a^11)*b^2)*x^4 - 34320*(16777216*a^8*b^1
1 + 6291456*a^8*b^10 + 196608*a^8*b^9 - 114688*a^8*b^8 - 2304*a^8*b^7 + 864
*a^8*b^6 - (262144*a^11 + 27*a^8)*b^5)*x^3 + 43008*(16777216*a^5*b^14 + 629
1456*a^5*b^13 + 196608*a^5*b^12 - 114688*a^5*b^11 - 2304*a^5*b^10 + 864*a^5
*b^9 - (262144*a^8 + 27*a^5)*b^8)*x^2 - 65536*(16777216*a^2*b^17 + 6291456*
a^2*b^16 + 196608*a^2*b^15 - 114688*a^2*b^14 - 2304*a^2*b^13 + 864*a^2*b^12
 - (262144*a^5 + 27*a^2)*b^11)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((1677721
6*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^4 - 262144*a^16 - 114688*a^13
*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.83, size = 396, normalized size = 1.07

$$-\frac{8388608\,b^{\frac{27}{2}}}{152108775\,a^{13}}+\frac{2\left(\frac{27\left(676039\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}-8817900\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b+53117350\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^{2}-195695500\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^{3}+492116625\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{4}-892371480\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{5}+1201269300\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{6}-1216870200\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{7}+929553625\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{8}-531173500\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{9}+223092870\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{10}-67603900\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{11}+16900975\sqrt{ax^{\frac{1}{3}}+b}\,b^{12}\right)b}{a^{12}}+\frac{13\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}}-18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b+119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^{2}-478056150\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^{3}+1320944625\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^{4}-2657429775\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{5}+4015671660\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{6}-4633467300\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^{7}+4106936925\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{8}-2788660875\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{9}+1434168450\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{10}-547591590\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{11}+152108775\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{12}-35102025\sqrt{ax^{\frac{1}{3}}+b}\,b^{13}\right)}{a^{12}}\right)}{152108775\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -8388608/152108775*b^(27/2)/a^13 + 2/152108775*(27*(676039*(a*x^(1/3) + b)^
(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)
*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17
/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)
^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3)
+ b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3)
 + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^
(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 18253053*(a*
x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478056150*(a
*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 - 26574297
75*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*b^6 - 463
3467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(11/2)*b^8
- 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b)^(7/2)*b
^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2
)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^(2/3))^(1/2), x)
```

3.168 $\int x^2 \sqrt{bx^{2/3}+ax}\, dx$

Optimal. Leaf size=283

$$\frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}}-921$$

```
[Out] 8192/46189*b^6*(b*x^(2/3)+a*x)^(3/2)/a^7-131072/1616615*b^9*(b*x^(2/3)+a*x)
^(3/2)/a^10/x+196608/1616615*b^8*(b*x^(2/3)+a*x)^(3/2)/a^9/x^(2/3)-49152/32
3323*b^7*(b*x^(2/3)+a*x)^(3/2)/a^8/x^(1/3)-9216/46189*b^5*x^(1/3)*(b*x^(2/3
)+a*x)^(3/2)/a^6+4608/20995*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^5-384/1615*
b^3*x*(b*x^(2/3)+a*x)^(3/2)/a^4+576/2261*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/
a^3-36/133*b*x^(5/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/7*x^2*(b*x^(2/3)+a*x)^(3/2
)/a
```

Rubi [A]
time = 0.30, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(ax+bx^{2/3}\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{133a^2}+\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8192*b^6*(b*x^(2/3) + a*x)^(3/2))/(46189*a^7) - (131072*b^9*(b*x^(2/3) + a
*x)^(3/2))/(1616615*a^10*x) + (196608*b^8*(b*x^(2/3) + a*x)^(3/2))/(1616615
*a^9*x^(2/3)) - (49152*b^7*(b*x^(2/3) + a*x)^(3/2))/(323323*a^8*x^(1/3)) -
(9216*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(46189*a^6) + (4608*b^4*x^(2/3)*
(b*x^(2/3) + a*x)^(3/2))/(20995*a^5) - (384*b^3*x*(b*x^(2/3) + a*x)^(3/2))/
(1615*a^4) + (576*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(2261*a^3) - (36*b*x
^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(133*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(3/2))
/(7*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
```

```
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} - \frac{(6b)\int x^{5/3}\sqrt{bx^{2/3}+ax}\,dx}{7a}$$

$$= -\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} + \frac{\left(96b^2\right)\int x^{4/3}\sqrt{bx^{2/3}+ax}\,dx}{133a^2}$$

$$= \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} - \frac{(192b^3)}{}$$

$$= -\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x}{}$$

$$= \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} -$$

$$= -\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}$$

$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} -$$

$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{46}{}$$

$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{921}{}$$

$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49}{}$$

Mathematica [A]

time = 0.09, size = 133, normalized size = 0.47

$$\frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(-65536b^9+98304ab^8\sqrt[3]{x}-122880a^2b^7x^{2/3}+143360a^3b^6x-161280a^4b^5x^{4/3}+177408a^5b^4x^{5/3}-192192a^6b^3x^2+205920a^7b^2x^{7/3}-218790a^8bx^{8/3}+230945a^9x^3\right)}{1616615a^{10}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2)*(-65536*b^9 + 98304*a*b^8*x^(1/3) - 122880*a^2*b
^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 177408*a^5*b^4*x^(
5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a^8*b*x^(8/3) +
 230945*a^9*x^3))/(1616615*a^10*x)
```

Maple [A]
time = 0.37, size = 123, normalized size = 0.43

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(230945a^9x^3-218790a^8b\,x^{\frac{8}{3}}+205920a^7b^2x^{\frac{7}{3}}-192192a^6b^3x^2+177408a^5b^4x^{\frac{5}{3}}-161280a^4\right.}{1616615x^{\frac{1}{3}}\,a^{10}}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(218790a^8b\,x^{\frac{8}{3}}-205920a^7b^2x^{\frac{7}{3}}-177408a^5b^4x^{\frac{5}{3}}+161280a^4b^5x^{\frac{4}{3}}-230945a^9x^3+12288\right.}{1616615x^{\frac{1}{3}}\,a^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1616615*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(218790*a^8*b*x^(8/3)-205920
*a^7*b^2*x^(7/3)-177408*a^5*b^4*x^(5/3)+161280*a^4*b^5*x^(4/3)-230945*a^9*x
^3+122880*a^2*b^7*x^(2/3)+192192*a^6*b^3*x^2-98304*a*b^8*x^(1/3)-143360*a^3
*b^6*x+65536*b^9)/x^(1/3)/a^10
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.52, size = 312, normalized size = 1.10

$$\frac{131072\,b^{\frac{21}{2}}}{1616615\,a^{10}}+\frac{2\left(\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}-122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b+554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^2-1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^3+2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^4-3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^5+2771340\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^6-1662804\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^7+692835\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^8-230945\sqrt{ax^{\frac{1}{3}}+b}\,b^9\right)b}{a^9}+\frac{5\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}-510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b+2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^2-7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^3+15668730\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^4-22221108\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^5+22632610\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^6-16628040\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^7+8729721\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^8-3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^9+969969\sqrt{ax^{\frac{1}{3}}+b}\,b^{10}\right)}{a^9}\right)}{1616615\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 131072/1616615*b^(21/2)/a^10 + 2/1616615*(21*(12155*(a*x^(1/3) + b)^(19/2)
- 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 149
2260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3233
230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804
*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt
(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510*(a*x^
(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(a*x^(1/
3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108*(a*x^(1
/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 16628040*(a*x^(1
/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(a*x^(1/3)
 + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^9)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(2/3))^(1/2), x)
```

3.169 $\int x\sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=195

$$-\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+\frac{48b^2\sqrt[3]{x}\,(b}{1}$$

```
[Out] -128/429*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4+2048/15015*b^6*(b*x^(2/3)+a*x)^(3/2)
/a^7/x-1024/5005*b^5*(b*x^(2/3)+a*x)^(3/2)/a^6/x^(2/3)+256/1001*b^4*(b*x^(2
/3)+a*x)^(3/2)/a^5/x^(1/3)+48/143*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^3-24/
65*b*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/5*x*(b*x^(2/3)+a*x)^(3/2)/a
```

Rubi [A]
time = 0.18, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2041, 2027, 2039}

$$\frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{65a^2}+\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*(b*x^(2/3) + a*x)^(3/2))/(429*a^4) + (2048*b^6*(b*x^(2/3) + a*x)^
(3/2))/(15015*a^7*x) - (1024*b^5*(b*x^(2/3) + a*x)^(3/2))/(5005*a^6*x^(2/3)
) + (256*b^4*(b*x^(2/3) + a*x)^(3/2))/(1001*a^5*x^(1/3)) + (48*b^2*x^(1/3)*
(b*x^(2/3) + a*x)^(3/2))/(143*a^3) - (24*b*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))
/(65*a^2) + (2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x\sqrt{bx^{2/3}+ax}\,dx = \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}-\frac{(4b)\int x^{2/3}\sqrt{bx^{2/3}+ax}\,dx}{5a}$$
$$= -\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}+\frac{(8b^2)\int\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}\,dx}{13a^2}$$
$$= \frac{48b^2\sqrt[3]{x}\,\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}-\frac{(64b^3)\int\sqrt{}}{}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\,\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x(b}{}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+\frac{48b^2\sqrt[3]{x}\,\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}}{}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}-\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+\frac{48b^2\sqrt[3]{x}}{}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4(b}{10}$$

Mathematica [A]
time = 0.06, size = 96, normalized size = 0.49

$$\frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(1024b^6-1536ab^5\sqrt[3]{x}\,+1920a^2b^4x^{2/3}-2240a^3b^3x+2520a^4b^2x^{4/3}-2772a^5bx^{5/3}+3003a^6x^2\right)}{15015a^7x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2)*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1920*a^2*b^4*x^
(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^(5/3) + 3003*a
^6*x^2))/(15015*a^7*x)
```

Maple [A]
time = 0.35, size = 90, normalized size = 0.46

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{6}x^{2}-2772a^{5}b\,x^{\frac{5}{3}}+2520a^{4}b^{2}x^{\frac{4}{3}}-2240a^{3}b^{3}x+1920a^{2}b^{4}x^{\frac{2}{3}}-1536a\,b^{5}x^{\frac{1}{3}}+1024b^{6}\right)}{15015x^{\frac{1}{3}}a^{7}}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(2772a^{5}b\,x^{\frac{5}{3}}-2520a^{4}b^{2}x^{\frac{4}{3}}-1920a^{2}b^{4}x^{\frac{2}{3}}-3003a^{6}x^{2}+1536a\,b^{5}x^{\frac{1}{3}}+2240a^{3}b^{3}x-1024b}{15015x^{\frac{1}{3}}a^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15015*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(2772*a^5*b*x^(5/3)-2520*a^4*b
^2*x^(4/3)-1920*a^2*b^4*x^(2/3)-3003*a^6*x^2+1536*a*b^5*x^(1/3)+2240*a^3*b^
3*x-1024*b^6)/x^(1/3)/a^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.20, size = 228, normalized size = 1.17

$$-\frac{2048\,b^{\frac{15}{2}}}{15015\,a^{7}}+\frac{2\left(\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{2}-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{3}+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{4}-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{5}+3003\sqrt{ax^{\frac{1}{3}}+b}\,b^{6}\right)b}{a^{6}}+\frac{7\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}-3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b+12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{2}-25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{3}+32175\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{4}-27027\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{5}+15015\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{6}-6435\sqrt{ax^{\frac{1}{3}}+b}\,b^{7}\right)}{a^{6}}\right)}{15015\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2048/15015*b^(15/2)/a^7 + 2/15015*(15*(231*(a*x^(1/3) + b)^(13/2) - 1638*(
a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3)
+ b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2
)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) + b)^(15/2)
 - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025
*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(
1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3)
 + b)*b^7)/a^6)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a\,x+b\,x^{2/3}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x*(a*x + b*x^(2/3))^(1/2), x)
```

3.170 $\int \sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=109

$$\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}$$

```
[Out] 2/3*(b*x^(2/3)+a*x)^(3/2)/a-32/105*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4/x+16/35*b^
2*(b*x^(2/3)+a*x)^(3/2)/a^3/x^(2/3)-4/7*b*(b*x^(2/3)+a*x)^(3/2)/a^2/x^(1/3)
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 2041, 2039}

$$-\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x}+\frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (32*b^3*(b*x^(2/3) + a*x)^(3/2))/(105*a
^4*x) + (16*b^2*(b*x^(2/3) + a*x)^(3/2))/(35*a^3*x^(2/3)) - (4*b*(b*x^(2/3)
 + a*x)^(3/2))/(7*a^2*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx^{2/3}+ax}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{(2b)\int \frac{\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}\,dx}{3a}\\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{\left(8b^2\right)\int \frac{\sqrt{bx^{2/3}+ax}}{x^{2/3}}\,dx}{21a^2}\\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}-\frac{\left(16b^3\right)\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx}{105a^3}\\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 0.68

$$\frac{2\sqrt{bx^{2/3}+ax}\left(-16b^4+8ab^3\sqrt[3]{x}-6a^2b^2x^{2/3}+5a^3bx+35a^4x^{4/3}\right)}{105a^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-16*b^4 + 8*a*b^3*x^(1/3) - 6*a^2*b^2*x^(2/3) + 5
*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^4*x^(1/3))
```

Maple [A]
time = 0.35, size = 57, normalized size = 0.52

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^3x-30a^2b\,x^{\frac{2}{3}}+24a\,b^2x^{\frac{1}{3}}-16b^3\right)}{105x^{\frac{1}{3}}a^4}$	57
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(30a^2b\,x^{\frac{2}{3}}-24a\,b^2x^{\frac{1}{3}}-35a^3x+16b^3\right)}{105x^{\frac{1}{3}}a^4}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(30*a^2*b*x^(2/3)-24*a*b^2*x^(1/
3)-35*a^3*x+16*b^3)/x^(1/3)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.61, size = 143, normalized size = 1.31

$$\frac{32\,b^{\frac{9}{2}}}{105\,a^{4}} + \frac{2\left(\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{2}-35\sqrt{ax^{\frac{1}{3}}+b}\,b^{3}\right)b}{a^{3}} + \frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{2}-420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{3}+315\sqrt{ax^{\frac{1}{3}}+b}\,b^{4}}{a^{3}}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 32/105*b^(9/2)/a^4 + 2/105*(9*(5*(a*x^(1/3) + b)^(7/2) - 21*(a*x^(1/3) + b)
^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3) + b)*b^3)*b/a^3
 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/2)*b + 378*(a*x^(1/3)
```

```
 + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*sqrt(a*x^(1/3) + b)*b
^4)/a^3)/a
```

Mupad [B]
time = 5.19, size = 40, normalized size = 0.37

$$\frac{3\,x\,\sqrt{a\,x+b\,x^{2/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},4;\;5;\;-\frac{a\,x^{1/3}}{b}\right)}{4\,\sqrt{\frac{a\,x^{1/3}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*(a*x + b*x^(2/3))^(1/2)*hypergeom([-1/2, 4], 5, -(a*x^(1/3))/b))/(4*((
a*x^(1/3))/b + 1)^(1/2))
```

3.171 $\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx$

Optimal. Leaf size=23

$$\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

```
[Out] 2*(b*x^(2/3)+a*x)^(3/2)/a/x
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 1.00

$$\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Maple [A]
time = 0.36, size = 27, normalized size = 1.17

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}a}$	27
default	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}a}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)*(b+a*x^(1/3))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3))/x, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x, x)
```

Giac [A]
time = 1.51, size = 23, normalized size = 1.00

$$\frac{2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}}{a}-\frac{2\,b^{\frac{3}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x, x)
```

3.172 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$

Optimal. Leaf size=90

$$-\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

```
[Out] 3/4*a^2*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3/2*(b*x^(2/
3)+a*x)^(1/2)/x-3/4*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(2/3)
```

Rubi [A]
time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2054, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(2*x) - (3*a*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(2/3)
) + (3*a^2*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}+\frac{1}{4}a\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}-\frac{a^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 76, normalized size = 0.84

$$-\frac{3\left(2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{4bx}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*(2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(4*b*x) + (3*a^2*ArcTanh[(Sqrt
[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Maple [A]
time = 0.35, size = 79, normalized size = 0.88

method	result	size

default	$-\frac{3\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\,a^{2}x^{\frac{2}{3}}+b^{\frac{5}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\right)}{4x\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}}$	79
derivativedivides	$\frac{3\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\,a^{2}x^{\frac{2}{3}}-b^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}-b^{\frac{5}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\right)}{4x\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*(b*x^(2/3)+a*x)^(1/2)*(b^(3/2)*(b+a*x^(1/3))^(3/2)-arctanh((b+a*x^(1/3
))^(1/2)/b^(1/2))*b*a^2*x^(2/3)+b^(5/2)*(b+a*x^(1/3))^(1/2))/x/(b+a*x^(1/3)
)^(1/2)/b^(5/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**2, x)
```

Giac [A]
time = 1.75, size = 72, normalized size = 0.80

$$-\frac{3\left(\frac{a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3+\sqrt{ax^{\frac{1}{3}}+b}\,a^3b}{a^2bx^{\frac{2}{3}}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
[Out] -3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3) +
 b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^2,x)
[Out] int((a*x + b*x^(2/3))^(1/2)/x^2, x)
```

3.173 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

Optimal. Leaf size=178

$$-\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}+\frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}}-\frac{21a^5\tanh^{-}}{}$$

```
[Out] -21/128*a^5*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/5*(b*x
^(2/3)+a*x)^(1/2)/x^2-3/40*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/80*a^2*(b*x^
(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/128*a^
4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A]

time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2054, 212}

$$-\frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}}+\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}}-\frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x}+\frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) - (3*a*Sqrt[b*x^(2/3) + a*x])/(40*b*x^(5
/3)) + (7*a^2*Sqrt[b*x^(2/3) + a*x])/(80*b^2*x^(4/3)) - (7*a^3*Sqrt[b*x^(2/
3) + a*x])/(64*b^3*x) + (21*a^4*Sqrt[b*x^(2/3) + a*x])/(128*b^4*x^(2/3)) -
(21*a^5*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}+\frac{1}{10}a\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}-\frac{(7a^2)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{80b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}+\frac{(7a^3)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+a}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}-(7$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}+21$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}+21$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}+21$$

Mathematica [A]
time = 0.22, size = 112, normalized size = 0.63

$$\frac{\sqrt{bx^{2/3}+ax}\left(-384b^4-48ab^3\sqrt[3]{x}+56a^2b^2x^{2/3}-70a^3bx+105a^4x^{4/3}\right)}{640b^4x^2}-\frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-384*b^4 - 48*a*b^3*x^(1/3) + 56*a^2*b^2*x^(2/3) -
70*a^3*b*x + 105*a^4*x^(4/3)))/(640*b^4*x^2) - (21*a^5*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Maple [A]
time = 0.35, size = 125, normalized size = 0.70

method	result
derivativedivides	$\frac{\sqrt{b x^{\frac{2}{3}}+ax}\left(105\left(b+a x^{\frac{1}{3}}\right)^{\frac{9}{2}} b^{\frac{9}{2}}-490\left(b+a x^{\frac{1}{3}}\right)^{\frac{7}{2}} b^{\frac{11}{2}}+896\left(b+a x^{\frac{1}{3}}\right)^{\frac{5}{2}} b^{\frac{13}{2}}-790\left(b+a x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{15}{2}}-105\,\operatorname{arctanh}\left(\right.\right.}{640x^{2}\sqrt{b+a x^{\frac{1}{3}}}\, b^{\frac{17}{2}}}$
default	$-\frac{\sqrt{b x^{\frac{2}{3}}+ax}\left(-105\left(b+a x^{\frac{1}{3}}\right)^{\frac{9}{2}} b^{\frac{9}{2}}+490\left(b+a x^{\frac{1}{3}}\right)^{\frac{7}{2}} b^{\frac{11}{2}}-896\left(b+a x^{\frac{1}{3}}\right)^{\frac{5}{2}} b^{\frac{13}{2}}+105\,\operatorname{arctanh}\left(\frac{\sqrt{b+a x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\right.}{640x^{2}\sqrt{b+a x^{\frac{1}{3}}}\, b^{\frac{17}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(b*x^(2/3)+a*x)^(1/2)*(-105*(b+a*x^(1/3))^(9/2)*b^(9/2)+490*(b+a*x^(
1/3))^(7/2)*b^(11/2)-896*(b+a*x^(1/3))^(5/2)*b^(13/2)+105*arctanh((b+a*x^(1
/3))^(1/2)/b^(1/2))*b^4*a^5*x^(5/3)+790*(b+a*x^(1/3))^(3/2)*b^(15/2)+105*(b
+a*x^(1/3))^(1/2)*b^(17/2))/x^2/(b+a*x^(1/3))^(1/2)/b^(17/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**3, x)
```

Giac [A]
time = 1.53, size = 126, normalized size = 0.71

$$\frac{\frac{105\, a^6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}} + b}}{\sqrt{-b}}\right)}{\sqrt{-b}\, b^4} + \frac{105 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^6 - 490 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^6 b + 896 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^6 b^2 - 790 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^6 b^3 - 105 \sqrt{ax^{\frac{1}{3}} + b}\, a^6 b^4}{a^5 b^4 x^{\frac{5}{3}}}}{640\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/640*(105*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(
a*x^(1/3) + b)^(9/2)*a^6 - 490*(a*x^(1/3) + b)^(7/2)*a^6*b + 896*(a*x^(1/3)
 + b)^(5/2)*a^6*b^2 - 790*(a*x^(1/3) + b)^(3/2)*a^6*b^3 - 105*sqrt(a*x^(1/3
) + b)*a^6*b^4)/(a^5*b^4*x^(5/3)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\, x + b\, x^{2/3}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^3, x)
```

3.174 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

Optimal. Leaf size=266

$$-\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-\frac{429a}{1}$$

```
[Out] 1287/16384*a^8*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/8*
(b*x^(2/3)+a*x)^(1/2)/x^3-3/112*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/448*a^
2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143/4480*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/
x^2+1287/35840*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5/3)-429/10240*a^5*(b*x^(2/
3)+a*x)^(1/2)/b^5/x^(4/3)+429/8192*a^6*(b*x^(2/3)+a*x)^(1/2)/b^6/x-1287/163
84*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A]
time = 0.32, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2054, 212}

$$\frac{1287a^8\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{16384b^{15/2}}-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}}+\frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x}-\frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}}+\frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}}-\frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2}+\frac{13a^2\sqrt{ax+bx^{2/3}}}{448b^2x^{7/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{112bx^{8/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) - (3*a*Sqrt[b*x^(2/3) + a*x])/(112*b*x^(
8/3)) + (13*a^2*Sqrt[b*x^(2/3) + a*x])/(448*b^2*x^(7/3)) - (143*a^3*Sqrt[b*
x^(2/3) + a*x])/(4480*b^3*x^2) + (1287*a^4*Sqrt[b*x^(2/3) + a*x])/(35840*b^
4*x^(5/3)) - (429*a^5*Sqrt[b*x^(2/3) + a*x])/(10240*b^5*x^(4/3)) + (429*a^6
*Sqrt[b*x^(2/3) + a*x])/(8192*b^6*x) - (1287*a^7*Sqrt[b*x^(2/3) + a*x])/(16
384*b^7*x^(2/3)) + (1287*a^8*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/(16384*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} + \frac{1}{16}a\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} - \frac{(13a^2)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{224b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} + \frac{(143a^3)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+}}}{2688b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} -$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} +$$

Mathematica [A]
time = 0.26, size = 149, normalized size = 0.56

$$\frac{\sqrt{bx^{2/3}+ax}\left(-215040b^7-15360ab^6\sqrt[3]{x}+16640a^2b^5x^{2/3}-18304a^3b^4x+20592a^4b^3x^{4/3}-24024a^5b^2x^{5/3}+30030a^6bx^2-45045a^7x^{7/3}\right)}{573440b^7x^3}+\frac{1287a^8\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-215040*b^7 - 15360*a*b^6*x^(1/3) + 16640*a^2*b^5*x
^(2/3) - 18304*a^3*b^4*x + 20592*a^4*b^3*x^(4/3) - 24024*a^5*b^2*x^(5/3) +
30030*a^6*b*x^2 - 45045*a^7*x^(7/3)))/(573440*b^7*x^3) + (1287*a^8*ArcTanh[
(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(16384*b^(15/2))
```

Maple [A]
time = 0.35, size = 167, normalized size = 0.63

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(45045\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{15}{2}}b^{\frac{15}{2}}-345345\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{13}{2}}b^{\frac{17}{2}}+1150149\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{19}{2}}-2167737\left(b+a\,x^{\frac{1}{3}}\right)\right.}{}$
default	$\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-45045\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{15}{2}}b^{\frac{15}{2}}+345345\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{13}{2}}b^{\frac{17}{2}}-1150149\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{19}{2}}+2167737\left(b+a\,x^{\frac{1}{3}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/573440*(b*x^(2/3)+a*x)^(1/2)*(-45045*(b+a*x^(1/3))^(15/2)*b^(15/2)+345345
*(b+a*x^(1/3))^(13/2)*b^(17/2)-1150149*(b+a*x^(1/3))^(11/2)*b^(19/2)+216773
7*(b+a*x^(1/3))^(9/2)*b^(21/2)-2518087*(b+a*x^(1/3))^(7/2)*b^(23/2)+1831739
*(b+a*x^(1/3))^(5/2)*b^(25/2)+45045*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^
7*a^8*x^(8/3)-801535*(b+a*x^(1/3))^(3/2)*b^(27/2)-45045*(b+a*x^(1/3))^(1/2)
*b^(29/2))/x^3/(b+a*x^(1/3))^(1/2)/b^(29/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^4, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**4, x)
```

Giac [A]
time = 1.64, size = 177, normalized size = 0.67

$$-\frac{\frac{45045\,a^{9}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{9}-345345\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{9}b+1150149\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{9}b^{2}-2167737\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{9}b^{3}+2518087\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{9}b^{4}-1831739\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{9}b^{5}+801535\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{9}b^{6}+45045\sqrt{ax^{\frac{1}{3}}+b}\,a^{9}b^{7}}{a^{8}b^{7}x^{\frac{8}{3}}}}{573440\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/573440*(45045*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
(45045*(a*x^(1/3) + b)^(15/2)*a^9 - 345345*(a*x^(1/3) + b)^(13/2)*a^9*b + 1
150149*(a*x^(1/3) + b)^(11/2)*a^9*b^2 - 2167737*(a*x^(1/3) + b)^(9/2)*a^9*b
^3 + 2518087*(a*x^(1/3) + b)^(7/2)*a^9*b^4 - 1831739*(a*x^(1/3) + b)^(5/2)*
a^9*b^5 + 801535*(a*x^(1/3) + b)^(3/2)*a^9*b^6 + 45045*sqrt(a*x^(1/3) + b)*
a^9*b^7)/(a^8*b^7*x^(8/3)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^4, x)
```

3.175 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

Optimal. Leaf size=354

$$-\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199}{2}$$

```
[Out] -12597/262144*a^11*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)-
3/11*(b*x^(2/3)+a*x)^(1/2)/x^4-3/220*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/
1320*a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)-323/21120*a^3*(b*x^(2/3)+a*x)^(
1/2)/b^3/x^3+323/19712*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(8/3)-4199/236544*a^
5*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+4199/215040*a^6*(b*x^(2/3)+a*x)^(1/2)/b
^6/x^2-12597/573440*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/3)+4199/163840*a^8*(
b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-4199/131072*a^9*(b*x^(2/3)+a*x)^(1/2)/b^9/
x+12597/262144*a^10*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A]

time = 0.44, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2054, 212}

$$-\frac{12597a^{11}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{262144b^{21/2}}+\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}-\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}}+\frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2}-\frac{4199a^5\sqrt{ax+bx^{2/3}}}{236544b^5x^{7/3}}+\frac{323a^4\sqrt{ax+bx^{2/3}}}{19712b^4x^{8/3}}-\frac{323a^3\sqrt{ax+bx^{2/3}}}{21120b^3x^3}+\frac{19a^2\sqrt{ax+bx^{2/3}}}{1320b^2x^{10/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{220bx^{11/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^
(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqr
t[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(1971
2*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4
199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3)
 + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^
8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*
Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b]
*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*b^(21/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} + \frac{1}{22}a\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} - \frac{(19a^2)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{440b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} + \frac{(323a^3)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}}}}{7920b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} -$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} +$$

Mathematica [A]

time = 0.33, size = 186, normalized size = 0.53

$$\frac{\sqrt{bx^{2/3}+ax}\left(-82575360b^{10}-4128768ab^9\sqrt[3]{x}+4358144a^2b^8x^{2/3}-4630528a^3b^7x+4961280a^4b^6x^{4/3}-5374720a^5b^5x^{5/3}+5912192a^6b^4x^2-6651216a^7b^3x^{7/3}+7759752a^8b^2x^{8/3}-9699690a^9bx^3+14549535a^{10}x^{10/3}\right)}{302776320b^{10}x^4}-\frac{12597a^{11}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-82575360*b^10 - 4128768*a*b^9*x^(1/3) + 4358144*a^
2*b^8*x^(2/3) - 4630528*a^3*b^7*x + 4961280*a^4*b^6*x^(4/3) - 5374720*a^5*b
^5*x^(5/3) + 5912192*a^6*b^4*x^2 - 6651216*a^7*b^3*x^(7/3) + 7759752*a^8*b^
2*x^(8/3) - 9699690*a^9*b*x^3 + 14549535*a^10*x^(10/3)))/(302776320*b^10*x^
4) - (12597*a^11*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*
b^(21/2))
```

Maple [A]
time = 0.36, size = 209, normalized size = 0.59

method	result
derivativedivides	$\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}} b^{\frac{21}{2}}-155195040\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}} b^{\frac{23}{2}}+749786037\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{25}{2}}-2163862272\right.}{}$
default	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}} b^{\frac{21}{2}}+155195040\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}} b^{\frac{23}{2}}-749786037\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{25}{2}}+21638622\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/302776320*(b*x^(2/3)+a*x)^(1/2)*(-14549535*(b+a*x^(1/3))^(21/2)*b^(21/2)
+155195040*(b+a*x^(1/3))^(19/2)*b^(23/2)-749786037*(b+a*x^(1/3))^(17/2)*b^(
25/2)+2163862272*(b+a*x^(1/3))^(15/2)*b^(27/2)-4139920070*(b+a*x^(1/3))^(13
/2)*b^(29/2)+5503713280*(b+a*x^(1/3))^(11/2)*b^(31/2)-5174056250*(b+a*x^(1/
3))^(9/2)*b^(33/2)+3424523520*(b+a*x^(1/3))^(7/2)*b^(35/2)-1551313995*(b+a*
x^(1/3))^(5/2)*b^(37/2)+14549535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*
a^11*x^(11/3)+450357600*(b+a*x^(1/3))^(3/2)*b^(39/2)+14549535*(b+a*x^(1/3))
^(1/2)*b^(41/2))/x^4/(b+a*x^(1/3))^(1/2)/b^(41/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^5, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**5, x)
```

Giac [A]
time = 1.64, size = 228, normalized size = 0.64

$$\frac{\frac{14549535\, a^{12} \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\, b^{10}} + \frac{14549535 \left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} a^{12} - 155195040 \left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} a^{12} b + 749786037 \left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^{12} b^2 - 2163862272 \left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^{12} b^3 + 4139920070 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^{12} b^4 - 5503713280 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^{12} b^5 + 5174056250 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^{12} b^6 - 3424523520 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^{12} b^7 + 1551313995 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^{12} b^8 - 450357600 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^{12} b^9 - 14549535 \sqrt{ax^{\frac{1}{3}}+b}\, a^{12} b^{10}}{a^{11} b^{10} x^{\frac{11}{3}}}}{302776320\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/302776320*(14549535*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b
^10) + (14549535*(a*x^(1/3) + b)^(21/2)*a^12 - 155195040*(a*x^(1/3) + b)^(1
9/2)*a^12*b + 749786037*(a*x^(1/3) + b)^(17/2)*a^12*b^2 - 2163862272*(a*x^(
1/3) + b)^(15/2)*a^12*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*a^12*b^4 - 55
03713280*(a*x^(1/3) + b)^(11/2)*a^12*b^5 + 5174056250*(a*x^(1/3) + b)^(9/2)
*a^12*b^6 - 3424523520*(a*x^(1/3) + b)^(7/2)*a^12*b^7 + 1551313995*(a*x^(1/
3) + b)^(5/2)*a^12*b^8 - 450357600*(a*x^(1/3) + b)^(3/2)*a^12*b^9 - 1454953
5*sqrt(a*x^(1/3) + b)*a^12*b^10)/(a^11*b^10*x^(11/3)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^5, x)
```

3.176 $\int x^2 \left(bx^{2/3}+ax\right)^{3/2} \, dx$

Optimal. Leaf size=343

$$\frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x}+$$

```
[Out] 45056/557175*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7-1048576/152108775*b^11*(b*x^(2/3
)+a*x)^(5/2)/a^12/x^(5/3)+524288/30421755*b^10*(b*x^(2/3)+a*x)^(5/2)/a^11/x
^(4/3)-131072/4345965*b^9*(b*x^(2/3)+a*x)^(5/2)/a^10/x+65536/1448655*b^8*(b
*x^(2/3)+a*x)^(5/2)/a^9/x^(2/3)-90112/1448655*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8
/x^(1/3)-11264/111435*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(5/2)/a^6+5632/45885*b^4*
x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^5-352/2415*b^3*x*(b*x^(2/3)+a*x)^(5/2)/a^4+
176/1035*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(5/2)/a^3-44/225*b*x^(5/3)*(b*x^(2/3)+
a*x)^(5/2)/a^2+2/9*x^2*(b*x^(2/3)+a*x)^(5/2)/a
```

Rubi [A]
time = 0.41, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^9x^{2/3}}-\frac{90112b^7\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^8\sqrt[3]{x}}+\frac{45056b^6\left(ax+bx^{2/3}\right)^{5/2}}{557175a^7}-\frac{11264b^5\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(ax+bx^{2/3}\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{225a^2}+\frac{2x^2\left(ax+bx^{2/3}\right)^{5/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3)
 + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5
/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(434596
5*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90
112*b^7*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^8*x^(1/3)) - (11264*b^5*x^(1/3)
*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (17
6*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3
) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx &= \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a} - \frac{(22b)\int x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{27a} \\
&= -\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a} + \frac{(88b^2)\int x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{135a^2} \\
&= \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a} - \frac{(176b \\
&= -\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} + \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \\
&= \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} - \frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} + \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \\
&= -\frac{11264b^5\sqrt[3]{x}\,\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} + \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} - \frac{352b^3x\left(bx^{2/3}+ax\right)}{2415a^4} \\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{11264b^5\sqrt[3]{x}\,\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} + \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5}}{45885a^5} \\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}} - \frac{11264b^5\sqrt[3]{x}\,\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} \\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} - \frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}} - 11 \\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} - 9 \\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} + \frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x} + \\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}} + \frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}}
\end{aligned}
$$

Mathematica [A]
time = 4.82, size = 168, normalized size = 0.49

$$\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(-524288b^{11}+1310720ab^{10}\sqrt[3]{x}-2293760a^2b^9x^{2/3}+3440640a^3b^8x-4730880a^4b^7x^{4/3}+6150144a^5b^6x^{5/3}-7687680a^6b^5x^2+9335040a^7b^4x^{7/3}-11085360a^8b^3x^{8/3}+12932920a^9b^2x^3-14872858a^{10}bx^{10/3}+16900975a^{11}x^{11/3}\right)}{152108775a^{12}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-524288*b^11 + 1310720*a*b^10*x
^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^(4
```

```
/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^(7/
3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*x^(1
0/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x)
```

Maple [A]
time = 0.36, size = 145, normalized size = 0.42

method	result
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(16900975 a^{11} x^{\frac{11}{3}}-14872858 a^{10} b\,x^{\frac{10}{3}}+12932920 a^{9} b^{2} x^{3}-11085360 a^{8} b^{3} x^{\frac{8}{3}}+9335040 a^{7} b^{4} x^{\frac{7}{3}}\right.}{1521087}$
default	$\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(16900975 a^{11} x^{\frac{11}{3}}-14872858 a^{10} b\,x^{\frac{10}{3}}+12932920 a^{9} b^{2} x^{3}-11085360 a^{8} b^{3} x^{\frac{8}{3}}+9335040 a^{7} b^{4} x^{\frac{7}{3}}\right.}{1521087}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/152108775*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(16900975*a^11*x^(11/3)-148
72858*a^10*b*x^(10/3)+12932920*a^9*b^2*x^3-11085360*a^8*b^3*x^(8/3)+9335040
*a^7*b^4*x^(7/3)-7687680*a^6*b^5*x^2+6150144*a^5*b^6*x^(5/3)-4730880*a^4*b^
7*x^(4/3)+3440640*a^3*b^8*x-2293760*a^2*b^9*x^(2/3)+1310720*a*b^10*x^(1/3)-
524288*b^11)/x/a^12
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + b*x**(2/3))**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. 2(255) = 510.
time = 1.15, size = 770, normalized size = 2.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/16900975*b*(524288*b^(25/2)/a^12 + (25*(88179*(a*x^(1/3) + b)^(23/2) - 10
62347*(a*x^(1/3) + b)^(21/2)*b + 5870865*(a*x^(1/3) + b)^(19/2)*b^2 - 19684
665*(a*x^(1/3) + b)^(17/2)*b^3 + 44618574*(a*x^(1/3) + b)^(15/2)*b^4 - 7207
6158*(a*x^(1/3) + b)^(13/2)*b^5 + 85180914*(a*x^(1/3) + b)^(11/2)*b^6 - 743
64290*(a*x^(1/3) + b)^(9/2)*b^7 + 47805615*(a*x^(1/3) + b)^(7/2)*b^8 - 2230
9287*(a*x^(1/3) + b)^(5/2)*b^9 + 7436429*(a*x^(1/3) + b)^(3/2)*b^10 - 20281
17*sqrt(a*x^(1/3) + b)*b^11)*b/a^11 + 3*(676039*(a*x^(1/3) + b)^(25/2) - 88
17900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)*b^2 - 1956
95500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17/2)*b^4 - 8
92371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)^(13/2)*b^6
 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3) + b)^(9/2)*
b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3) + b)^(5/2)
*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^(1/3) + b)*
b^12)/a^11)/a) - 2/152108775*a*(4194304*b^(27/2)/a^13 - (27*(676039*(a*x^(1
/3) + b)^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) +
b)^(21/2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3)
 + b)^(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(
1/3) + b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a
*x^(1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(
a*x^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*
sqrt(a*x^(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 182
53053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478
056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 -
 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*
b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(1
1/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b
)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3)
+ b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x^2*(a*x + b*x^(2/3))^(3/2), x)
```

3.177 $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=255

$$-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5(\ldots}{461\ldots}$$

```
[Out] -256/1615*b^3*(b*x^(2/3)+a*x)^(5/2)/a^4+65536/4849845*b^8*(b*x^(2/3)+a*x)^(
5/2)/a^9/x^(5/3)-32768/969969*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8/x^(4/3)+8192/13
8567*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7/x-4096/46189*b^5*(b*x^(2/3)+a*x)^(5/2)/a
^6/x^(2/3)+512/4199*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(1/3)+64/323*b^2*x^(1/3
)*(b*x^(2/3)+a*x)^(5/2)/a^3-32/133*b*x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^2+2/7*
x*(b*x^(2/3)+a*x)^(5/2)/a
```

Rubi [A]
time = 0.28, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {2041, 2027, 2039}

$$\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax+bx^{2/3}\right)^{5/2}}{4199a^5\sqrt[3]{x}}-\frac{256b^3\left(ax+bx^{2/3}\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{133a^2}+\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-256*b^3*(b*x^(2/3) + a*x)^(5/2))/(1615*a^4) + (65536*b^8*(b*x^(2/3) + a*x
)^(5/2))/(4849845*a^9*x^(5/3)) - (32768*b^7*(b*x^(2/3) + a*x)^(5/2))/(96996
9*a^8*x^(4/3)) + (8192*b^6*(b*x^(2/3) + a*x)^(5/2))/(138567*a^7*x) - (4096*
b^5*(b*x^(2/3) + a*x)^(5/2))/(46189*a^6*x^(2/3)) + (512*b^4*(b*x^(2/3) + a*
x)^(5/2))/(4199*a^5*x^(1/3)) + (64*b^2*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(32
3*a^3) - (32*b*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(133*a^2) + (2*x*(b*x^(2/3)
 + a*x)^(5/2))/(7*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
```

```
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(bx^{2/3}+ax\right)^{3/2}dx &= \frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{(16b)\int x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}dx}{21a}\\
&= -\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{(32b^2)\int\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}dx}{57a^2}\\
&= \frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{(128b^3}{}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x}{}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32b}{}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{}}{}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b}{}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{409}{}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{81}{}
\end{aligned}$$

Mathematica [A]
time = 4.86, size = 131, normalized size = 0.51

$$\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(32768b^8-81920ab^7\sqrt[3]{x}+143360a^2b^6x^{2/3}-215040a^3b^5x+295680a^4b^4x^{4/3}-384384a^5b^3x^{5/3}+480480a^6b^2x^2-583440a^7bx^{7/3}+692835a^8x^{8/3}\right)}{4849845a^9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(32768*b^8 - 81920*a*b^7*x^(1/3)
 + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 384
384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835*a^
8*x^(8/3)))/(4849845*a^9*x)
```

Maple [A]
time = 0.36, size = 112, normalized size = 0.44

method	result
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(692835a^{8}x^{\frac{8}{3}}-583440a^{7}b\,x^{\frac{7}{3}}+480480a^{6}b^{2}x^{2}-384384a^{5}b^{3}x^{\frac{5}{3}}+295680a^{4}b^{4}x^{\frac{4}{3}}-215040a^{3}b^{5}x+}{4849845x\,a^{9}}$
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(692835a^{8}x^{\frac{8}{3}}-583440a^{7}b\,x^{\frac{7}{3}}+480480a^{6}b^{2}x^{2}-384384a^{5}b^{3}x^{\frac{5}{3}}+295680a^{4}b^{4}x^{\frac{4}{3}}-215040a^{3}b^{5}x+}{4849845x\,a^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/4849845*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(692835*a^8*x^(8/3)-583440*a^
7*b*x^(7/3)+480480*a^6*b^2*x^2-384384*a^5*b^3*x^(5/3)+295680*a^4*b^4*x^(4/3
)-215040*a^3*b^5*x+143360*a^2*b^6*x^(2/3)-81920*x^(1/3)*a*b^7+32768*b^8)/x/
a^9
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x*(a*x + b*x**(2/3))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(189) = 378.
time = 1.67, size = 602, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/692835*b*(32768*b^(19/2)/a^9 - (19*(6435*(a*x^(1/3) + b)^(17/2) - 58344*
(a*x^(1/3) + b)^(15/2)*b + 235620*(a*x^(1/3) + b)^(13/2)*b^2 - 556920*(a*x^
(1/3) + b)^(11/2)*b^3 + 850850*(a*x^(1/3) + b)^(9/2)*b^4 - 875160*(a*x^(1/3
) + b)^(7/2)*b^5 + 612612*(a*x^(1/3) + b)^(5/2)*b^6 - 291720*(a*x^(1/3) + b
)^(3/2)*b^7 + 109395*sqrt(a*x^(1/3) + b)*b^8)*b/a^8 + 9*(12155*(a*x^(1/3) +
 b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2
)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)
*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^
6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 -
230945*sqrt(a*x^(1/3) + b)*b^9)/a^8)/a) + 2/1616615*a*(65536*b^(21/2)/a^10
+ (21*(12155*(a*x^(1/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554
268*(a*x^(1/3) + b)^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 26453
70*(a*x^(1/3) + b)^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340
*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*
x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(
a*x^(1/3) + b)^(21/2) - 510510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3
) + b)^(17/2)*b^2 - 7759752*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3
) + b)^(13/2)*b^4 - 22221108*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/
3) + b)^(9/2)*b^6 - 16628040*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3)
 + b)^(5/2)*b^8 - 3233230*(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3)
 + b)*b^10)/a^9)/a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x*(a*x + b*x^(2/3))^(3/2), x)
```

3.178 $\int \left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=169

$$\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}$$

```
[Out] 2/5*(b*x^(2/3)+a*x)^(5/2)/a-512/15015*b^5*(b*x^(2/3)+a*x)^(5/2)/a^6/x^(5/3)
+256/3003*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(4/3)-64/429*b^3*(b*x^(2/3)+a*x)^
(5/2)/a^4/x+32/143*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(2/3)-4/13*b*(b*x^(2/3)+
a*x)^(5/2)/a^2/x^(1/3)
```

Rubi [A]
time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2027, 2041, 2039}

$$-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(1501
5*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64
*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2)
)/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{(2b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{\sqrt[3]{x}}\,dx}{3a}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} + \frac{(16b^2)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{2/3}}\,dx}{39a^2}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} - \frac{(32b^3)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}}{143a^3}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} + \frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} + \frac{32b^2\left(bx^{2/3}+ax\right)}{143a^3x^{2/3}}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3\left(bx^{2/3}+ax\right)}{429a^4x}$$

Mathematica [A]
time = 4.76, size = 94, normalized size = 0.56

$$\frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(-256b^5+640ab^4\sqrt[3]{x}-1120a^2b^3x^{2/3}+1680a^3b^2x-2310a^4bx^{4/3}+3003a^5x^{5/3}\right)}{15015a^6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2),x]
[Out] (2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-256*b^5 + 640*a*b^4*x^(1/3) -
1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(5/
3)))/(15015*a^6*x)
```

Maple [A]
time = 0.36, size = 79, normalized size = 0.47

method	result	size

derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{5}x^{\frac{5}{3}}-2310a^{4}b\,x^{\frac{4}{3}}+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640a\,b^{4}x^{\frac{1}{3}}-256b^{5}\right)}{15015x\,a^{6}}$	79
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{5}x^{\frac{5}{3}}-2310a^{4}b\,x^{\frac{4}{3}}+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640a\,b^{4}x^{\frac{1}{3}}-256b^{5}\right)}{15015x\,a^{6}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15015*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(3003*a^5*x^(5/3)-2310*a^4*b*x^
(4/3)+1680*a^3*b^2*x-1120*a^2*b^3*x^(2/3)+640*a*b^4*x^(1/3)-256*b^5)/x/a^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(125) = 250$.
time = 287.48, size = 768, normalized size = 4.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] 2/15015*(4*(805306368*b^13 + 167772160*b^12 + 786432*(64*a^3 - 3)*b^10 - 15
728640*b^11 - 4096*(11264*a^3 - 53)*b^9 + 4372368*a^9 - 1536*(5504*a^3 + 1)
*b^8 - 48*(242810880*a^6 + 114688*a^3 + 3)*b^7 - 1792*(1353984*a^6 - 103*a^
3)*b^6 + 192*(1152384*a^6 - 23*a^3)*b^5 - 3*(3633315840*a^9 - 12027392*a^6
- 15*a^3)*b^4 - 112*(35389440*a^9 + 29281*a^6)*b^3 - 819*(368640*a^9 - 31*a
^6)*b^2 + 693*(40960*a^9 + 3*a^6)*b)*x + (3003*(16777216*a^7*b^6 + 6291456*
a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2304*a^7*b^2 + 86
4*a^7*b - 27*a^7)*x^3 - 70*(16777216*a^4*b^9 + 6291456*a^4*b^8 + 196608*a^4
*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (262144*a^7 + 27*a^4)*
b^3)*x^2 + 128*(16777216*a*b^12 + 6291456*a*b^11 + 196608*a*b^10 - 114688*a
*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b^6)*x - 16*(268435456*
b^13 + 100663296*b^12 + 3145728*b^11 - 1835008*b^10 - 36864*b^9 - 16*(26214
4*a^3 + 27)*b^7 + 13824*b^8 - 231*(16777216*a^6*b^7 + 6291456*a^6*b^6 + 196
608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6*b^3 + 864*a^6*b^2 - (262144*a^9 + 2
```

```
7*a^6)*b)*x^2 - 5*(16777216*a^3*b^10 + 6291456*a^3*b^9 + 196608*a^3*b^8 - 1
14688*a^3*b^7 - 2304*a^3*b^6 + 864*a^3*b^5 - (262144*a^6 + 27*a^3)*b^4)*x)*
x^(2/3) + 3*(21*(16777216*a^5*b^8 + 6291456*a^5*b^7 + 196608*a^5*b^6 - 1146
88*a^5*b^5 - 2304*a^5*b^4 + 864*a^5*b^3 - (262144*a^8 + 27*a^5)*b^2)*x^2 -
32*(16777216*a^2*b^11 + 6291456*a^2*b^10 + 196608*a^2*b^9 - 114688*a^2*b^8
- 2304*a^2*b^7 + 864*a^2*b^6 - (262144*a^5 + 27*a^2)*b^5)*x)*x^(1/3))*sqrt(
a*x + b*x^(2/3)))/((16777216*a^6*b^6 + 6291456*a^6*b^5 + 196608*a^6*b^4 - 2
62144*a^9 - 114688*a^6*b^3 - 2304*a^6*b^2 + 864*a^6*b - 27*a^6)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(125) = 250.
time = 2.26, size = 434, normalized size = 2.57

$$\frac{2}{3003}\, b \left(\frac{256\, b^{\frac{13}{2}}}{a^{6}} + \frac{\frac{13\left(63\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} - 385\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} b + 990\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} b^{2} - 1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} b^{3} + 1155\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} b^{4} - 693\sqrt{ax^{\frac{1}{3}}+b}\, b^{5}\right) b}{a^{5}} + \frac{3\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} - 1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} b + 5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} b^{2} - 8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} b^{3} + 9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} b^{4} - 6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} b^{5} + 3003\sqrt{ax^{\frac{1}{3}}+b}\, b^{6}\right)}{a^{5}}}{a}\right) - \frac{2}{15015}\, a \left(\frac{1024\, b^{\frac{15}{2}}}{a^{7}} - \frac{\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} - 1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} b + 5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} b^{2} - 8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} b^{3} + 9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} b^{4} - 6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} b^{5} + 3003\sqrt{ax^{\frac{1}{3}}+b}\, b^{6}\right) b}{a^{6}} + \frac{7\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} - 3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} b + 12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} b^{2} - 25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} b^{3} + 32175\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} b^{4} - 27027\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} b^{5} + 15015\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} b^{6} - 6435\sqrt{ax^{\frac{1}{3}}+b}\, b^{7}\right)}{a^{6}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3003*b*(256*b^(13/2)/a^6 + (13*(63*(a*x^(1/3) + b)^(11/2) - 385*(a*x^(1/3
) + b)^(9/2)*b + 990*(a*x^(1/3) + b)^(7/2)*b^2 - 1386*(a*x^(1/3) + b)^(5/2)
*b^3 + 1155*(a*x^(1/3) + b)^(3/2)*b^4 - 693*sqrt(a*x^(1/3) + b)*b^5)*b/a^5
+ 3*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x
^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) +
b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^
6)/a^5)/a) - 2/15015*a*(1024*b^(15/2)/a^7 - (15*(231*(a*x^(1/3) + b)^(13/2)
 - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a
*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3)
+ b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) +
b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^
2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 270
27*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(
a*x^(1/3) + b)*b^7)/a^6)/a)
```

Mupad [B]

time = 5.14, size = 40, normalized size = 0.24

$$\frac{x\left(a\,x+b\,x^{2/3}\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},6;\ 7;\ -\frac{a\,x^{1/3}}{b}\right)}{2\left(\frac{a\,x^{1/3}}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] (x*(a*x + b*x^(2/3))^(3/2)*hypergeom([-3/2, 6], 7, -(a*x^(1/3))/b))/(2*((a*
x^(1/3))/b + 1)^(3/2))
```

3.179 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=84

$$\frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}$$

```
[Out] 16/105*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(5/3)-8/21*b*(b*x^(2/3)+a*x)^(5/2)/a
^2/x^(4/3)+2/3*(b*x^(2/3)+a*x)^(5/2)/a/x
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (16*b^2*(b*x^(2/3) + a*x)^(5/2))/(105*a^3*x^(5/3)) - (8*b*(b*x^(2/3) + a*x)
^(5/2))/(21*a^2*x^(4/3)) + (2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax} - \frac{(4b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{4/3}}\,dx}{9a}$$
$$= -\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}} + \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax} + \frac{(8b^2)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{5/3}}\,dx}{63a^2}$$
$$= \frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}} - \frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}} + \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}$$

Mathematica [A]
time = 4.75, size = 59, normalized size = 0.70

$$\frac{2\left(b+a\sqrt[3]{x}\right)\left(8b^2-20ab\sqrt[3]{x}+35a^2x^{2/3}\right)\left(bx^{2/3}+ax\right)^{3/2}}{105a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*(b + a*x^(1/3))*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*(b*x^(2/3) + a
*x)^(3/2))/(105*a^3*x)
```

Maple [A]
time = 0.46, size = 48, normalized size = 0.57

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^2x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^2\right)}{105x\,a^3}$	48
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^2x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^2\right)}{105x\,a^3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(35*a^2*x^(2/3)-20*a*b*x^(1/3)+8*
b^2)/x/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(62) = 124.
time = 234.82, size = 501, normalized size = 5.96

$$-\frac{\left(\left(201326592\,b^{10}+41943040\,b^{9}+196608\,(6784\,a^{3}-3)b^{7}-3932160\,b^{8}+1024\,(257536\,a^{3}+53)b^{6}-407680\,a^{6}-384\,(72704\,a^{3}+1)b^{5}+12\,(94371840\,a^{6}-437248\,a^{3}-3)b^{4}+896\,(442368\,a^{6}+449\,a^{3})b^{3}+24\,(1105920\,a^{6}-151\,a^{3})b^{2}-15\,(253952\,a^{6}+15\,a^{3})b\right)x-2\left(35\,(16777216\,a^{4}b^{6}+6291456\,a^{4}b^{5}+196608\,a^{4}b^{4}-262144\,a^{7}-114688\,a^{4}b^{3}-2304\,a^{4}b^{2}+864\,a^{4}b-27\,a^{4})x^{2}+3\,(16777216\,a^{2}b^{8}+6291456\,a^{2}b^{7}+196608\,a^{2}b^{6}-114688\,a^{2}b^{5}-2304\,a^{2}b^{4}+864\,a^{2}b^{3}-(262144\,a^{5}+27\,a^{2})b^{2})x^{\frac{4}{3}}-4\,(16777216\,ab^{9}+6291456\,ab^{8}+196608\,ab^{7}-114688\,ab^{6}-2304\,ab^{5}+864\,ab^{4}-(262144\,a^{4}+27\,a)b^{3})x+2\,(67108864\,b^{10}+25165824\,b^{9}+786432\,b^{8}-458752\,b^{7}-9216\,b^{6}-4\,(262144\,a^{3}+27)b^{4}+3456\,b^{5}+25\,(16777216\,a^{3}b^{7}+6291456\,a^{3}b^{6}+196608\,a^{3}b^{5}-114688\,a^{3}b^{4}-2304\,a^{3}b^{3}+864\,a^{3}b^{2}-(262144\,a^{6}+27\,a^{3})b)x)x^{\frac{2}{3}}\right)\right)\sqrt{ax+bx^{\frac{2}{3}}}}{105\,(16777216\,a^{3}b^{6}+6291456\,a^{3}b^{5}+196608\,a^{3}b^{4}-262144\,a^{6}-114688\,a^{3}b^{3}-2304\,a^{3}b^{2}+864\,a^{3}b-27\,a^{3})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/105*((201326592*b^10 + 41943040*b^9 + 196608*(6784*a^3 - 3)*b^7 - 393216
0*b^8 + 1024*(257536*a^3 + 53)*b^6 - 407680*a^6 - 384*(72704*a^3 + 1)*b^5 +
 12*(94371840*a^6 - 437248*a^3 - 3)*b^4 + 896*(442368*a^6 + 449*a^3)*b^3 +
24*(1105920*a^6 - 151*a^3)*b^2 - 15*(253952*a^6 + 15*a^3)*b)*x - 2*(35*(167
77216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*
b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 3*(16777216*a^2*b^8 + 629145
6*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^3 -
(262144*a^5 + 27*a^2)*b^2)*x^(4/3) - 4*(16777216*a*b^9 + 6291456*a*b^8 + 19
6608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27*a)*b^
3)*x + 2*(67108864*b^10 + 25165824*b^9 + 786432*b^8 - 458752*b^7 - 9216*b^6
 - 4*(262144*a^3 + 27)*b^4 + 3456*b^5 + 25*(16777216*a^3*b^7 + 6291456*a^3*
b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3*b^2 - (26214
4*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^3*b^6 +
6291456*a^3*b^5 + 196608*a^3*b^4 - 262144*a^6 - 114688*a^3*b^3 - 2304*a^3*b
^2 + 864*a^3*b - 27*a^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(62) = 124.
time = 1.84, size = 265, normalized size = 3.15

$$-\frac{2}{35}\,b\left(\frac{8\,b^{\frac{7}{2}}}{a^{3}}-\frac{\frac{7\left(3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}-10\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b+15\sqrt{ax^{\frac{1}{3}}+b}\,b^{2}\right)b}{a^{2}}+\frac{3\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{2}-35\sqrt{ax^{\frac{1}{3}}+b}\,b^{3}\right)}{a^{2}}}{a}\right)+\frac{2}{105}\,a\left(\frac{16\,b^{\frac{9}{2}}}{a^{4}}+\frac{\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{2}-35\sqrt{ax^{\frac{1}{3}}+b}\,b^{3}\right)b}{a^{3}}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{2}-420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{3}+315\sqrt{ax^{\frac{1}{3}}+b}\,b^{4}}{a^{3}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -2/35*b*(8*b^(7/2)/a^3 - (7*(3*(a*x^(1/3) + b)^(5/2) - 10*(a*x^(1/3) + b)^(
3/2)*b + 15*sqrt(a*x^(1/3) + b)*b^2)*b/a^2 + 3*(5*(a*x^(1/3) + b)^(7/2) - 2
1*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3
) + b)*b^3)/a^2)/a) + 2/105*a*(16*b^(9/2)/a^4 + (9*(5*(a*x^(1/3) + b)^(7/2)
 - 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^
(1/3) + b)*b^3)*b/a^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/
2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*
sqrt(a*x^(1/3) + b)*b^4)/a^3)/a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x, x)
```

3.180 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=78

$$\frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)$$

```
[Out] 2*(b*x^(2/3)+a*x)^(3/2)/x-6*b^(3/2)*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)
^(1/2))+6*b*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2046, 2054, 212}

$$-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(
3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b\int \frac{\sqrt{bx^{2/3}+ax}}{x^{4/3}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-\left(6b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)
\end{aligned}
$$

Mathematica [A]
time = 10.08, size = 88, normalized size = 1.13

$$
\frac{2\sqrt{bx^{2/3}+ax}\left(\sqrt{b+a\sqrt[3]{x}}\left(4b+a\sqrt[3]{x}\right)-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)\right)}{\sqrt{b+a\sqrt[3]{x}}\,\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2)
*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

Maple [A]
time = 0.40, size = 69, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(-3b^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)+\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}+3b\sqrt{b+a\,x^{\frac{1}{3}}}\right)}{x\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}}$	67
default	$-\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)-\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}-3b\sqrt{b+a\,x^{\frac{1}{3}}}\right)}{x\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^(2/3)+a*x)^(3/2)*(3*b^(3/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))-(b
+a*x^(1/3))^(3/2)-3*b*(b+a*x^(1/3))^(1/2))/x/(b+a*x^(1/3))^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)
```

Giac [**A**]
time = 1.54, size = 83, normalized size = 1.06

$$\frac{6\, b^2 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}} + b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} + 6\sqrt{ax^{\frac{1}{3}} + b}\, b - \frac{2\left(3\, b^2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 4\sqrt{-b}\, b^{\frac{3}{2}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3/
2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-
b)*b^(3/2))/sqrt(-b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

3.181 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}$$

```
[Out] -(b*x^(2/3)+a*x)^(3/2)/x^2+3/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^
(1/2))/b^(3/2)-3/4*a*(b*x^(2/3)+a*x)^(1/2)/x-3/8*a^2*(b*x^(2/3)+a*x)^(1/2)/
b/x^(2/3)
```

Rubi [A]
time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2045, 2050, 2054, 212}

$$\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(4*x) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/x^2 + (3*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(8*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{2}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{8}a^2\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}-\frac{a^3\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}}{16b}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{(3a^3)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.}{8}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{}}{\sqrt{bx^{2/3}}}\right.}{8b^{3/2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.06, size = 61, normalized size = 0.54

$$\frac{6a^3\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\ {}_2F_1\left(\frac{5}{2},4;\frac{7}{2};1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (6*a^3*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 4, 7/
2, 1 + (a*x^(1/3))/b])/(5*b^4*x^(1/3))
```

Maple [A]
time = 0.39, size = 93, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(3\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}} b^{\frac{3}{2}}-3\,\mathrm{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{3} x+8\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{5}{2}}-3\sqrt{b+a\,x^{\frac{1}{3}}}\, b^{\frac{7}{2}}\right)}{8 x^{2}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{5}{2}}}$	93
default	$\frac{\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(-3\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}} b^{\frac{3}{2}}-8\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{5}{2}}+3\,\mathrm{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{3} x+3\sqrt{b+a\,x^{\frac{1}{3}}}\, b^{\frac{7}{2}}\right)}{8 x^{2}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{5}{2}}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/8*(b*x^(2/3)+a*x)^(3/2)*(-3*(b+a*x^(1/3))^(5/2)*b^(3/2)-8*(b+a*x^(1/3))^(
3/2)*b^(5/2)+3*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*a^3*x+3*(b+a*x^(1/3))
^(1/2)*b^(7/2))/x^2/(b+a*x^(1/3))^(3/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)
```

Giac [A]
time = 2.58, size = 92, normalized size = 0.81

$$-\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^4+8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^4b-3\sqrt{ax^{\frac{1}{3}}+b}\,a^4b^2}{a^3bx}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^4*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1/
3) + b)^(5/2)*a^4 + 8*(a*x^(1/3) + b)^(3/2)*a^4*b - 3*sqrt(a*x^(1/3) + b)*a
^4*b^2)/(a^3*b*x))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

3.182 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=203

$$-\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}}-\frac{(bx^{2/3}+}{2x}$$

```
[Out] -1/2*(b*x^(2/3)+a*x)^(3/2)/x^3-21/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3
)+a*x)^(1/2))/b^(9/2)-3/20*a*(b*x^(2/3)+a*x)^(1/2)/x^2-3/160*a^2*(b*x^(2/3)
+a*x)^(1/2)/b/x^(5/3)+7/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/256*a^4
*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A]

time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2054, 212}

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}}+\frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}}-\frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x}+\frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}-\frac{3a\sqrt{ax+bx^{2/3}}}{20x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(20*x^2) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(160*
b*x^(5/3)) + (7*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^2*x^(4/3)) - (7*a^4*Sqrt[
b*x^(2/3) + a*x])/(256*b^3*x) + (21*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^4*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/(2*x^3) - (21*a^6*ArcTanh[(Sqrt[b]*x^(1/3))
/Sqrt[b*x^(2/3) + a*x]])/(512*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{4}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{40}a^2\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}-\frac{(7a^3)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+}}}{320b}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}-$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.05, size = 61, normalized size = 0.30

$$-\frac{6a^6\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\ {}_2F_1\left(\frac{5}{2},7;\frac{7}{2};1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-6*a^6*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))
```

Maple [A]
time = 0.36, size = 139, normalized size = 0.68

method	result
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(105\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{9}{2}}-595\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}b^{\frac{11}{2}}+1386\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}b^{\frac{13}{2}}-1686\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}b^{\frac{15}{2}}-105\,\mathrm{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{2560x^{3}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$
default	$-\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(-105\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{9}{2}}+595\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}b^{\frac{11}{2}}-1386\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}b^{\frac{13}{2}}+1686\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}b^{\frac{15}{2}}+595\left(b+a\,x^{\frac{1}{3}}\right.\right.}{2560x^{3}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2560*(b*x^(2/3)+a*x)^(3/2)*(-105*(b+a*x^(1/3))^(11/2)*b^(9/2)+595*(b+a*x
^(1/3))^(9/2)*b^(11/2)-1386*(b+a*x^(1/3))^(7/2)*b^(13/2)+1686*(b+a*x^(1/3))
^(5/2)*b^(15/2)+595*(b+a*x^(1/3))^(3/2)*b^(17/2)-105*(b+a*x^(1/3))^(1/2)*b^
(19/2)+105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^4*a^6*x^2)/x^3/(b+a*x^(1/
3))^(3/2)/b^(17/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**4, x)
```

Giac [A]
time = 1.49, size = 143, normalized size = 0.70

$$\frac{\frac{105\,a^7\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^7 - 595\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^7b + 1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^7b^2 - 1686\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^7b^3 - 595\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^7b^4 + 105\sqrt{ax^{\frac{1}{3}}+b}\,a^7b^5}{a^6b^4x^2}}{2560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*
(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^(1
/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(1/3
) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

3.183 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=291

$$-\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}}-\frac{143a^6}{20}$$

```
[Out] -1/3*(b*x^(2/3)+a*x)^(3/2)/x^4+429/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^(
2/3)+a*x)^(1/2))/b^(15/2)-1/16*a*(b*x^(2/3)+a*x)^(1/2)/x^3-1/224*a^2*(b*x^(
2/3)+a*x)^(1/2)/b/x^(8/3)+13/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143
/26880*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+429/71680*a^5*(b*x^(2/3)+a*x)^(1/2
)/b^4/x^(5/3)-143/20480*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+143/16384*a^7
*(b*x^(2/3)+a*x)^(1/2)/b^6/x-429/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3
)
```

Rubi [A]

time = 0.35, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2054, 212}

$$\frac{429a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{15/2}}-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}}+\frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x}-\frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}}-\frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2}+\frac{13a^3\sqrt{ax+bx^{2/3}}}{2688b^2x^{7/3}}-\frac{a^2\sqrt{ax+bx^{2/3}}}{224bx^{8/3}}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4}-\frac{a\sqrt{ax+bx^{2/3}}}{16x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] -1/16*(a*Sqrt[b*x^(2/3) + a*x])/x^3 - (a^2*Sqrt[b*x^(2/3) + a*x])/(224*b*x^
(8/3)) + (13*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^2*x^(7/3)) - (143*a^4*Sqrt[
b*x^(2/3) + a*x])/(26880*b^3*x^2) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(71680*
b^4*x^(5/3)) - (143*a^6*Sqrt[b*x^(2/3) + a*x])/(20480*b^5*x^(4/3)) + (143*a
^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^6*x) - (429*a^8*Sqrt[b*x^(2/3) + a*x])/(
32768*b^7*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(3*x^4) + (429*a^9*ArcTanh[(Sq
rt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
```

```
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} + \frac{1}{6}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} + \frac{1}{96}a^2\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} - \frac{(13a^3)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}}{1344b}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} + \frac{(14}{}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} -$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} +$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} +$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} +$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} +$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} +$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.06, size = 61, normalized size = 0.21

$$\frac{6a^9\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\;{}_2F_1\left(\frac{5}{2},10;\frac{7}{2};1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))
```

Maple [A]
time = 0.34, size = 181, normalized size = 0.62

method	result
derivativedivides	$-\frac{\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(45045\left(b+a x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{15}{2}}-390390\left(b+a x^{\frac{1}{3}}\right)^{\frac{15}{2}} b^{\frac{17}{2}}+1495494\left(b+a x^{\frac{1}{3}}\right)^{\frac{13}{2}} b^{\frac{19}{2}}-3317886\left(b+a x^{\frac{1}{3}}\right)^{\frac{11}{2}} b\right.}{}$
default	$\frac{\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(-45045\left(b+a x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{15}{2}}+390390\left(b+a x^{\frac{1}{3}}\right)^{\frac{15}{2}} b^{\frac{17}{2}}-1495494\left(b+a x^{\frac{1}{3}}\right)^{\frac{13}{2}} b^{\frac{19}{2}}+3317886\left(b+a x^{\frac{1}{3}}\right)^{\frac{11}{2}} b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3440640*(b*x^(2/3)+a*x)^(3/2)*(-45045*(b+a*x^(1/3))^(17/2)*b^(15/2)+39039
0*(b+a*x^(1/3))^(15/2)*b^(17/2)-1495494*(b+a*x^(1/3))^(13/2)*b^(19/2)+33178
86*(b+a*x^(1/3))^(11/2)*b^(21/2)-4685824*(b+a*x^(1/3))^(9/2)*b^(23/2)+43498
26*(b+a*x^(1/3))^(7/2)*b^(25/2)-2633274*(b+a*x^(1/3))^(5/2)*b^(27/2)-390390
*(b+a*x^(1/3))^(3/2)*b^(29/2)+45045*(b+a*x^(1/3))^(1/2)*b^(31/2)+45045*arct
anh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^9*x^3)/x^4/(b+a*x^(1/3))^(3/2)/b^(29
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**5, x)
```

Giac [A]
time = 1.71, size = 194, normalized size = 0.67

$$-\frac{\frac{45045\, a^{10} \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\, b^{7}} + \frac{45045 \left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} a^{10} - 390390 \left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} a^{10} b + 1495494 \left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} a^{10} b^{2} - 3317886 \left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} a^{10} b^{3} + 4685824 \left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} a^{10} b^{4} - 4349826 \left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}} a^{10} b^{5} + 2633274 \left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}} a^{10} b^{6} + 390390 \left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} a^{10} b^{7} - 45045 \sqrt{ax^{\frac{1}{3}}+b}\, a^{10} b^{8}}{a^{9} b^{7} x^{3}}}{3440640\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(45045*a^10*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7)
+ (45045*(a*x^(1/3) + b)^(17/2)*a^10 - 390390*(a*x^(1/3) + b)^(15/2)*a^10*b
 + 1495494*(a*x^(1/3) + b)^(13/2)*a^10*b^2 - 3317886*(a*x^(1/3) + b)^(11/2)
*a^10*b^3 + 4685824*(a*x^(1/3) + b)^(9/2)*a^10*b^4 - 4349826*(a*x^(1/3) + b
)^(7/2)*a^10*b^5 + 2633274*(a*x^(1/3) + b)^(5/2)*a^10*b^6 + 390390*(a*x^(1/
3) + b)^(3/2)*a^10*b^7 - 45045*sqrt(a*x^(1/3) + b)*a^10*b^8)/(a^9*b^7*x^3))
/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

3.184 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=379

$$-\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}}-41$$

```
[Out] -1/4*(b*x^(2/3)+a*x)^(3/2)/x^5-12597/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/(
b*x^(2/3)+a*x)^(1/2))/b^(21/2)-3/88*a*(b*x^(2/3)+a*x)^(1/2)/x^4-3/1760*a^2*
(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/10560*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(
10/3)-323/168960*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/157696*a^5*(b*x^(2/3
)+a*x)^(1/2)/b^4/x^(8/3)-4199/1892352*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)
+4199/1720320*a^7*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-12597/4587520*a^8*(b*x^(2/3
)+a*x)^(1/2)/b^7/x^(5/3)+4199/1310720*a^9*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)
-4199/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^9/x+12597/2097152*a^11*(b*x^(2/3
)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A]
time = 0.48, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2054, 212}

$$-\frac{12597a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{21/2}}+\frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}}-\frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x}+\frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}}-\frac{12597a^8\sqrt{ax+bx^{2/3}}}{4587520b^7x^{5/3}}+\frac{4199a^7\sqrt{ax+bx^{2/3}}}{1720320b^6x^2}-\frac{4199a^6\sqrt{ax+bx^{2/3}}}{1892352b^5x^{7/3}}+\frac{323a^5\sqrt{ax+bx^{2/3}}}{157696b^4x^{8/3}}-\frac{323a^4\sqrt{ax+bx^{2/3}}}{168960b^3x^3}+\frac{19a^3\sqrt{ax+bx^{2/3}}}{10560b^2x^{10/3}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{1760bx^{11/3}}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5}-\frac{3a\sqrt{ax+bx^{2/3}}}{88x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760
*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a
^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x]
)/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7
/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt
[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])
/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x)
+ (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) +
a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) +
 a*x]])/(2097152*b^(21/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} + \frac{1}{8}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} + \frac{1}{176}a^2\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} - \frac{(19a^3)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}}}}{3520b}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} +$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}$$

Mathematica **[C]** Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.06, size = 61, normalized size = 0.16

$$-\frac{6a^{12}\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\ {}_2F_1\left(\frac{5}{2},13;\frac{7}{2};1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{13}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13,
 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))
```

Maple [A]
time = 0.36, size = 223, normalized size = 0.59

method	result
derivativedivides	$\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{23}{2}}b^{\frac{21}{2}}-169744575\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}}b^{\frac{23}{2}}+904981077\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}}b^{\frac{25}{2}}-2913648309\left(b+a\right.\right.$
default	$-\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(-14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{23}{2}}b^{\frac{21}{2}}+169744575\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}}b^{\frac{23}{2}}-904981077\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}}b^{\frac{25}{2}}+2913648309\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2422210560*(b*x^(2/3)+a*x)^(3/2)*(-14549535*(b+a*x^(1/3))^(23/2)*b^(21/2
)+169744575*(b+a*x^(1/3))^(21/2)*b^(23/2)-904981077*(b+a*x^(1/3))^(19/2)*b^
(25/2)+2913648309*(b+a*x^(1/3))^(17/2)*b^(27/2)-6303782342*(b+a*x^(1/3))^(1
5/2)*b^(29/2)+9643633350*(b+a*x^(1/3))^(13/2)*b^(31/2)-10677769530*(b+a*x^(
1/3))^(11/2)*b^(33/2)+8598579770*(b+a*x^(1/3))^(9/2)*b^(35/2)-4975837515*(b
+a*x^(1/3))^(7/2)*b^(37/2)+2001671595*(b+a*x^(1/3))^(5/2)*b^(39/2)+16974457
5*(b+a*x^(1/3))^(3/2)*b^(41/2)-14549535*(b+a*x^(1/3))^(1/2)*b^(43/2)+145495
35*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*a^12*x^4)/x^5/(b+a*x^(1/3))^(3
/2)/b^(41/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4496 deep
```

Giac [**A**]
time = 1.70, size = 245, normalized size = 0.65

$$\frac{\frac{14549535\,a^{13}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{13}-169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{13}b+904981077\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{13}b^{2}-2913648309\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{13}b^{3}+6303782342\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{13}b^{4}-9643633350\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{13}b^{5}+10677769530\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{13}b^{6}-8598579770\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{13}b^{7}+4975837515\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{13}b^{8}-2001671595\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{13}b^{9}-169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{13}b^{10}+14549535\sqrt{ax^{\frac{1}{3}}+b}\,a^{13}b^{11}}{a^{12}b^{10}x^{4}}}{2422210560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
[Out] 1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*
b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)^(
21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a*x^
(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4 - 9
643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b)^(11
/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515*(a*x^
(1/3) + b)^(7/2)*a^13*b^8 - 2001671595*(a*x^(1/3) + b)^(5/2)*a^13*b^9 - 169
744575*(a*x^(1/3) + b)^(3/2)*a^13*b^10 + 14549535*sqrt(a*x^(1/3) + b)*a^13*
b^11)/(a^12*b^10*x^4))/a
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^6,x)
[Out] int((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

3.185 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=401

$$\frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}}-\frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}}-\frac{2097152b^{11}\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/}}}{2340135a^{11}}$$

```
[Out] 8388608/11700675*b^12*(b*x^(2/3)+a*x)^(1/2)/a^13-16777216/11700675*b^13*(b*
x^(2/3)+a*x)^(1/2)/a^14/x^(1/3)-2097152/3900225*b^11*x^(1/3)*(b*x^(2/3)+a*x
)^(1/2)/a^12+1048576/2340135*b^10*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^11-131072
/334305*b^9*x*(b*x^(2/3)+a*x)^(1/2)/a^10+65536/185725*b^8*x^(4/3)*(b*x^(2/3
)+a*x)^(1/2)/a^9-180224/557175*b^7*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+117145
6/3900225*b^6*x^2*(b*x^(2/3)+a*x)^(1/2)/a^7-73216/260015*b^5*x^(7/3)*(b*x^(
2/3)+a*x)^(1/2)/a^6+36608/137655*b^4*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-9152
/36225*b^3*x^3*(b*x^(2/3)+a*x)^(1/2)/a^4+416/1725*b^2*x^(10/3)*(b*x^(2/3)+a
*x)^(1/2)/a^3-52/225*b*x^(11/3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(2/3
)+a*x)^(1/2)/a
```

Rubi [A]

time = 0.49, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\,\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}}-\frac{131072b^9x\sqrt{ax+bx^{2/3}}}{334305a^{10}}+\frac{65536b^8x^{4/3}\sqrt{ax+bx^{2/3}}}{185725a^9}-\frac{180224b^7x^{5/3}\sqrt{ax+bx^{2/3}}}{557175a^8}+\frac{1171456b^6x^2\sqrt{ax+bx^{2/3}}}{3900225a^7}-\frac{73216b^5x^{7/3}\sqrt{ax+bx^{2/3}}}{260015a^6}+\frac{36608b^4x^{8/3}\sqrt{ax+bx^{2/3}}}{137655a^5}-\frac{9152b^3x^3\sqrt{ax+bx^{2/3}}}{36225a^4}+\frac{416b^2x^{10/3}\sqrt{ax+bx^{2/3}}}{1725a^3}-\frac{52bx^{11/3}\sqrt{ax+bx^{2/3}}}{225a^2}+\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[
b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^
(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (655
36*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sq
rt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])
/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (
36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[
b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1
725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*
x^(2/3) + a*x])/(9*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(26b)\int \frac{x^{11/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{27a}$$

$$= -\frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{(208b^2)\int \frac{x^{10/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{225a^2}$$

$$= \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(4576b^3)\int}{5}$$

$$= -\frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4}{}$$

$$= \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52}{}$$

$$= -\frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} +$$

$$= \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5}$$

$$= -\frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+a}}{260015a^6}$$

$$= \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7}$$

$$= -\frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8}$$

$$= \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9}$$

$$= -\frac{2097152b^{11}\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}}}{334305a^{10}}$$

$$= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{2097152b^{11}\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+}}{2340135a^{11}}$$

$$= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}} - \frac{2097152b^{11}\sqrt[3]{x}\,\sqrt{bx^{2/3}+a}}{3900225a^{12}}$$

Mathematica [A]

time = 0.11, size = 185, normalized size = 0.46

$$\frac{2\sqrt{bx^{2/3}+ax}\,\left(-8388608b^{13}+4194304ab^{12}\sqrt[3]{x}-3145728a^2b^{11}x^{2/3}+2621440a^3b^{10}x-2293760a^4b^9x^{4/3}+2064384a^5b^8x^{5/3}-1892352a^6b^7x^2+1757184a^7b^6x^{7/3}-1647360a^8b^5x^{8/3}+1555840a^9b^4x^3-1478048a^{10}b^3x^{10/3}+1410864a^{11}b^2x^{11/3}-1352078a^{12}bx^4+1300075a^{13}x^{13/3}\right)}{11700675a^{14}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728*
a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384*a
^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360*a^
8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 1410864*a
^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700675*a
^14*x^(1/3))
```

Maple [A]
time = 0.36, size = 167, normalized size = 0.42

method	result
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^4+1410864a^{11}b^2x^{\frac{11}{3}}-1478048a^{10}b^3x^{\frac{10}{3}}+1555840a^9b^4x^3-1647360a^8\right.}{117}$
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^4+1410864a^{11}b^2x^{\frac{11}{3}}-1478048a^{10}b^3x^{\frac{10}{3}}+1555840a^9b^4x^3-1647360a^8\right.}{117}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11700675*x^(1/3)*(b+a*x^(1/3))*(1300075*a^13*x^(13/3)-1352078*a^12*b*x^4+
1410864*a^11*b^2*x^(11/3)-1478048*a^10*b^3*x^(10/3)+1555840*a^9*b^4*x^3-164
7360*a^8*b^5*x^(8/3)+1757184*a^7*b^6*x^(7/3)-1892352*a^6*b^7*x^2+2064384*a^
5*b^8*x^(5/3)-2293760*a^4*b^9*x^(4/3)+2621440*a^3*b^10*x-3145728*a^2*b^11*x
^(2/3)+4194304*a*b^12*x^(1/3)-8388608*b^13)/(b*x^(2/3)+a*x)^(1/2)/a^14
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(299) = 598$.
time = 270.07, size = 1294, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/11700675*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(64*
a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 + 151
43273600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6 - 114
688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*(48816*
a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10 - 734
0032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b^8 + 1228
8*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(43743616*a^12 +
89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 3264*(3608543232*a^1
5 + 64599808*a^12 + 2145*a^9)*b^4 + 578816*(13049856*a^15 - 27313*a^12)*b^3
 + 217056*(6211584*a^15 + 2353*a^12)*b^2 - 156009*(2547712*a^15 + 39*a^12)*
b)*x + 2*(1300075*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^4 -
 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x^5
- 1478048*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*b^7 - 114688*
a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a^10)*b^3)*x^4
+ 1757184*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7*b^10 - 114688*
a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^7)*b^6)*x^3 - 22
93760*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b^13 - 114688*a^4*
b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^4)*b^9)*x^2 + 4194
304*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 - 114688*a*b^15 - 230
4*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x - 2*(70368744177664*b^1
9 + 26388279066624*b^18 + 824633720832*b^17 - 481036337152*b^16 - 966367641
6*b^15 - 4194304*(262144*a^3 + 27)*b^13 + 3623878656*b^14 + 676039*(1677721
6*a^12*b^7 + 6291456*a^12*b^6 + 196608*a^12*b^5 - 114688*a^12*b^4 - 2304*a^
12*b^3 + 864*a^12*b^2 - (262144*a^15 + 27*a^12)*b)*x^4 - 777920*(16777216*a
^9*b^10 + 6291456*a^9*b^9 + 196608*a^9*b^8 - 114688*a^9*b^7 - 2304*a^9*b^6
+ 864*a^9*b^5 - (262144*a^12 + 27*a^9)*b^4)*x^3 + 946176*(16777216*a^6*b^13
 + 6291456*a^6*b^12 + 196608*a^6*b^11 - 114688*a^6*b^10 - 2304*a^6*b^9 + 86
4*a^6*b^8 - (262144*a^9 + 27*a^6)*b^7)*x^2 - 1310720*(16777216*a^3*b^16 + 6
291456*a^3*b^15 + 196608*a^3*b^14 - 114688*a^3*b^13 - 2304*a^3*b^12 + 864*a
^3*b^11 - (262144*a^6 + 27*a^3)*b^10)*x)*x^(2/3) + 48*(29393*(16777216*a^11
*b^8 + 6291456*a^11*b^7 + 196608*a^11*b^6 - 114688*a^11*b^5 - 2304*a^11*b^4
 + 864*a^11*b^3 - (262144*a^14 + 27*a^11)*b^2)*x^4 - 34320*(16777216*a^8*b^
11 + 6291456*a^8*b^10 + 196608*a^8*b^9 - 114688*a^8*b^8 - 2304*a^8*b^7 + 86
4*a^8*b^6 - (262144*a^11 + 27*a^8)*b^5)*x^3 + 43008*(16777216*a^5*b^14 + 62
91456*a^5*b^13 + 196608*a^5*b^12 - 114688*a^5*b^11 - 2304*a^5*b^10 + 864*a^
5*b^9 - (262144*a^8 + 27*a^5)*b^8)*x^2 - 65536*(16777216*a^2*b^17 + 6291456
*a^2*b^16 + 196608*a^2*b^15 - 114688*a^2*b^14 - 2304*a^2*b^13 + 864*a^2*b^1
2 - (262144*a^5 + 27*a^2)*b^11)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((167772
16*a^14*b^6 + 6291456*a^14*b^5 + 196608*a^14*b^4 - 262144*a^17 - 114688*a^1
4*b^3 - 2304*a^14*b^2 + 864*a^14*b - 27*a^14)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.35, size = 206, normalized size = 0.51

$$\frac{16777216\,b^{\frac{27}{2}}}{11700675\,a^{14}} + \frac{2\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}} - 18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b + 119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^2 - 478056150\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^3 + 1320944625\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^4 - 2657429775\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^5 + 4015671660\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^6 - 4633467300\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^7 + 4106936925\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^8 - 2788660875\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^9 + 1434168450\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{10} - 547591590\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{11} + 152108775\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{12} - 35102025\sqrt{ax^{\frac{1}{3}}+b}\,b^{13}\right)}{11700675\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/2
) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^
2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2
)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^
(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3)
+ b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1
/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x
^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^14
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(2/3))^(1/2), x)
```

3.186 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=313

$$-\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}}+\frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}}+\frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8}$$

```
[Out] -262144/323323*b^9*(b*x^(2/3)+a*x)^(1/2)/a^10+524288/323323*b^10*(b*x^(2/3)
+a*x)^(1/2)/a^11/x^(1/3)+196608/323323*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^
9-163840/323323*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+20480/46189*b^6*x*(b*
x^(2/3)+a*x)^(1/2)/a^7-18432/46189*b^5*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+15
36/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-768/2261*b^3*x^2*(b*x^(2/3)+a
*x)^(1/2)/a^4+720/2261*b^2*x^(7/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-40/133*b*x^(8/
3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A]
time = 0.36, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}}-\frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}}+\frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8}+\frac{20480b^6x\sqrt{ax+bx^{2/3}}}{46189a^7}-\frac{18432b^5x^{4/3}\sqrt{ax+bx^{2/3}}}{46189a^6}+\frac{1536b^4x^{5/3}\sqrt{ax+bx^{2/3}}}{4199a^5}-\frac{768b^3x^2\sqrt{ax+bx^{2/3}}}{2261a^4}+\frac{720b^2x^{7/3}\sqrt{ax+bx^{2/3}}}{2261a^3}-\frac{40bx^{8/3}\sqrt{ax+bx^{2/3}}}{133a^2}+\frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(
2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a
*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8)
 + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/
(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7
/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]
)/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
```

```
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{(20b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a}$$

$$= -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} + \frac{(120b^2)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^2}$$

$$= \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \cdots$$

$$= -\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \cdots$$

$$= \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \cdots$$

$$= -\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \cdots$$

$$= \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \cdots$$

$$= -\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} \cdots$$

$$= \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} \cdots$$

$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} \cdots$$

$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \cdots$$

Mathematica [A]
time = 0.09, size = 148, normalized size = 0.47

$$\frac{2\sqrt{bx^{2/3}+ax}\left(262144b^{10} - 131072ab^9\sqrt[3]{x} + 98304a^2b^8x^{2/3} - 81920a^3b^7x + 71680a^4b^6x^{4/3} - 64512a^5b^5x^{5/3} + 59136a^6b^4x^2 - 54912a^7b^3x^{7/3} + 51480a^8b^2x^{8/3} - 48620a^9bx^3 + 46189a^{10}x^{10/3}\right)}{323323a^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b^
8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/3)
```

```
 + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4862
0*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

Maple [A]
time = 0.36, size = 134, normalized size = 0.43

method	result
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136x^{2}a^{6}b^{4}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}b^{6}x\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^{11}}$
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136x^{2}a^{6}b^{4}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}b^{6}x\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/323323*x^(1/3)*(b+a*x^(1/3))*(46189*a^10*x^(10/3)-48620*a^9*b*x^3+51480*a
^8*b^2*x^(8/3)-54912*a^7*b^3*x^(7/3)+59136*x^2*a^6*b^4-64512*a^5*b^5*x^(5/3
)+71680*a^4*b^6*x^(4/3)-81920*a^3*b^7*x+98304*a^2*b^8*x^(2/3)-131072*a*b^9*
x^(1/3)+262144*b^10)/(b*x^(2/3)+a*x)^(1/2)/a^11
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(233) = 466$.
time = 242.12, size = 1031, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/323323*((3298534883328*b^16 + 687194767360*b^15 + 3221225472*(64*a^3 - 3
)*b^13 - 64424509440*b^14 - 16777216*(11264*a^3 - 53)*b^12 - 269004736*a^12
 - 6291456*(5504*a^3 + 1)*b^11 + 196608*(3194880*a^6 - 114688*a^3 - 3)*b^10
 + 7340032*(18816*a^6 + 103*a^3)*b^9 - 786432*(48816*a^6 + 23*a^3)*b^8 - 12
288*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^7 - 114688*(1349120*a^9 + 3439*a
^6)*b^6 + 3913728*(5600*a^9 + 3*a^6)*b^5 - 2112*(101384192*a^12 + 1958400*a
```

```
^9 + 63*a^6)*b^4 - 36608*(3784704*a^12 - 8101*a^9)*b^3 - 109824*(226688*a^1
2 + 85*a^9)*b^2 + 7293*(974848*a^12 + 15*a^9)*b)*x - (46189*(16777216*a^10*
b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 114688*a^10*b^3 -
2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 - 54912*(16777216*a^7*b^9 + 62914
56*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5 + 864*a^7*b^4 -
 (262144*a^10 + 27*a^7)*b^3)*x^3 + 71680*(16777216*a^4*b^12 + 6291456*a^4*b
^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 864*a^4*b^7 - (2621
44*a^7 + 27*a^4)*b^6)*x^2 - 131072*(16777216*a*b^15 + 6291456*a*b^14 + 1966
08*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (262144*a^4 + 27*a)*
b^9)*x + 4*(1099511627776*b^16 + 412316860416*b^15 + 12884901888*b^14 - 751
6192768*b^13 - 150994944*b^12 - 65536*(262144*a^3 + 27)*b^10 + 56623104*b^1
1 - 12155*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*a^9*b^5 - 114688*a^9
*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*a^9)*b)*x^3 + 14784*(
16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b^8 - 114688*a^6*b^7 - 230
4*a^6*b^6 + 864*a^6*b^5 - (262144*a^9 + 27*a^6)*b^4)*x^2 - 20480*(16777216*
a^3*b^13 + 6291456*a^3*b^12 + 196608*a^3*b^11 - 114688*a^3*b^10 - 2304*a^3*
b^9 + 864*a^3*b^8 - (262144*a^6 + 27*a^3)*b^7)*x)*x^(2/3) + 24*(2145*(16777
216*a^8*b^8 + 6291456*a^8*b^7 + 196608*a^8*b^6 - 114688*a^8*b^5 - 2304*a^8*
b^4 + 864*a^8*b^3 - (262144*a^11 + 27*a^8)*b^2)*x^3 - 2688*(16777216*a^5*b^
11 + 6291456*a^5*b^10 + 196608*a^5*b^9 - 114688*a^5*b^8 - 2304*a^5*b^7 + 86
4*a^5*b^6 - (262144*a^8 + 27*a^5)*b^5)*x^2 + 4096*(16777216*a^2*b^14 + 6291
456*a^2*b^13 + 196608*a^2*b^12 - 114688*a^2*b^11 - 2304*a^2*b^10 + 864*a^2*
b^9 - (262144*a^5 + 27*a^2)*b^8)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((16777
216*a^11*b^6 + 6291456*a^11*b^5 + 196608*a^11*b^4 - 262144*a^14 - 114688*a^
11*b^3 - 2304*a^11*b^2 + 864*a^11*b - 27*a^11)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.13, size = 164, normalized size = 0.52

$$-\frac{524288\,b^{\frac{21}{2}}}{323323\,a^{11}} + \frac{2\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} - 510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b + 2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^2 - 7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^3 + 15668730\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^4 - 22221108\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^5 + 22632610\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^6 - 16628040\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^7 + 8729721\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^8 - 3233230\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^9 + 969969\sqrt{ax^{\frac{1}{3}}+b}\,b^{10}\right)}{323323\,a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 510
510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752
*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222110
8*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 1662804
0*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(
a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(1/2), x)
```

3.187 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=225

$$\frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7}-\frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}}-\frac{512b^5\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5}-\frac{224b^3x\sqrt{bx}}{429}$$

```
[Out] 2048/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^7-4096/2145*b^7*(b*x^(2/3)+a*x)^(1/2)
/a^8/x^(1/3)-512/715*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+256/429*b^4*x^(2
/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-224/429*b^3*x*(b*x^(2/3)+a*x)^(1/2)/a^4+336/7
15*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-28/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1
/2)/a^2+2/5*x^2*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A]
time = 0.23, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}-\frac{512b^5\sqrt[3]{x}\,\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}+\frac{336b^2x^{4/3}\sqrt{ax+bx^{2/3}}}{715a^3}-\frac{28bx^{5/3}\sqrt{ax+bx^{2/3}}}{65a^2}+\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*
x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6)
+ (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(
2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3)
- (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a
*x])/(5*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(14b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a}$$

$$= -\frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + \frac{(56b^2)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^2}$$

$$= \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(112b^3)\int -}{1}$$

$$= -\frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{}}{}$$

$$= \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/}}{}$$

$$= -\frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336}{}$$

$$= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3}{}$$

$$= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/}}{}$$

Mathematica [A]
time = 0.08, size = 111, normalized size = 0.49

$$\frac{2\sqrt{bx^{2/3}+ax}\left(-2048b^7+1024ab^6\sqrt[3]{x}-768a^2b^5x^{2/3}+640a^3b^4x-560a^4b^3x^{4/3}+504a^5b^2x^{5/3}-462a^6bx^2+429a^7x^{7/3}\right)}{2145a^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(2
/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6*b
*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))

Maple [A]
time = 0.35, size = 101, normalized size = 0.45

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{7}x^{\frac{7}{3}}-462a^{6}b\,x^{2}+504a^{5}b^{2}x^{\frac{5}{3}}-560a^{4}b^{3}x^{\frac{4}{3}}+640a^{3}b^{4}x-768a^{2}b^{5}x^{\frac{2}{3}}+1024a\,b^{6}x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^{8}}$	101
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{7}x^{\frac{7}{3}}-462a^{6}b\,x^{2}+504a^{5}b^{2}x^{\frac{5}{3}}-560a^{4}b^{3}x^{\frac{4}{3}}+640a^{3}b^{4}x-768a^{2}b^{5}x^{\frac{2}{3}}+1024a\,b^{6}x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^{8}}$	101

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/2145*x^(1/3)*(b+a*x^(1/3))*(429*a^7*x^(7/3)-462*a^6*b*x^2+504*a^5*b^2*x^(
5/3)-560*a^4*b^3*x^(4/3)+640*a^3*b^4*x-768*a^2*b^5*x^(2/3)+1024*a*b^6*x^(1/
3)-2048*b^7)/(b*x^(2/3)+a*x)^(1/2)/a^8

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(a*x + b*x^(2/3)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(167) = 334$.
time = 176.97, size = 768, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] 1/2145*((51539607552*b^13 + 10737418240*b^12 + 50331648*(64*a^3 - 3)*b^10 -
 1006632960*b^11 - 262144*(11264*a^3 - 53)*b^9 + 4996992*a^9 - 98304*(5504*
a^3 + 1)*b^8 + 3072*(3194880*a^6 - 114688*a^3 - 3)*b^7 + 114688*(18816*a^6
+ 103*a^3)*b^6 - 12288*(48816*a^6 + 23*a^3)*b^5 + 192*(21626880*a^9 + 49587
2*a^6 + 15*a^3)*b^4 + 256*(10690560*a^9 - 24073*a^6)*b^3 + 3744*(133120*a^9
 + 49*a^6)*b^2 - 297*(450560*a^9 + 7*a^6)*b)*x + 2*(429*(16777216*a^7*b^6 +

```
 6291456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2304*a^7
*b^2 + 864*a^7*b - 27*a^7)*x^3 - 560*(16777216*a^4*b^9 + 6291456*a^4*b^8 +
196608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (262144*a^7
+ 27*a^4)*b^3)*x^2 + 1024*(16777216*a*b^12 + 6291456*a*b^11 + 196608*a*b^10
 - 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b^6)*x - 2*(
17179869184*b^13 + 6442450944*b^12 + 201326592*b^11 - 117440512*b^10 - 2359
296*b^9 - 1024*(262144*a^3 + 27)*b^7 + 884736*b^8 + 231*(16777216*a^6*b^7 +
 6291456*a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6*b^3 + 864*a^6
*b^2 - (262144*a^9 + 27*a^6)*b)*x^2 - 320*(16777216*a^3*b^10 + 6291456*a^3*
b^9 + 196608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 864*a^3*b^5 - (26214
4*a^6 + 27*a^3)*b^4)*x)*x^(2/3) + 24*(21*(16777216*a^5*b^8 + 6291456*a^5*b^
7 + 196608*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 864*a^5*b^3 - (262144*
a^8 + 27*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291456*a^2*b^10 + 196608*
a^2*b^9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2*b^6 - (262144*a^5 + 27*a^
2)*b^5)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^8*b^6 + 6291456*a^8
*b^5 + 196608*a^8*b^4 - 262144*a^11 - 114688*a^8*b^3 - 2304*a^8*b^2 + 864*a
^8*b - 27*a^8)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x**2/sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.16, size = 122, normalized size = 0.54

$$\frac{4096\, b^{\frac{15}{2}}}{2145\, a^8} + \frac{2\left(429\left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}} - 3465\left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}} b + 12285\left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} b^2 - 25025\left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} b^3 + 32175\left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} b^4 - 27027\left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} b^5 + 15015\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} b^6 - 6435\sqrt{ax^{\frac{1}{3}} + b}\, b^7\right)}{2145\, a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
[Out] 4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/
3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)
^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*
b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^8
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a\,x + b\,x^{2/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(2/3))^(1/2),x)
[Out] int(x^2/(a*x + b*x^(2/3))^(1/2), x)
```

3.188 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=137

$$-\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4}+\frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}}+\frac{32b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{35a^3}-\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2}+\frac{2x\sqrt{bx^{2/3}+ax}}{3a}$$

```
[Out] -128/105*b^3*(b*x^(2/3)+a*x)^(1/2)/a^4+256/105*b^4*(b*x^(2/3)+a*x)^(1/2)/a^
5/x^(1/3)+32/35*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-16/21*b*x^(2/3)*(b*x^
(2/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2041, 2027, 2039}

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4}+\frac{32b^2\sqrt[3]{x}\,\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*Sqrt[b*x^(2/3) + a*x])/(105*a^4) + (256*b^4*Sqrt[b*x^(2/3) + a*x]
)/(105*a^5*x^(1/3)) + (32*b^2*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(35*a^3) - (16
*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^2) + (2*x*Sqrt[b*x^(2/3) + a*x])/(3
*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{(8b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a}$$

$$= -\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \frac{(16b^2)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}$$

$$= \frac{32b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{(64b^3)\int \frac{\quad}{\sqrt{bx^{2/}}}}{105a}$$

$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{32b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}}}{3a}$$

$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}} + \frac{32b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx}}{21a}$$

Mathematica [A]
time = 0.06, size = 74, normalized size = 0.54

$$\frac{2\sqrt{bx^{2/3}+ax}\left(128b^4-64ab^3\sqrt[3]{x}+48a^2b^2x^{2/3}-40a^3bx+35a^4x^{4/3}\right)}{105a^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3) -
 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

Maple [A]
time = 0.34, size = 68, normalized size = 0.50

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^4x^{\frac{4}{3}}-40a^3bx+48a^2b^2x^{\frac{2}{3}}-64a\,b^3x^{\frac{1}{3}}+128b^4\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^5}$	68

default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^4x^{\frac{4}{3}}-40a^3bx+48a^2b^2x^{\frac{2}{3}}-64a\,b^3x^{\frac{1}{3}}+128b^4\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^5}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*x^(1/3)*(b+a*x^(1/3))*(35*a^4*x^(4/3)-40*a^3*b*x+48*a^2*b^2*x^(2/3)-6
4*a*b^3*x^(1/3)+128*b^4)/(b*x^(2/3)+a*x)^(1/2)/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(101) = 202.
time = 271.30, size = 502, normalized size = 3.66

$$-\frac{2\left(2\left(805306368b^{10}+167772160b^9+786432(64a^3-3)b^7-15728640b^8-4096(11264a^3-53)b^6-101920a^6-1536(5504a^3+1)b^5-48(1966080a^6+114688a^3+3)b^4-1792(36864a^6-103a^3)b^3-192(65280a^6+23a^3)b^2+15(188416a^6+3a^3)b\right)x-\left(35(16777216a^4b^6+6291456a^4b^5+196608a^4b^4-262144a^7-114688a^4b^3-2304a^4b^2+864a^4b-27a^4)x^2+48(16777216a^2b^8+6291456a^2b^7+196608a^2b^6-114688a^2b^5-2304a^2b^4+864a^2b^3-(262144a^5+27a^2)b^2)x^{\frac{4}{3}}-64(16777216ab^9+6291456ab^8+196608ab^7-114688ab^6-2304ab^5+864ab^4-(262144a^4+27a)b^3)x+8(268435456b^{10}+100663296b^9+3145728b^8-1835008b^7-36864b^6-16(262144a^3+27)b^4+13824b^5-5(16777216a^3b^7+6291456a^3b^6+196608a^3b^5-114688a^3b^4-2304a^3b^3+864a^3b^2-(262144a^6+27a^3)b)x)x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{2}{3}}}\right)}{105(16777216a^5b^6+6291456a^5b^5+196608a^5b^4-262144a^8-114688a^5b^3-2304a^5b^2+864a^5b-27a^5)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(2*(805306368*b^10 + 167772160*b^9 + 786432*(64*a^3 - 3)*b^7 - 15728
640*b^8 - 4096*(11264*a^3 - 53)*b^6 - 101920*a^6 - 1536*(5504*a^3 + 1)*b^5
 - 48*(1966080*a^6 + 114688*a^3 + 3)*b^4 - 1792*(36864*a^6 - 103*a^3)*b^3 -
192*(65280*a^6 + 23*a^3)*b^2 + 15*(188416*a^6 + 3*a^3)*b)*x - (35*(16777216
*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*b^3 -
 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 48*(16777216*a^2*b^8 + 6291456*a^
2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^3 - (262
144*a^5 + 27*a^2)*b^2)*x^(4/3) - 64*(16777216*a*b^9 + 6291456*a*b^8 + 19660
8*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27*a)*b^3)*
x + 8*(268435456*b^10 + 100663296*b^9 + 3145728*b^8 - 1835008*b^7 - 36864*b
^6 - 16*(262144*a^3 + 27)*b^4 + 13824*b^5 - 5*(16777216*a^3*b^7 + 6291456*a
^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3*b^2 - (26
2144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^5*b^6
 + 6291456*a^5*b^5 + 196608*a^5*b^4 - 262144*a^8 - 114688*a^5*b^3 - 2304*a^
5*b^2 + 864*a^5*b - 27*a^5)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.35, size = 80, normalized size = 0.58

$$-\frac{256\,b^{\frac{9}{2}}}{105\,a^5} + \frac{2\left(35\left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} - 180\left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}}b + 378\left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}}b^2 - 420\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}}b^3 + 315\sqrt{ax^{\frac{1}{3}} + b}\,b^4\right)}{105\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b
)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 +
 315*sqrt(a*x^(1/3) + b)*b^4)/a^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a\,x + b\,x^{2/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(2/3))^(1/2), x)
```

3.189 $\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

```
[Out] 2*(b*x^(2/3)+a*x)^(1/2)/a-4*b*(b*x^(2/3)+a*x)^(1/2)/a^2/x^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2027, 2039}

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx &= \frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{(2b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a}\\ &= \frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.77

$$\frac{2\left(-2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^(2/3) + a*x],x]
[Out] (2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Maple [A]
time = 0.34, size = 36, normalized size = 0.77

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(a\,x^{\frac{1}{3}}-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^{2}}$	36
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(a\,x^{\frac{1}{3}}-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\;a^{2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/3)*(b+a*x^(1/3))*(a*x^(1/3)-2*b)/(b*x^(2/3)+a*x)^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*x + b*x^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(37) = 74.
time = 211.26, size = 238, normalized size = 5.06

$$\frac{(50331648\,b^7+10485760\,b^6+49152\,(512\,a^3-3)b^4-983040\,b^5+256\,(24576\,a^3+53)b^3+11648\,a^3-96\,(2048\,a^3+1)b^2-3\,(155648\,a^3+3)b)x+2\left((16777216\,ab^6+6291456\,ab^5+196608\,ab^4-262144\,a^4-114688\,ab^3-2304\,ab^2+864\,ab-27\,a)x-2\,(16777216\,b^7+6291456\,b^6+196608\,b^5-114688\,b^4-2304\,b^3-(262144\,a^3+27)b+864\,b^2)x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{2}{3}}}}{(16777216\,a^2b^6+6291456\,a^2b^5+196608\,a^2b^4-262144\,a^5-114688\,a^2b^3-2304\,a^2b^2+864\,a^2b-27\,a^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] ((50331648*b^7 + 10485760*b^6 + 49152*(512*a^3 - 3)*b^4 - 983040*b^5 + 256*
(24576*a^3 + 53)*b^3 + 11648*a^3 - 96*(2048*a^3 + 1)*b^2 - 3*(155648*a^3 +
3)*b)*x + 2*((16777216*a*b^6 + 6291456*a*b^5 + 196608*a*b^4 - 262144*a^4 -
114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x - 2*(16777216*b^7 + 6291456*b
^6 + 196608*b^5 - 114688*b^4 - 2304*b^3 - (262144*a^3 + 27)*b + 864*b^2)*x^
(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^2*b^6 + 6291456*a^2*b^5 + 196608
*a^2*b^4 - 262144*a^5 - 114688*a^2*b^3 - 2304*a^2*b^2 + 864*a^2*b - 27*a^2)
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(2/3)), x)
```

Giac [A]
time = 1.22, size = 36, normalized size = 0.77

$$\frac{4\, b^{\frac{3}{2}}}{a^2} + \frac{2\left(\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} - 3\, \sqrt{ax^{\frac{1}{3}} + b}\, b\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2
```

Mupad [B]
time = 5.22, size = 40, normalized size = 0.85

$$\frac{3\, x\, \sqrt{\frac{a\, x^{1/3}}{b} + 1}\, {}_2\mathrm{F}_1\left(\frac{1}{2}, 2;\ 3;\ -\frac{a\, x^{1/3}}{b}\right)}{2\, \sqrt{a\, x + b\, x^{2/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*((a*x^(1/3))/b + 1)^(1/2)*hypergeom([1/2, 2], 3, -(a*x^(1/3))/b))/(2*(
a*x + b*x^(2/3))^(1/2))
```

3.190 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=61

$$-\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

```
[Out] 3*a*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3*(b*x^(2/3)+a*x
)^(1/2)/b/x^(2/3)
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2054, 212}

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}-\frac{a\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{2b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{(3a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 61, normalized size = 1.00

$$-\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

Maple [**A**]
time = 0.34, size = 61, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{3\sqrt{b+a\,x^{\frac{1}{3}}}\left(\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{5}{2}}}$	61
default	$\frac{3\sqrt{b+a\,x^{\frac{1}{3}}}\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}-\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{3}{2}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{5}{2}}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*(b+a*x^(1/3))^(1/2)*(arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*a*x^(1/3)-(b+
a*x^(1/3))^(1/2)*b^(3/2))/(b*x^(2/3)+a*x)^(1/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)
```

Giac [A]
time = 1.27, size = 51, normalized size = 0.84

$$-\frac{3\left(\frac{a^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\sqrt{ax^{\frac{1}{3}}+b}\,a}{bx^{\frac{1}{3}}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -3*(a^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3)
+ b)*a/(b*x^(1/3)))/a
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(2/3))^(1/2)), x)
```

3.191 $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=153

$$-\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}}+\frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}}-\frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x}+\frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}}-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

```
[Out] -105/64*a^4*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/4*(b*x
^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/8*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-35/32*a
^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x+105/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A]
time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2054, 212}

$$-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}+\frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}}-\frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x}+\frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) + (7*a*Sqrt[b*x^(2/3) + a*x])/(8*b
^2*x^(4/3)) - (35*a^2*Sqrt[b*x^(2/3) + a*x])/(32*b^3*x) + (105*a^3*Sqrt[b*x
^(2/3) + a*x])/(64*b^4*x^(2/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/(64*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} - \frac{(7a)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} + \frac{(35a^2)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{48b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} - \frac{(35a^3)\int \frac{1}{x\sqrt{bx^{2/3}+}}}{64b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}}$$

Mathematica [A]
time = 0.17, size = 101, normalized size = 0.66

$$\frac{\sqrt{bx^{2/3}+ax}\left(-48b^3+56ab^2\sqrt[3]{x}-70a^2bx^{2/3}+105a^3x\right)}{64b^4x^{5/3}} - \frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-48*b^3 + 56*a*b^2*x^(1/3) - 70*a^2*b*x^(2/3) + 105
*a^3*x))/(64*b^4*x^(5/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/
3) + a*x]])/(64*b^(9/2))
```

Maple [A]
time = 0.34, size = 126, normalized size = 0.82

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(48\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{9}{2}}-56\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a\,x^{\frac{1}{3}}+70\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}a^{2}x^{\frac{2}{3}}-105\sqrt{b+a\,x^{\frac{1}{3}}}\right.}{64x\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$
default	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(105x^{\frac{7}{3}}\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{4}b-56x^{\frac{4}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a-105x^{2}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{3}{2}}a^{3}+70\right.}{64x^{2}\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/x^2*(b+a*x^(1/3))^(1/2)*(105*x^(7/3)*arctanh((b+a*x^(1/3))^(1/2)/b^(1
/2))*a^4*b-56*x^(4/3)*(b+a*x^(1/3))^(1/2)*b^(7/2)*a-105*x^2*(b+a*x^(1/3))^(
1/2)*b^(3/2)*a^3+70*x^(5/3)*(b+a*x^(1/3))^(1/2)*b^(5/2)*a^2+48*(b+a*x^(1/3)
)^(1/2)*b^(9/2)*x)/(b*x^(2/3)+a*x)^(1/2)/b^(11/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)
```

Giac [A]
time = 1.04, size = 109, normalized size = 0.71

$$\frac{\frac{105\,a^5\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^5-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^5b+511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^5b^2-279\sqrt{ax^{\frac{1}{3}}+b}\,a^5b^3}{a^4b^4x^{\frac{4}{3}}}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a
*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3)
+ b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)), x)
```

3.192 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=241

$$-\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}+\frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}}-\frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2}+\frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}}-\frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}}+42$$

```
[Out] 1287/1024*a^7*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/7*(
b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/28*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-14
3/280*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+1287/2240*a^3*(b*x^(2/3)+a*x)^(1/2)
/b^4/x^(5/3)-429/640*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+429/512*a^5*(b*x
^(2/3)+a*x)^(1/2)/b^6/x-1287/1024*a^6*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A]

time = 0.28, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2054, 212}

$$\frac{1287a^7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{1024b^{15/2}}-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}-\frac{143a^2\sqrt{ax+bx^{2/3}}}{280b^3x^2}+\frac{13a\sqrt{ax+bx^{2/3}}}{28b^2x^{7/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) + (13*a*Sqrt[b*x^(2/3) + a*x])/(28
*b^2*x^(7/3)) - (143*a^2*Sqrt[b*x^(2/3) + a*x])/(280*b^3*x^2) + (1287*a^3*S
qrt[b*x^(2/3) + a*x])/(2240*b^4*x^(5/3)) - (429*a^4*Sqrt[b*x^(2/3) + a*x])/
(640*b^5*x^(4/3)) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^6*x) - (1287*a^6
*Sqrt[b*x^(2/3) + a*x])/(1024*b^7*x^(2/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} - \frac{(13a)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{14b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} + \frac{(143a^2)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{168b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} - \frac{(429a^3)\int \frac{}{x^2\sqrt{bx^{2}}}}{560b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+}}{2240b^4x^{5/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+}}{2240b^4x^{5/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+}}{2240b^4x^{5/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+}}{2240b^4x^{5/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+}}{2240b^4x^{5/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+}}{2240b^4x^{5/3}}$$

Mathematica [A]
time = 0.19, size = 138, normalized size = 0.57

$$\frac{\sqrt{bx^{2/3}+ax}\left(-15360b^6+16640ab^5\sqrt[3]{x}-18304a^2b^4x^{2/3}+20592a^3b^3x-24024a^4b^2x^{4/3}+30030a^5bx^{5/3}-45045a^6x^2\right)}{35840b^7x^{8/3}}+\frac{1287a^7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

[Out] (Sqrt[b*x^(2/3) + a*x]*(-15360*b^6 + 16640*a*b^5*x^(1/3) - 18304*a^2*b^4*x^(2/3) + 20592*a^3*b^3*x - 24024*a^4*b^2*x^(4/3) + 30030*a^5*b*x^(5/3) - 45045*a^6*x^2))/(35840*b^7*x^(8/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))

Maple [A]
time = 0.34, size = 188, normalized size = 0.78

method	result
derivativedivides	$-\sqrt{b+a x^{\frac{1}{3}}}\,\left(45045\sqrt{b+a x^{\frac{1}{3}}}\, b^{\frac{3}{2}} a^{6} x^{2}-45045\,\mathrm{arctanh}\left(\frac{\sqrt{b+a x^{\frac{1}{3}}}}{\sqrt{b}}\right) a^{7} b x^{\frac{7}{3}}-30030\sqrt{b+a x^{\frac{1}{3}}}\, b^{\frac{5}{2}} \cdots\right.$
default	$\sqrt{b+a x^{\frac{1}{3}}}\,\left(45045 x^{\frac{13}{3}}\,\mathrm{arctanh}\left(\frac{\sqrt{b+a x^{\frac{1}{3}}}}{\sqrt{b}}\right) a^{7} b-24024 x^{\frac{10}{3}}\sqrt{b+a x^{\frac{1}{3}}}\, b^{\frac{7}{2}} a^{4}-45045 x^{4}\sqrt{b+a x^{\frac{1}{3}}} \cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/35840/x^4*(b+a*x^(1/3))^(1/2)*(45045*x^(13/3)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^7*b-24024*x^(10/3)*(b+a*x^(1/3))^(1/2)*b^(7/2)*a^4-45045*x^4*(b+a*x^(1/3))^(1/2)*b^(3/2)*a^6+16640*x^(7/3)*(b+a*x^(1/3))^(1/2)*b^(13/2)*a+30030*x^(11/3)*(b+a*x^(1/3))^(1/2)*b^(5/2)*a^5+20592*x^3*(b+a*x^(1/3))^(1/2)*b^(9/2)*a^3-18304*x^(8/3)*(b+a*x^(1/3))^(1/2)*b^(11/2)*a^2-15360*(b+a*x^(1/3))^(1/2)*b^(15/2)*x^2)/(b*x^(2/3)+a*x)^(1/2)/b^(17/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)
```

Giac [A]

time = 1.96, size = 160, normalized size = 0.66

$$-\frac{\frac{45045\,a^8\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^8-300300\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^8b+849849\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^8b^2-1317888\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^8b^3+1200199\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^8b^4-631540\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^8b^5+169995\sqrt{ax^{\frac{1}{3}}+b}\,a^8b^6}{a^7b^7x^{\frac{7}{3}}}}{35840\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35840*(45045*a^8*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + (
45045*(a*x^(1/3) + b)^(13/2)*a^8 - 300300*(a*x^(1/3) + b)^(11/2)*a^8*b + 84
9849*(a*x^(1/3) + b)^(9/2)*a^8*b^2 - 1317888*(a*x^(1/3) + b)^(7/2)*a^8*b^3
+ 1200199*(a*x^(1/3) + b)^(5/2)*a^8*b^4 - 631540*(a*x^(1/3) + b)^(3/2)*a^8*
b^5 + 169995*sqrt(a*x^(1/3) + b)*a^8*b^6)/(a^7*b^7*x^(7/3)))/a
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)), x)
```

3.193 $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=329

$$-\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}}+46$$

```
[Out] -138567/131072*a^10*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)
-3/10*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/60*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^
(10/3)-323/960*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/896*a^3*(b*x^(2/3)+a*x
)^(1/2)/b^4/x^(8/3)-4199/10752*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+46189/
107520*a^5*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-138567/286720*a^6*(b*x^(2/3)+a*x)^
(1/2)/b^7/x^(5/3)+46189/81920*a^7*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-46189/6
5536*a^8*(b*x^(2/3)+a*x)^(1/2)/b^9/x+138567/131072*a^9*(b*x^(2/3)+a*x)^(1/2
)/b^10/x^(2/3)
```

Rubi [A]

time = 0.39, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2054, 212}

$$-\frac{138567a^{10}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{131072b^{21/2}}+\frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}}-\frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x}+\frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}}-\frac{138567a^6\sqrt{ax+bx^{2/3}}}{286720b^7x^{5/3}}+\frac{46189a^5\sqrt{ax+bx^{2/3}}}{107520b^6x^2}-\frac{4199a^4\sqrt{ax+bx^{2/3}}}{10752b^5x^{7/3}}+\frac{323a^3\sqrt{ax+bx^{2/3}}}{896b^4x^{8/3}}-\frac{323a^2\sqrt{ax+bx^{2/3}}}{960b^3x^3}+\frac{19a\sqrt{ax+bx^{2/3}}}{60b^2x^{10/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(
60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3
*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x]
)/(10752*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2)
- (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt
[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/
(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) -
(138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2
1/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} - \frac{(19a)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{20b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} + \frac{(323a^2)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{360b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} - \frac{(323a^3)\int \frac{1}{x^3\sqrt{bx^{2/}}}}{384b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}$$

Mathematica [A]
time = 0.24, size = 175, normalized size = 0.53

$$\frac{\sqrt{bx^{2/3}+ax}\left(-4128768b^9+4358144ab^8\sqrt[3]{x}-4630528a^2b^7x^{2/3}+4961280a^3b^6x-5374720a^4b^5x^{4/3}+5912192a^5b^4x^{5/3}-6651216a^6b^3x^2+7759752a^7b^2x^{7/3}-9699690a^8bx^{8/3}+14549535a^9x^3\right)}{13762560b^{10}x^{11/3}} - \frac{138567a^{10}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-4128768*b^9 + 4358144*a*b^8*x^(1/3) - 4630528*a^2*
b^7*x^(2/3) + 4961280*a^3*b^6*x - 5374720*a^4*b^5*x^(4/3) + 5912192*a^5*b^4
*x^(5/3) - 6651216*a^6*b^3*x^2 + 7759752*a^7*b^2*x^(7/3) - 9699690*a^8*b*x^
(8/3) + 14549535*a^9*x^3))/(13762560*b^10*x^(11/3)) - (138567*a^10*ArcTanh[
(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(21/2))
```

Maple [A]
time = 0.34, size = 248, normalized size = 0.75

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(4128768\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{21}{2}}-4358144\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{19}{2}}a\,x^{\frac{1}{3}}+4630528\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{17}{2}}a^{2}x^{\frac{2}{3}}\right.}{}$
default	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(14549535\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)x^{\frac{19}{3}}a^{10}b-7759752x^{\frac{16}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a^{7}-14549535x^{6}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/13762560/x^6*(b+a*x^(1/3))^(1/2)*(14549535*arctanh((b+a*x^(1/3))^(1/2)/b
^(1/2))*x^(19/3)*a^10*b-7759752*x^(16/3)*(b+a*x^(1/3))^(1/2)*b^(7/2)*a^7-14
549535*x^6*(b+a*x^(1/3))^(1/2)*b^(3/2)*a^9+5374720*x^(13/3)*(b+a*x^(1/3))^(
1/2)*b^(13/2)*a^4+9699690*x^(17/3)*(b+a*x^(1/3))^(1/2)*b^(5/2)*a^8-4358144*
x^(10/3)*(b+a*x^(1/3))^(1/2)*b^(19/2)*a+6651216*x^5*(b+a*x^(1/3))^(1/2)*b^(
9/2)*a^6-5912192*x^(14/3)*(b+a*x^(1/3))^(1/2)*b^(11/2)*a^5-4961280*x^4*(b+a
*x^(1/3))^(1/2)*b^(15/2)*a^3+4630528*x^(11/3)*(b+a*x^(1/3))^(1/2)*b^(17/2)*
a^2+4128768*(b+a*x^(1/3))^(1/2)*b^(21/2)*x^3)/(b*x^(2/3)+a*x)^(1/2)/b^(23/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)

[Out] Integral(1/(x**4*sqrt(a*x + b*x**(2/3))), x)

Giac [A]
time = 1.39, size = 211, normalized size = 0.64

$$\frac{\frac{14549535\,a^{11}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{11}-140645505\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{11}b+609140532\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{11}b^{2}-1554721740\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{11}b^{3}+2585198330\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{11}b^{4}-2918514950\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{11}b^{5}+2255541300\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{11}b^{6}-1168982220\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{11}b^{7}+382331775\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{11}b^{8}-68025825\sqrt{ax^{\frac{1}{3}}+b}\,a^{11}b^{9}}{a^{10}b^{10}x^{\frac{10}{3}}}}{13762560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")

[Out] 1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(17/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x^(1/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 - 2918514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7/2)*a^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^(1/3) + b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b^10*x^(10/3)))/a

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)),x)

[Out] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)), x)

3.194 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}}+\frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}}+\frac{393216b^8\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{29393a^{10}}-3276\cdots$$

```
[Out] -6*x^4/a/(b*x^(2/3)+a*x)^(1/2)-524288/29393*b^9*(b*x^(2/3)+a*x)^(1/2)/a^11+
1048576/29393*b^10*(b*x^(2/3)+a*x)^(1/2)/a^12/x^(1/3)+393216/29393*b^8*x^(1
/3)*(b*x^(2/3)+a*x)^(1/2)/a^10-327680/29393*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/
2)/a^9+40960/4199*b^6*x*(b*x^(2/3)+a*x)^(1/2)/a^8-36864/4199*b^5*x^(4/3)*(b
*x^(2/3)+a*x)^(1/2)/a^7+33792/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-16
896/2261*b^3*x^2*(b*x^(2/3)+a*x)^(1/2)/a^5+15840/2261*b^2*x^(7/3)*(b*x^(2/3
)+a*x)^(1/2)/a^4-880/133*b*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+44/7*x^3*(b*x^
(2/3)+a*x)^(1/2)/a^2
```

Rubi [A]
time = 0.39, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2040, 2041, 2027, 2039}

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\,\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}+\frac{40960b^6x\sqrt{ax+bx^{2/3}}}{4199a^8}-\frac{36864b^5x^{4/3}\sqrt{ax+bx^{2/3}}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{ax+bx^{2/3}}}{4199a^6}-\frac{16896b^3x^2\sqrt{ax+bx^{2/3}}}{2261a^5}+\frac{15840b^2x^{7/3}\sqrt{ax+bx^{2/3}}}{2261a^4}-\frac{880bx^{8/3}\sqrt{ax+bx^{2/3}}}{133a^3}+\frac{44x^3\sqrt{ax+bx^{2/3}}}{7a^2}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) - (524288*b^9*Sqrt[b*x^(2/3) + a*x])/(29
393*a^11) + (1048576*b^10*Sqrt[b*x^(2/3) + a*x])/(29393*a^12*x^(1/3)) + (39
3216*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(29393*a^10) - (327680*b^7*x^(2/3)*
Sqrt[b*x^(2/3) + a*x])/(29393*a^9) + (40960*b^6*x*Sqrt[b*x^(2/3) + a*x])/(4
199*a^8) - (36864*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^7) + (33792*b^
4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^6) - (16896*b^3*x^2*Sqrt[b*x^(2/3)
 + a*x])/(2261*a^5) + (15840*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^4)
- (880*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^3) + (44*x^3*Sqrt[b*x^(2/3)
+ a*x])/(7*a^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{22\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} - \frac{(440b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} + \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}} - \cdots$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}} + \frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}} + \cdots$$

Mathematica [A]

time = 4.18, size = 161, normalized size = 0.48

$$\frac{2\sqrt[3]{x}\left(524288b^{11}+262144ab^{10}\sqrt[3]{x}-65536a^2b^9x^{2/3}+32768a^3b^8x-20480a^4b^7x^{4/3}+14336a^5b^6x^{5/3}-10752a^6b^5x^2+8448a^7b^4x^{7/3}-6864a^8b^3x^{8/3}+5720a^9b^2x^3-4862a^{10}bx^{10/3}+4199a^{11}x^{11/3}\right)}{29393a^{12}\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) + 3
2768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^6*
b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3 -
4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[b*x^(2/3) + a*
x])
```

Maple [A]
time = 0.39, size = 143, normalized size = 0.43

method	result
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720a^{9}b^{2}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-2048}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720a^{9}b^{2}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-2048}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/29393*x*(b+a*x^(1/3))*(4199*a^11*x^(11/3)-4862*a^10*b*x^(10/3)+5720*a^9*b
^2*x^3-6864*a^8*b^3*x^(8/3)+8448*a^7*b^4*x^(7/3)-10752*a^6*b^5*x^2+14336*a^
5*b^6*x^(5/3)-20480*a^4*b^7*x^(4/3)+32768*a^3*b^8*x-65536*a^2*b^9*x^(2/3)+2
62144*a*b^10*x^(1/3)+524288*b^11)/(b*x^(2/3)+a*x)^(3/2)/a^12
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2566 vs. 2(252) = 504.
time = 197.19, size = 2566, normalized size = 7.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

[Out] -1/29393*((6442450944*a^3*b^19 + 5368709120*a^3*b^18 - 2013265920*a^3*b^17 - 6113744*a^18 + 402653184*(17*a^6 - 3*a^3)*b^16 + 8388608*(464*a^6 + 53*a^3)*b^15 - 12582912*(246*a^6 + a^3)*b^14 + 1572864*(1036*a^9 - 2560*a^6 - 3*a^3)*b^13 - 524288*(758*a^9 - 1569*a^6)*b^12 - 393216*(5803*a^9 + 124*a^6)*b^11 + 98304*(1315*a^12 - 20924*a^9 - 33*a^6)*b^10 - 57344*(2264*a^12 - 3153*a^9)*b^9 - 6144*(83789*a^12 + 2066*a^9)*b^8 - 1536*(46256*a^15 - 159272*a^12 - 267*a^9)*b^7 - 128*(264488*a^15 + 382229*a^12)*b^6 + 9984*(15547*a^15 + 482*a^12)*b^5 - 24*(2376192*a^18 + 4735792*a^15 + 7887*a^12)*b^4 - 1664*(107856*a^18 - 16759*a^15)*b^3 - 156*(935424*a^18 + 17935*a^15)*b^2 + 663*(97664*a^18 + 123*a^15)*b)*x^2 + (6442450944*b^22 + 5368709120*b^21 + 402653184*(17*a^3 - 3)*b^19 - 2013265920*b^20 + 8388608*(464*a^3 + 53)*b^18 - 6113744*a^15*b^3 - 12582912*(246*a^3 + 1)*b^17 + 1572864*(1036*a^6 - 2560*a^3 - 3)*b^16 - 524288*(758*a^6 - 1569*a^3)*b^15 - 393216*(5803*a^6 + 124*a^3)*b^14 + 98304*(1315*a^9 - 20924*a^6 - 33*a^3)*b^13 - 57344*(2264*a^9 - 3153*a^6)*b^12 - 6144*(83789*a^9 + 2066*a^6)*b^11 - 1536*(46256*a^12 - 159272*a^9 - 267*a^6)*b^10 - 128*(264488*a^12 + 382229*a^9)*b^9 + 9984*(15547*a^12 + 482*a^9)*b^8 - 24*(2376192*a^15 + 4735792*a^12 + 7887*a^9)*b^7 - 1664*(107856*a^15 - 16759*a^12)*b^6 - 156*(935424*a^15 + 17935*a^12)*b^5 + 663*(97664*a^15 + 123*a^12)*b^4)*x - 2*(4199*(4096*a^13*b^9 + 6144*a^13*b^8 + 768*a^13*b^7 - 4096*a^19 - 144*a^16*b^2 + 216*a^16*b - 27*a^16 + 256*(16*a^16 - 7*a^13)*b^6 + 48*(128*a^16 - 3*a^13)*b^5 + 24*(32*a^16 + 9*a^13)*b^4 - (5888*a^16 + 27*a^13)*b^3)*x^5 - 17446*(4096*a^10*b^12 + 6144*a^10*b^11 + 768*a^10*b^10 - 144*a^13*b^5 + 216*a^13*b^4 + 256*(16*a^13 - 7*a^10)*b^9 + 48*(128*a^13 - 3*a^10)*b^8 + 24*(32*a^13 + 9*a^10)*b^7 - (5888*a^13 + 27*a^10)*b^6 - (4096*a^16 + 27*a^13)*b^3)*x^4 + 33536*(4096*a^7*b^15 + 6144*a^7*b^14 + 768*a^7*b^13 - 144*a^10*b^8 + 216*a^10*b^7 + 256*(16*a^10 - 7*a^7)*b^12 + 48*(128*a^10 - 3*a^7)*b^11 + 24*(32*a^10 + 9*a^7)*b^10 - (5888*a^10 + 27*a^7)*b^9 - (4096*a^13 + 27*a^10)*b^6)*x^3 - 118784*(4096*a^4*b^18 + 6144*a^4*b^17 + 768*a^4*b^16 - 144*a^7*b^11 + 216*a^7*b^10 + 256*(16*a^7 - 7*a^4)*b^15 + 48*(128*a^7 - 3*a^4)*b^14 + 24*(32*a^7 + 9*a^4)*b^13 - (5888*a^7 + 27*a^4)*b^12 - (4096*a^10 + 27*a^7)*b^9)*x^2 - 262144*(4096*a*b^21 + 6144*a*b^20 + 768*a*b^19 + 256*(16*a^4 - 7*a)*b^18 - 144*a^4*b^14 + 48*(128*a^4 - 3*a)*b^17 + 216*a^4*b^13 + 24*(32*a^4 + 9*a)*b^16 - (5888*a^4 + 27*a)*b^15 - (4096*a^7 + 27*a^4)*b^12)*x + (2147483648*b^22 + 3221225472*b^21 + 134217728*(16*a^3 - 7)*b^19 + 402653184*b^20 + 25165824*(128*a^3 - 3)*b^18 - 75497472*a^3*b^15 + 12582912*(32*a^3 + 9)*b^17 + 113246208*a^3*b^14 - 524288*(5888*a^3 + 27)*b^16 - 524288*(4096*a^6 + 27*a^3)*b^13 - 9061*(4096*a^12*b^10 + 6144*a^12*b^9 + 768*a^12*b^8 - 144*a^15*b^3 + 216*a^15*b^2 + 256*(16*a^15 - 7*a^12)*b^7 + 48*(128*a^15 - 3*a^12)*b^6 + 24*(32*a^15 + 9*a^12)*b^5 - (5888*a^15 + 27*a^12)*b^4 - (4096*a^18 + 27*a^15)*b)*x^4 + 21032*(4096*a^9*b^13 + 6144*a^9*b^12 + 768*a^9*b^11 - 144*a^12*b^6 + 216*a^12*b^5 + 256*(16*a^12 - 7*a^9)*b^10 + 48*(128*a^12 - 3*a^9)*b^9 + 24*(32*a^12 + 9*a^9)*b^8 - (5888*a^12 + 27*a^9)*b^7 - (4096*a^15 + 27*a^12)*b^4)*x^3 - 45568*(4096*a^6*b^16 + 6144*a^6*b^15 + 768*a^6*b^14 - 144*a^9*b^9 + 216*a^9*b^8 + 256*(16*a^9 - 7*a^6)*b^13 + 48*(128*a^9 - 3*a^6)*b^12 + 24*(32*a^9 + 9*a^6)*b^11 -

```
(5888*a^9 + 27*a^6)*b^10 - (4096*a^12 + 27*a^9)*b^7)*x^2 + 360448*(4096*a^3
*b^19 + 6144*a^3*b^18 + 768*a^3*b^17 - 144*a^6*b^12 + 216*a^6*b^11 + 256*(1
6*a^6 - 7*a^3)*b^16 + 48*(128*a^6 - 3*a^3)*b^15 + 24*(32*a^6 + 9*a^3)*b^14
- (5888*a^6 + 27*a^3)*b^13 - (4096*a^9 + 27*a^6)*b^10)*x)*x^(2/3) + 3*(4927
*(4096*a^11*b^11 + 6144*a^11*b^10 + 768*a^11*b^9 - 144*a^14*b^4 + 216*a^14*
b^3 + 256*(16*a^14 - 7*a^11)*b^8 + 48*(128*a^14 - 3*a^11)*b^7 + 24*(32*a^14
 + 9*a^11)*b^6 - (5888*a^14 + 27*a^11)*b^5 - (4096*a^17 + 27*a^14)*b^2)*x^4
 - 8688*(4096*a^8*b^14 + 6144*a^8*b^13 + 768*a^8*b^12 - 144*a^11*b^7 + 216*
a^11*b^6 + 256*(16*a^11 - 7*a^8)*b^11 + 48*(128*a^11 - 3*a^8)*b^10 + 24*(32
*a^11 + 9*a^8)*b^9 - (5888*a^11 + 27*a^8)*b^8 - (4096*a^14 + 27*a^11)*b^5)*
x^3 + 22528*(4096*a^5*b^17 + 6144*a^5*b^16 + 768*a^5*b^15 - 144*a^8*b^10 +
216*a^8*b^9 + 256*(16*a^8 - 7*a^5)*b^14 + 48*(128*a^8 - 3*a^5)*b^13 + 24*(3
2*a^8 + 9*a^5)*b^12 - (5888*a^8 + 27*a^5)*b^11 - (4096*a^11 + 27*a^8)*b^8)*
x^2 + 65536*(4096*a^2*b^20 + 6144*a^2*b^19 + 768*a^2*b^18 - 144*a^5*b^13 +
256*(16*a^5 - 7*a^2)*b^17 + 216*a^5*b^12 + 48*(128*a^5 - 3*a^2)*b^16 + 24*(
32*a^5 + 9*a^2)*b^15 - (5888*a^5 + 27*a^2)*b^14 - (4096*a^8 + 27*a^5)*b^11)
*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((4096*a^15*b^9 + 6144*a^15*b^8 + 768*a
^15*b^7 - 4096*a^21 - 144*a^18*b^2 + 216*a^18*b - 27*a^18 + 256*(16*a^18 -
7*a^15)*b^6 + 48*(128*a^18 - 3*a^15)*b^5 + 24*(32*a^18 + 9*a^15)*b^4 - (588
8*a^18 + 27*a^15)*b^3)*x^2 + (4096*a^12*b^12 + 6144*a^12*b^11 + 768*a^12*b^
10 - 144*a^15*b^5 + 216*a^15*b^4 + 256*(16*a^15 - 7*a^12)*b^9 + 48*(128*a^1
5 - 3*a^12)*b^8 + 24*(32*a^15 + 9*a^12)*b^7 - (...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x**4/(a*x + b*x**(2/3))**(3/2), x)
```

Giac [A]
time = 2.09, size = 214, normalized size = 0.64

$$-\frac{1048576\, b^{\frac{21}{2}}}{29393\, a^{12}} + \frac{6\, b^{11}}{\sqrt{ax^{\frac{1}{3}} + b}\, a^{12}} + \frac{2\left(4199\left(ax^{\frac{1}{3}} + b\right)^{\frac{21}{2}} a^{240} - 51051\left(ax^{\frac{1}{3}} + b\right)^{\frac{19}{2}} a^{240} b + 285285\left(ax^{\frac{1}{3}} + b\right)^{\frac{17}{2}} a^{240} b^2 - 969969\left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}} a^{240} b^3 + 2238390\left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}} a^{240} b^4 - 3703518\left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} a^{240} b^5 + 4526522\left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} a^{240} b^6 - 4157010\left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} a^{240} b^7 + 2909907\left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} a^{240} b^8 - 1616615\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} a^{240} b^9 + 969969\sqrt{ax^{\frac{1}{3}} + b}\, a^{240} b^{10}\right)}{29393\, a^{252}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
[Out] -1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393*
(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b +
 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)*a^
```

```
240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3) + b
)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010*(a*x
^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8 - 161
6615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^240*b^1
0)/a^252
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

3.195 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8}-\frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{b}}{429a}$$

```
[Out] -6*x^3/a/(b*x^(2/3)+a*x)^(1/2)+32768/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^8-655
36/2145*b^7*(b*x^(2/3)+a*x)^(1/2)/a^9/x^(1/3)-8192/715*b^5*x^(1/3)*(b*x^(2/
3)+a*x)^(1/2)/a^7+4096/429*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-3584/429*b
^3*x*(b*x^(2/3)+a*x)^(1/2)/a^5+5376/715*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a
^4-448/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+32/5*x^2*(b*x^(2/3)+a*x)^(1/2
)/a^2
```

Rubi [A]
time = 0.27, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2040, 2041, 2027, 2039}

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{ax+bx^{2/3}}}{429a^5}+\frac{5376b^2x^{4/3}\sqrt{ax+bx^{2/3}}}{715a^4}-\frac{448bx^{5/3}\sqrt{ax+bx^{2/3}}}{65a^3}+\frac{32x^2\sqrt{ax+bx^{2/3}}}{5a^2}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (32768*b^6*Sqrt[b*x^(2/3) + a*x])/(214
5*a^8) - (65536*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^9*x^(1/3)) - (8192*b^5*x
^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^7) + (4096*b^4*x^(2/3)*Sqrt[b*x^(2/3)
+ a*x])/(429*a^6) - (3584*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^5) + (5376*b^
2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^4) - (448*b*x^(5/3)*Sqrt[b*x^(2/3)
+ a*x])/(65*a^3) + (32*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
```

```
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{16\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} - \frac{(224b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a^2}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} + \frac{(896b^2)\int \frac{x}{\sqrt{bx^{2/}}}}{65a^3}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2}}}{5a}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{}}{65}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}}{7}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - 3584b^3x$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7} + \frac{4096b^4x^{2/3}}{4}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8} - \frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}} - \frac{8192b^5\sqrt[3]{x}\sqrt{}}{715}$$

Mathematica [A]
time = 4.12, size = 122, normalized size = 0.49

$$\frac{2\left(-32768b^8\sqrt[3]{x} - 16384ab^7x^{2/3} + 4096a^2b^6x - 2048a^3b^5x^{4/3} + 1280a^4b^4x^{5/3} - 896a^5b^3x^2 + 672a^6b^2x^{7/3} - 528a^7bx^{8/3} + 429a^8x^3\right)}{2145a^9\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b^
5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3) -
528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])
```

Maple [A]
time = 0.38, size = 110, normalized size = 0.44

method	result
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{8}x^{\frac{8}{3}}-528a^{7}b\,x^{\frac{7}{3}}+672a^{6}b^{2}x^{2}-896a^{5}b^{3}x^{\frac{5}{3}}+1280a^{4}b^{4}x^{\frac{4}{3}}-2048a^{3}b^{5}x+4096a^{2}b^{6}x^{\frac{2}{3}}-16384x^{\frac{1}{3}}a\,b^{7}-\right.}{2145\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{9}}$
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(429a^{8}x^{\frac{8}{3}}-528a^{7}b\,x^{\frac{7}{3}}+672a^{6}b^{2}x^{2}-896a^{5}b^{3}x^{\frac{5}{3}}+1280a^{4}b^{4}x^{\frac{4}{3}}-2048a^{3}b^{5}x+4096a^{2}b^{6}x^{\frac{2}{3}}-16384x^{\frac{1}{3}}a\,b^{7}-\right.}{2145\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/2145*x*(b+a*x^(1/3))*(429*a^8*x^(8/3)-528*a^7*b*x^(7/3)+672*a^6*b^2*x^2-8
96*a^5*b^3*x^(5/3)+1280*a^4*b^4*x^(4/3)-2048*a^3*b^5*x+4096*a^2*b^6*x^(2/3)
-16384*x^(1/3)*a*b^7-32768*b^8)/(b*x^(2/3)+a*x)^(3/2)/a^9
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2083 vs. $2(186) = 372$.
time = 224.79, size = 2083, normalized size = 8.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2145*((402653184*a^3*b^16 + 335544320*a^3*b^15 - 125829120*a^3*b^14 + 624
624*a^15 + 25165824*(17*a^6 - 3*a^3)*b^13 + 524288*(464*a^6 + 53*a^3)*b^12
- 786432*(246*a^6 + a^3)*b^11 + 98304*(1036*a^9 - 2560*a^6 - 3*a^3)*b^10 -
32768*(758*a^9 - 1569*a^6)*b^9 - 24576*(5803*a^9 + 124*a^6)*b^8 + 6144*(600
*a^12 - 20924*a^9 - 33*a^6)*b^7 - 1536*(7666*a^12 - 7357*a^9)*b^6 - 768*(40
107*a^12 + 1033*a^9)*b^5 + 96*(63360*a^15 + 167852*a^12 + 267*a^9)*b^4 + 32
*(613440*a^15 - 105031*a^12)*b^3 + 468*(34560*a^15 + 661*a^12)*b^2 - 99*(68
480*a^15 + 87*a^12)*b)*x^2 + (402653184*b^19 + 335544320*b^18 + 25165824*(1
7*a^3 - 3)*b^16 - 125829120*b^17 + 524288*(464*a^3 + 53)*b^15 + 624624*a^12
*b^3 - 786432*(246*a^3 + 1)*b^14 + 98304*(1036*a^6 - 2560*a^3 - 3)*b^13 - 3
```

```
2768*(758*a^6 - 1569*a^3)*b^12 - 24576*(5803*a^6 + 124*a^3)*b^11 + 6144*(60
0*a^9 - 20924*a^6 - 33*a^3)*b^10 - 1536*(7666*a^9 - 7357*a^6)*b^9 - 768*(40
107*a^9 + 1033*a^6)*b^8 + 96*(63360*a^12 + 167852*a^9 + 267*a^6)*b^7 + 32*(
613440*a^12 - 105031*a^9)*b^6 + 468*(34560*a^12 + 661*a^9)*b^5 - 99*(68480*
a^12 + 87*a^9)*b^4)*x + 2*(429*(4096*a^10*b^9 + 6144*a^10*b^8 + 768*a^10*b^
7 - 4096*a^16 - 144*a^13*b^2 + 216*a^13*b - 27*a^13 + 256*(16*a^13 - 7*a^10
)*b^6 + 48*(128*a^13 - 3*a^10)*b^5 + 24*(32*a^13 + 9*a^10)*b^4 - (5888*a^13
 + 27*a^10)*b^3)*x^4 - 2096*(4096*a^7*b^12 + 6144*a^7*b^11 + 768*a^7*b^10 -
 144*a^10*b^5 + 216*a^10*b^4 + 256*(16*a^10 - 7*a^7)*b^9 + 48*(128*a^10 - 3
*a^7)*b^8 + 24*(32*a^10 + 9*a^7)*b^7 - (5888*a^10 + 27*a^7)*b^6 - (4096*a^1
3 + 27*a^10)*b^3)*x^3 + 7424*(4096*a^4*b^15 + 6144*a^4*b^14 + 768*a^4*b^13
- 144*a^7*b^8 + 216*a^7*b^7 + 256*(16*a^7 - 7*a^4)*b^12 + 48*(128*a^7 - 3*a
^4)*b^11 + 24*(32*a^7 + 9*a^4)*b^10 - (5888*a^7 + 27*a^4)*b^9 - (4096*a^10
+ 27*a^7)*b^6)*x^2 + 16384*(4096*a*b^18 + 6144*a*b^17 + 768*a*b^16 + 256*(1
6*a^4 - 7*a)*b^15 - 144*a^4*b^11 + 48*(128*a^4 - 3*a)*b^14 + 216*a^4*b^10 +
 24*(32*a^4 + 9*a)*b^13 - (5888*a^4 + 27*a)*b^12 - (4096*a^7 + 27*a^4)*b^9)
*x - (134217728*b^19 + 201326592*b^18 + 8388608*(16*a^3 - 7)*b^16 + 2516582
4*b^17 + 1572864*(128*a^3 - 3)*b^15 - 4718592*a^3*b^12 + 786432*(32*a^3 + 9
)*b^14 + 7077888*a^3*b^11 - 32768*(5888*a^3 + 27)*b^13 - 32768*(4096*a^6 +
27*a^3)*b^10 + 957*(4096*a^9*b^10 + 6144*a^9*b^9 + 768*a^9*b^8 - 144*a^12*b
^3 + 216*a^12*b^2 + 256*(16*a^12 - 7*a^9)*b^7 + 48*(128*a^12 - 3*a^9)*b^6 +
 24*(32*a^12 + 9*a^9)*b^5 - (5888*a^12 + 27*a^9)*b^4 - (4096*a^15 + 27*a^12
)*b)*x^3 - 2848*(4096*a^6*b^13 + 6144*a^6*b^12 + 768*a^6*b^11 - 144*a^9*b^6
 + 216*a^9*b^5 + 256*(16*a^9 - 7*a^6)*b^10 + 48*(128*a^9 - 3*a^6)*b^9 + 24*
(32*a^9 + 9*a^6)*b^8 - (5888*a^9 + 27*a^6)*b^7 - (4096*a^12 + 27*a^9)*b^4)*
x^2 + 22528*(4096*a^3*b^16 + 6144*a^3*b^15 + 768*a^3*b^14 - 144*a^6*b^9 + 2
16*a^6*b^8 + 256*(16*a^6 - 7*a^3)*b^13 + 48*(128*a^6 - 3*a^3)*b^12 + 24*(32
*a^6 + 9*a^3)*b^11 - (5888*a^6 + 27*a^3)*b^10 - (4096*a^9 + 27*a^6)*b^7)*x)
*x^(2/3) + 3*(543*(4096*a^8*b^11 + 6144*a^8*b^10 + 768*a^8*b^9 - 144*a^11*b
^4 + 216*a^11*b^3 + 256*(16*a^11 - 7*a^8)*b^8 + 48*(128*a^11 - 3*a^8)*b^7 +
 24*(32*a^11 + 9*a^8)*b^6 - (5888*a^11 + 27*a^8)*b^5 - (4096*a^14 + 27*a^11
)*b^2)*x^3 - 1408*(4096*a^5*b^14 + 6144*a^5*b^13 + 768*a^5*b^12 - 144*a^8*b
^7 + 216*a^8*b^6 + 256*(16*a^8 - 7*a^5)*b^11 + 48*(128*a^8 - 3*a^5)*b^10 +
24*(32*a^8 + 9*a^5)*b^9 - (5888*a^8 + 27*a^5)*b^8 - (4096*a^11 + 27*a^8)*b^
5)*x^2 - 4096*(4096*a^2*b^17 + 6144*a^2*b^16 + 768*a^2*b^15 - 144*a^5*b^10
+ 256*(16*a^5 - 7*a^2)*b^14 + 216*a^5*b^9 + 48*(128*a^5 - 3*a^2)*b^13 + 24*
(32*a^5 + 9*a^2)*b^12 - (5888*a^5 + 27*a^2)*b^11 - (4096*a^8 + 27*a^5)*b^8)
*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((4096*a^12*b^9 + 6144*a^12*b^8 + 768*a
^12*b^7 - 4096*a^18 - 144*a^15*b^2 + 216*a^15*b - 27*a^15 + 256*(16*a^15 -
7*a^12)*b^6 + 48*(128*a^15 - 3*a^12)*b^5 + 24*(32*a^15 + 9*a^12)*b^4 - (588
8*a^15 + 27*a^12)*b^3)*x^2 + (4096*a^9*b^12 + 6144*a^9*b^11 + 768*a^9*b^10
- 144*a^12*b^5 + 216*a^12*b^4 + 256*(16*a^12 - 7*a^9)*b^9 + 48*(128*a^12 -
3*a^9)*b^8 + 24*(32*a^12 + 9*a^9)*b^7 - (5888*a^12 + 27*a^9)*b^6 - (4096*a^
15 + 27*a^12)*b^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)
```

Giac [A]
time = 1.90, size = 163, normalized size = 0.66

$$\frac{65536\,b^{\frac{15}{2}}}{2145\,a^9}-\frac{6\,b^8}{\sqrt{ax^{\frac{1}{3}}+b}\,a^9}+\frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{126}-3960\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{126}b+16380\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{126}b^2-40040\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{126}b^3+64350\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{126}b^4-72072\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{126}b^5+60060\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{126}b^6-51480\sqrt{ax^{\frac{1}{3}}+b}\,a^{126}b^7\right)}{2145\,a^{135}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 65536/2145*b^(15/2)/a^9 - 6*b^8/(sqrt(a*x^(1/3) + b)*a^9) + 2/2145*(429*(a*
x^(1/3) + b)^(15/2)*a^126 - 3960*(a*x^(1/3) + b)^(13/2)*a^126*b + 16380*(a*
x^(1/3) + b)^(11/2)*a^126*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^126*b^3 + 643
50*(a*x^(1/3) + b)^(7/2)*a^126*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^126*b^5
+ 60060*(a*x^(1/3) + b)^(3/2)*a^126*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^126*b
^7)/a^135
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

3.196 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}-\frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5}+\frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{7a^4}-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}$$

```
[Out] -6*x^2/a/(b*x^(2/3)+a*x)^(1/2)-256/21*b^3*(b*x^(2/3)+a*x)^(1/2)/a^5+512/21*
b^4*(b*x^(2/3)+a*x)^(1/2)/a^6/x^(1/3)+64/7*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2
)/a^4-160/21*b*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+20/3*x*(b*x^(2/3)+a*x)^(1/
2)/a^2
```

Rubi [A]
time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2040, 2041, 2027, 2039}

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}}-\frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5}+\frac{64b^2\sqrt[3]{x}\,\sqrt{ax+bx^{2/3}}}{7a^4}-\frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3}+\frac{20x\sqrt{ax+bx^{2/3}}}{3a^2}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) - (256*b^3*Sqrt[b*x^(2/3) + a*x])/(21*a^
5) + (512*b^4*Sqrt[b*x^(2/3) + a*x])/(21*a^6*x^(1/3)) + (64*b^2*x^(1/3)*Sqr
t[b*x^(2/3) + a*x])/(7*a^4) - (160*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^3
) + (20*x*Sqrt[b*x^(2/3) + a*x])/(3*a^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{10\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} - \frac{(80b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a^2}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} + \frac{(160b^2)\int \frac{\sqrt{}}{\sqrt{bx^2}}}{21a^3}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{64b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}}}{3a^2}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{64b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx}}{21a^3}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}} + \frac{64b^2\sqrt[3]{x}\,\sqrt{bx^{2/3}}}{7a^4}$$

Mathematica [A]
time = 4.37, size = 85, normalized size = 0.53

$$\frac{512b^5\sqrt[3]{x}+256ab^4x^{2/3}-64a^2b^3x+32a^3b^2x^{4/3}-20a^4bx^{5/3}+14a^5x^2}{21a^6\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) -
20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*Sqrt[b*x^(2/3) + a*x])
```

Maple [A]
time = 0.36, size = 77, normalized size = 0.48

method	result	size
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(7a^{5}x^{\frac{5}{3}}-10a^{4}b\,x^{\frac{4}{3}}+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128a\,b^{4}x^{\frac{1}{3}}+256b^{5}\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{6}}$	77
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(7a^{5}x^{\frac{5}{3}}-10a^{4}b\,x^{\frac{4}{3}}+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128a\,b^{4}x^{\frac{1}{3}}+256b^{5}\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{6}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*x*(b+a*x^(1/3))*(7*a^5*x^(5/3)-10*a^4*b*x^(4/3)+16*a^3*b^2*x-32*a^2*b^
3*x^(2/3)+128*a*b^4*x^(1/3)+256*b^5)/(b*x^(2/3)+a*x)^(3/2)/a^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. 2(120) = 240.
time = 192.10, size = 1598, normalized size = 9.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/21*((3145728*a^3*b^13 + 2621440*a^3*b^12 - 983040*a^3*b^11 - 10192*a^12
+ 196608*(17*a^6 - 3*a^3)*b^10 + 4096*(464*a^6 + 53*a^3)*b^9 - 6144*(246*a^
6 + a^3)*b^8 + 768*(1120*a^9 - 2560*a^6 - 3*a^3)*b^7 - 256*(548*a^9 - 1569*
a^6)*b^6 - 768*(1477*a^9 + 31*a^6)*b^5 - 48*(2304*a^12 + 21176*a^9 + 33*a^6
```

```
)*b^4 - 4032*(96*a^12 - 23*a^9)*b^3 - 12*(27648*a^12 + 527*a^9)*b^2 + 3*(39
296*a^12 + 51*a^9)*b)*x^2 + (3145728*b^16 + 2621440*b^15 + 196608*(17*a^3 -
 3)*b^13 - 983040*b^14 + 4096*(464*a^3 + 53)*b^12 - 10192*a^9*b^3 - 6144*(2
46*a^3 + 1)*b^11 + 768*(1120*a^6 - 2560*a^3 - 3)*b^10 - 256*(548*a^6 - 1569
*a^3)*b^9 - 768*(1477*a^6 + 31*a^3)*b^8 - 48*(2304*a^9 + 21176*a^6 + 33*a^3
)*b^7 - 4032*(96*a^9 - 23*a^6)*b^6 - 12*(27648*a^9 + 527*a^6)*b^5 + 3*(3929
6*a^9 + 51*a^6)*b^4)*x - 2*(7*(4096*a^7*b^9 + 6144*a^7*b^8 + 768*a^7*b^7 -
4096*a^13 - 144*a^10*b^2 + 216*a^10*b - 27*a^10 + 256*(16*a^10 - 7*a^7)*b^6
 + 48*(128*a^10 - 3*a^7)*b^5 + 24*(32*a^10 + 9*a^7)*b^4 - (5888*a^10 + 27*a
^7)*b^3)*x^3 - 58*(4096*a^4*b^12 + 6144*a^4*b^11 + 768*a^4*b^10 - 144*a^7*b
^5 + 216*a^7*b^4 + 256*(16*a^7 - 7*a^4)*b^9 + 48*(128*a^7 - 3*a^4)*b^8 + 24
*(32*a^7 + 9*a^4)*b^7 - (5888*a^7 + 27*a^4)*b^6 - (4096*a^10 + 27*a^7)*b^3)
*x^2 - 128*(4096*a*b^15 + 6144*a*b^14 + 768*a*b^13 + 256*(16*a^4 - 7*a)*b^1
2 - 144*a^4*b^8 + 48*(128*a^4 - 3*a)*b^11 + 216*a^4*b^7 + 24*(32*a^4 + 9*a)
*b^10 - (5888*a^4 + 27*a)*b^9 - (4096*a^7 + 27*a^4)*b^6)*x + (1048576*b^16
+ 1572864*b^15 + 65536*(16*a^3 - 7)*b^13 + 196608*b^14 + 12288*(128*a^3 - 3
)*b^12 - 36864*a^3*b^9 + 6144*(32*a^3 + 9)*b^11 + 55296*a^3*b^8 - 256*(5888
*a^3 + 27)*b^10 - 256*(4096*a^6 + 27*a^3)*b^7 - 17*(4096*a^6*b^10 + 6144*a^
6*b^9 + 768*a^6*b^8 - 144*a^9*b^3 + 216*a^9*b^2 + 256*(16*a^9 - 7*a^6)*b^7
+ 48*(128*a^9 - 3*a^6)*b^6 + 24*(32*a^9 + 9*a^6)*b^5 - (5888*a^9 + 27*a^6)*
b^4 - (4096*a^12 + 27*a^9)*b)*x^2 + 176*(4096*a^3*b^13 + 6144*a^3*b^12 + 76
8*a^3*b^11 - 144*a^6*b^6 + 216*a^6*b^5 + 256*(16*a^6 - 7*a^3)*b^10 + 48*(12
8*a^6 - 3*a^3)*b^9 + 24*(32*a^6 + 9*a^3)*b^8 - (5888*a^6 + 27*a^3)*b^7 - (4
096*a^9 + 27*a^6)*b^4)*x)*x^(2/3) + 3*(11*(4096*a^5*b^11 + 6144*a^5*b^10 +
768*a^5*b^9 - 144*a^8*b^4 + 216*a^8*b^3 + 256*(16*a^8 - 7*a^5)*b^8 + 48*(12
8*a^8 - 3*a^5)*b^7 + 24*(32*a^8 + 9*a^5)*b^6 - (5888*a^8 + 27*a^5)*b^5 - (4
096*a^11 + 27*a^8)*b^2)*x^2 + 32*(4096*a^2*b^14 + 6144*a^2*b^13 + 768*a^2*b
^12 - 144*a^5*b^7 + 256*(16*a^5 - 7*a^2)*b^11 + 216*a^5*b^6 + 48*(128*a^5 -
 3*a^2)*b^10 + 24*(32*a^5 + 9*a^2)*b^9 - (5888*a^5 + 27*a^2)*b^8 - (4096*a^
8 + 27*a^5)*b^5)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((4096*a^9*b^9 + 6144*a
^9*b^8 + 768*a^9*b^7 - 4096*a^15 - 144*a^12*b^2 + 216*a^12*b - 27*a^12 + 25
6*(16*a^12 - 7*a^9)*b^6 + 48*(128*a^12 - 3*a^9)*b^5 + 24*(32*a^12 + 9*a^9)*
b^4 - (5888*a^12 + 27*a^9)*b^3)*x^2 + (4096*a^6*b^12 + 6144*a^6*b^11 + 768*
a^6*b^10 - 144*a^9*b^5 + 216*a^9*b^4 + 256*(16*a^9 - 7*a^6)*b^9 + 48*(128*a
^9 - 3*a^6)*b^8 + 24*(32*a^9 + 9*a^6)*b^7 - (5888*a^9 + 27*a^6)*b^6 - (4096
*a^12 + 27*a^9)*b^3)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)
```

Giac [A]
time = 1.94, size = 112, normalized size = 0.70

$$-\frac{512\,b^{\frac{9}{2}}}{21\,a^{6}}+\frac{6\,b^{5}}{\sqrt{ax^{\frac{1}{3}}+b}\;a^{6}}+\frac{2\left(7\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{48}-45\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{48}b+126\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{48}b^{2}-210\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{48}b^{3}+315\sqrt{ax^{\frac{1}{3}}+b}\;a^{48}b^{4}\right)}{21\,a^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3)
+ b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(5/
2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) + b)*
a^48*b^4)/a^54
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

3.197 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}$$

```
[Out] -6*x/a/(b*x^(2/3)+a*x)^(1/2)+8*(b*x^(2/3)+a*x)^(1/2)/a^2-16*b*(b*x^(2/3)+a*
x)^(1/2)/a^3/x^(1/3)
```

Rubi [A]

time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2040, 2027, 2039}

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (8*Sqrt[b*x^(2/3) + a*x])/a^2 - (16*b*Sq
rt[b*x^(2/3) + a*x])/(a^3*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
```

```
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{4\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\ &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{(8b)\int \frac{1}{\sqrt[3]{x}\,\sqrt{bx^{2/3}+ax}}\,dx}{3a^2}\\ &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}\end{aligned}$$

Mathematica [A]
time = 4.13, size = 45, normalized size = 0.66

$$\frac{2\left(-8b^2\sqrt[3]{x}-4abx^{2/3}+a^2x\right)}{a^3\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^(2/3) + a*x)^(3/2),x]
[Out] (2*(-8*b^2*x^(1/3) - 4*a*b*x^(2/3) + a^2*x))/(a^3*Sqrt[b*x^(2/3) + a*x])
```

Maple [A]
time = 0.35, size = 45, normalized size = 0.66

method	result	size
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(a^2x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^2\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^3}$	45
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(a^2x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^2\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^3}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*x*(b+a*x^(1/3))*(a^2*x^(2/3)-4*a*b*x^(1/3)-8*b^2)/(b*x^(2/3)+a*x)^(3/2)/a
^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*x^(2/3))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(54) = 108.
time = 214.17, size = 1107, normalized size = 16.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] ((98304*a^3*b^10 + 81920*a^3*b^9 - 30720*a^3*b^8 + 1456*a^9 + 6144*(16*a^6
- 3*a^3)*b^7 + 6784*(8*a^6 + a^3)*b^6 - 192*(236*a^6 + a^3)*b^5 + 24*(1536*
a^9 - 2512*a^6 - 3*a^3)*b^4 + 32*(576*a^9 + 379*a^6)*b^3 - 12*(2304*a^9 + 6
1*a^6)*b^2 - 3*(10112*a^9 + 15*a^6)*b)*x^2 + (98304*b^13 + 81920*b^12 + 614
4*(16*a^3 - 3)*b^10 - 30720*b^11 + 6784*(8*a^3 + 1)*b^9 + 1456*a^6*b^3 - 19
2*(236*a^3 + 1)*b^8 + 24*(1536*a^6 - 2512*a^3 - 3)*b^7 + 32*(576*a^6 + 379*
a^3)*b^6 - 12*(2304*a^6 + 61*a^3)*b^5 - 3*(10112*a^6 + 15*a^3)*b^4)*x + 2*(
(4096*a^4*b^9 + 6144*a^4*b^8 + 768*a^4*b^7 - 4096*a^10 - 144*a^7*b^2 + 216*
a^7*b - 27*a^7 + 256*(16*a^7 - 7*a^4)*b^6 + 48*(128*a^7 - 3*a^4)*b^5 + 24*(
32*a^7 + 9*a^4)*b^4 - (5888*a^7 + 27*a^4)*b^3)*x^2 - 3*(4096*a^2*b^11 + 614
4*a^2*b^10 + 768*a^2*b^9 - 144*a^5*b^4 + 256*(16*a^5 - 7*a^2)*b^8 + 216*a^5
*b^3 + 48*(128*a^5 - 3*a^2)*b^7 + 24*(32*a^5 + 9*a^2)*b^6 - (5888*a^5 + 27*
a^2)*b^5 - (4096*a^8 + 27*a^5)*b^2)*x^(4/3) + 4*(4096*a*b^12 + 6144*a*b^11
+ 768*a*b^10 + 256*(16*a^4 - 7*a)*b^9 - 144*a^4*b^5 + 48*(128*a^4 - 3*a)*b^
8 + 216*a^4*b^4 + 24*(32*a^4 + 9*a)*b^7 - (5888*a^4 + 27*a)*b^6 - (4096*a^7
 + 27*a^4)*b^3)*x - (32768*b^13 + 49152*b^12 + 2048*(16*a^3 - 7)*b^10 + 614
4*b^11 + 384*(128*a^3 - 3)*b^9 - 1152*a^3*b^6 + 192*(32*a^3 + 9)*b^8 + 1728
*a^3*b^5 - 8*(5888*a^3 + 27)*b^7 - 8*(4096*a^6 + 27*a^3)*b^4 + 5*(4096*a^3*
b^10 + 6144*a^3*b^9 + 768*a^3*b^8 - 144*a^6*b^3 + 216*a^6*b^2 + 256*(16*a^6
 - 7*a^3)*b^7 + 48*(128*a^6 - 3*a^3)*b^6 + 24*(32*a^6 + 9*a^3)*b^5 - (5888*
a^6 + 27*a^3)*b^4 - (4096*a^9 + 27*a^6)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)
))/((4096*a^6*b^9 + 6144*a^6*b^8 + 768*a^6*b^7 - 4096*a^12 - 144*a^9*b^2 +
216*a^9*b - 27*a^9 + 256*(16*a^9 - 7*a^6)*b^6 + 48*(128*a^9 - 3*a^6)*b^5 +
24*(32*a^9 + 9*a^6)*b^4 - (5888*a^9 + 27*a^6)*b^3)*x^2 + (4096*a^3*b^12 + 6
144*a^3*b^11 + 768*a^3*b^10 - 144*a^6*b^5 + 216*a^6*b^4 + 256*(16*a^6 - 7*a
^3)*b^9 + 48*(128*a^6 - 3*a^3)*b^8 + 24*(32*a^6 + 9*a^3)*b^7 - (5888*a^6 +
27*a^3)*b^6 - (4096*a^9 + 27*a^6)*b^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*x**(2/3))**(3/2), x)
```

Giac [A]
time = 1.63, size = 60, normalized size = 0.88

$$\frac{16\, b^{\frac{3}{2}}}{a^3} - \frac{6\, b^2}{\sqrt{ax^{\frac{1}{3}} + b}\; a^3} + \frac{2\left(\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} a^6 - 6\sqrt{ax^{\frac{1}{3}} + b}\; a^6 b\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 16*b^(3/2)/a^3 - 6*b^2/(sqrt(a*x^(1/3) + b)*a^3) + 2*((a*x^(1/3) + b)^(3/2)
*a^6 - 6*sqrt(a*x^(1/3) + b)*a^6*b)/a^9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a\, x + b\, x^{2/3}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x/(a*x + b*x^(2/3))^(3/2), x)
```

3.198 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

```
[Out] -6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)+6*x^(1/3)/b/(b*x^
(2/3)+a*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2031, 2054, 212}

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}} + \frac{\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}} - \frac{6\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}$$

$$= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}} - \frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 1.55, size = 71, normalized size = 1.18

$$\frac{6\sqrt{bx^{2/3}+ax}}{b\left(b+a\sqrt[3]{x}\right)\sqrt[3]{x}} - \frac{6\tanh^{-1}\left(\frac{\sqrt{bx^{2/3}+ax}}{\sqrt{b}\sqrt[3]{x}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*Sqrt[b*x^(2/3) + a*x])/(b*(b + a*x^(1/3))*x^(1/3)) - (6*ArcTanh[Sqrt[b*x
^(2/3) + a*x]/(Sqrt[b]*x^(1/3))])/b^(3/2)
```

Maple [A]
time = 0.34, size = 56, normalized size = 0.93

method	result	size
derivativedivides	$-\frac{6x\left(b+a\,x^{\frac{1}{3}}\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\sqrt{b+a\,x^{\frac{1}{3}}}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56
default	$-\frac{6x\left(b+a\,x^{\frac{1}{3}}\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\sqrt{b+a\,x^{\frac{1}{3}}}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*x*(b+a*x^(1/3))*(arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*(b+a*x^(1/3))^(1
/2)-b^(3/2))/(b*x^(2/3)+a*x)^(3/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(-3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral((a*x + b*x**(2/3))**(-3/2), x)
```

Giac [A]
time = 1.84, size = 71, normalized size = 1.18

$$\frac{6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}} + b}}{\sqrt{-b}}\right)}{\sqrt{-b}\, b} - \frac{6\left(\sqrt{b} \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + \sqrt{-b}\right)}{\sqrt{-b}\, b^{\frac{3}{2}}} + \frac{6}{\sqrt{ax^{\frac{1}{3}} + b}\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sqr
t(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)
```

Mupad [B]
time = 5.36, size = 40, normalized size = 0.67

$$-\frac{2\,x\,{\left(\frac{b}{a\,x^{1/3}}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{3}{2};\ \frac{5}{2};\ -\frac{b}{a\,x^{1/3}}\right)}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] -(2*x*(b/(a*x^(1/3)) + 1)^(3/2)*hypergeom([3/2, 3/2], 5/2, -b/(a*x^(1/3))))
/(a*x + b*x^(2/3))^(3/2)
```

3.199 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}+\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+a}}\right)}{8b^{9/2}}$$

```
[Out] 105/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)+6/b/x^(2/3
)/(b*x^(2/3)+a*x)^(1/2)-7*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)+35/4*a*(b*x^(2/
3)+a*x)^(1/2)/b^3/x-105/8*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A]
time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2048, 2050, 2054, 212}

$$\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}}-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}}+\frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x}-\frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3
)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x
])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/(8*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}+\frac{7\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}-\frac{(35a)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{6b^2}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}+\frac{(35a^2)\int \frac{1}{x\sqrt{bx^{2/3}+a}}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}$$

Mathematica [A]
time = 4.21, size = 110, normalized size = 0.75

$$\frac{-\sqrt{b}\left(8b^3-14ab^2\sqrt[3]{x}+35a^2bx^{2/3}+105a^3x\right)+105a^3\sqrt{b+a\sqrt[3]{x}}\,x\tanh^{-1}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)}{8b^{9/2}x^{2/3}\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-(Sqrt[b]*(8*b^3 - 14*a*b^2*x^(1/3) + 35*a^2*b*x^(2/3) + 105*a^3*x)) + 105
*a^3*Sqrt[b + a*x^(1/3)]*x*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/(8*b^(9/2)
*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
```

Maple [A]
time = 0.35, size = 88, normalized size = 0.60

method	result	s
derivativedivides	$\frac{\left(b+a x^{\frac{1}{3}}\right)\left(105 \sqrt{b+a x^{\frac{1}{3}}}\, \operatorname{arctanh}\left(\frac{\sqrt{b+a x^{\frac{1}{3}}}}{\sqrt{b}}\right) a^{3} x+14 b^{\frac{5}{2}} a x^{\frac{1}{3}}-35 b^{\frac{3}{2}} a^{2} x^{\frac{2}{3}}-105 a^{3} x \sqrt{b}-8 b^{\frac{7}{2}}\right)}{8\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{9}{2}}}$	8
default	$-\frac{\left(b+a x^{\frac{1}{3}}\right)\left(105 a^{3} x \sqrt{b}+35 b^{\frac{3}{2}} a^{2} x^{\frac{2}{3}}-14 b^{\frac{5}{2}} a x^{\frac{1}{3}}-105 \sqrt{b+a x^{\frac{1}{3}}}\, \operatorname{arctanh}\left(\frac{\sqrt{b+a x^{\frac{1}{3}}}}{\sqrt{b}}\right) a^{3} x+8 b^{\frac{7}{2}}\right)}{8\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{9}{2}}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(b+a*x^(1/3))*(105*a^3*x*b^(1/2)+35*b^(3/2)*a^2*x^(2/3)-14*b^(5/2)*a*x
^(1/3)-105*(b+a*x^(1/3))^(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^3*x+8
*b^(7/2))/(b*x^(2/3)+a*x)^(3/2)/b^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)
```

Giac [A]
time = 1.26, size = 105, normalized size = 0.72

$$-\frac{105\,a^{3}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}\,b^{4}}-\frac{6\,a^{3}}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{4}}-\frac{57\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{3}-136\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{3}b+87\,\sqrt{ax^{\frac{1}{3}}+b}\,a^{3}b^{2}}{8\,a^{3}b^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sqr
t(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3)
+ b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(2/3))^(3/2)), x)
```

3.200 $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}-$$

```
[Out] -9009/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)+6/b/x
^(5/3)/(b*x^(2/3)+a*x)^(1/2)-13/2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)+143/20*
a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2-1287/160*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5
/3)+3003/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)-3003/256*a^4*(b*x^(2/3)+
a*x)^(1/2)/b^6/x+9009/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A]

time = 0.28, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2048, 2050, 2054, 212}

$$-\frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}}+\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}}-\frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}}+\frac{143a\sqrt{ax+bx^{2/3}}}{20b^3x^2}-\frac{13\sqrt{ax+bx^{2/3}}}{2b^2x^{7/3}}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) - (13*Sqrt[b*x^(2/3) + a*x])/(2*b^2*x^(
7/3)) + (143*a*Sqrt[b*x^(2/3) + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^(2/
3) + a*x])/(160*b^4*x^(5/3)) + (3003*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^5*x^
(4/3)) - (3003*a^4*Sqrt[b*x^(2/3) + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^
(2/3) + a*x])/(512*b^7*x^(2/3)) - (9009*a^6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/(512*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
 !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
```

```
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^2 (bx^{2/3}+ax)^{3/2}}\,dx = \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} + \frac{13\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} - \frac{(143a)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{12b^2}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} + \frac{(429a^2)\int \frac{1}{x^2\sqrt{bx}}}{40b}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+}}{160b^4x^{5/3}}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+}}{160b^4x^{5/3}}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+}}{160b^4x^{5/3}}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+}}{160b^4x^{5/3}}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+}}{160b^4x^{5/3}}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} - \frac{1287a^2\sqrt{bx^{2/3}+}}{160b^4x^{5/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.08, size = 48, normalized size = 0.20

$$\frac{6a^6\sqrt[3]{x}\ {}_2F_1\left(-\frac{1}{2},7;\frac{1}{2};1+\frac{a\sqrt[3]{x}}{b}\right)}{b^7\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
[Out] (6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sqr
t[b*x^(2/3) + a*x])
```

Maple [A]
time = 0.35, size = 126, normalized size = 0.53

method	result
derivativedivides	$-\frac{\left(b+a x^{\frac{1}{3}}\right)\left(2288 b^{\frac{9}{2}} a^{2} x^{\frac{2}{3}}-3432 b^{\frac{7}{2}} a^{3} x+6006 b^{\frac{5}{2}} a^{4} x^{\frac{4}{3}}-15015 b^{\frac{3}{2}} a^{5} x^{\frac{5}{3}}-45045 a^{6} x^{2} \sqrt{b}-1664 b^{\frac{11}{2}} a x^{\frac{1}{3}}+1280 b^{\frac{13}{2}}+450\right.}{2560 x \left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{15}{2}}}$
default	$\frac{\left(b+a x^{\frac{1}{3}}\right)\left(-2288 b^{\frac{9}{2}} a^{2} x^{\frac{2}{3}}+3432 b^{\frac{7}{2}} a^{3} x-6006 b^{\frac{5}{2}} a^{4} x^{\frac{4}{3}}+15015 b^{\frac{3}{2}} a^{5} x^{\frac{5}{3}}+45045 a^{6} x^{2} \sqrt{b}+1664 b^{\frac{11}{2}} a x^{\frac{1}{3}}-1280 b^{\frac{13}{2}}-450\right.}{2560 x \left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} b^{\frac{15}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2560*(b+a*x^(1/3))*(-2288*b^(9/2)*a^2*x^(2/3)+3432*b^(7/2)*a^3*x-6006*b^(
5/2)*a^4*x^(4/3)+15015*b^(3/2)*a^5*x^(5/3)+45045*a^6*x^2*b^(1/2)+1664*b^(11
/2)*a*x^(1/3)-1280*b^(13/2)-45045*(b+a*x^(1/3))^(1/2)*arctanh((b+a*x^(1/3))
^(1/2)/b^(1/2))*a^6*x^2)/x/(b*x^(2/3)+a*x)^(3/2)/b^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)
```

Giac [A]
time = 1.45, size = 156, normalized size = 0.66

$$\frac{9009\,a^6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{512\,\sqrt{-b}\,b^7}+\frac{6\,a^6}{\sqrt{ax^{\frac{1}{3}}+b}\,b^7}+\frac{29685\,\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^6-163095\,\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6b+364194\,\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b^2-416094\,\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^3+246505\,\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6b^4-62475\,\sqrt{ax^{\frac{1}{3}}+b}\,a^6b^5}{2560\,a^6b^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(s
qrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 163095
*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 41609
4*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4 - 62
475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)), x)
```

3.201 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=324

$$\frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}}-$$

```
[Out] 692835/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)+6/
b/x^(8/3)/(b*x^(2/3)+a*x)^(1/2)-19/3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)+323
/48*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3-1615/224*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/
x^(8/3)+20995/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)-46189/5376*a^4*(b*
x^(2/3)+a*x)^(1/2)/b^6/x^2+138567/14336*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/
3)-46189/4096*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)+230945/16384*a^7*(b*x^(
2/3)+a*x)^(1/2)/b^9/x-692835/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A]
time = 0.39, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2048, 2050, 2054, 212}

$$\frac{692835a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{21/2}}-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}}+\frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x}-\frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}}+\frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}}-\frac{46189a^4\sqrt{ax+bx^{2/3}}}{5376b^6x^2}+\frac{20995a^3\sqrt{ax+bx^{2/3}}}{2688b^5x^{7/3}}-\frac{1615a^2\sqrt{ax+bx^{2/3}}}{224b^4x^{8/3}}+\frac{323a\sqrt{ax+bx^{2/3}}}{48b^3x^3}-\frac{19\sqrt{ax+bx^{2/3}}}{3b^2x^{10/3}}+\frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) - (19*Sqrt[b*x^(2/3) + a*x])/(3*b^2*x^(
10/3)) + (323*a*Sqrt[b*x^(2/3) + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^(2
/3) + a*x])/(224*b^4*x^(8/3)) + (20995*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^5
*x^(7/3)) - (46189*a^4*Sqrt[b*x^(2/3) + a*x])/(5376*b^6*x^2) + (138567*a^5*
Sqrt[b*x^(2/3) + a*x])/(14336*b^7*x^(5/3)) - (46189*a^6*Sqrt[b*x^(2/3) + a*
x])/(4096*b^8*x^(4/3)) + (230945*a^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^9*x) -
 (692835*a^8*Sqrt[b*x^(2/3) + a*x])/(32768*b^10*x^(2/3)) + (692835*a^9*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(21/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
```

```
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} + \frac{19\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} - \frac{(323a)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{18b^2}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} + \frac{(1615a^2)\int \frac{}{x^3\sqrt{bx}}}{96b^3}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+}}{224b^4x^{8/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.07, size = 48, normalized size = 0.15

$$-\frac{6a^9\sqrt[3]{x}\ {}_2F_1\left(-\frac{1}{2},10;\frac{1}{2};1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{10}\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^9*x^(1/3)*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^(1/3))/b])/(b^10*
Sqrt[b*x^(2/3) + a*x])
```

Maple [A]
time = 0.33, size = 159, normalized size = 0.49

method	result
derivativedivides	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(413440b^{\frac{13}{2}}a^{3}x-537472b^{\frac{11}{2}}a^{4}x^{\frac{4}{3}}+739024b^{\frac{9}{2}}a^{5}x^{\frac{5}{3}}-1108536b^{\frac{7}{2}}a^{6}x^{2}+1939938b^{\frac{5}{2}}a^{7}x^{\frac{7}{3}}-4849845b^{\frac{3}{2}}a^{8}x^{\frac{8}{3}}+\right.}{688128}$
default	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-413440b^{\frac{13}{2}}a^{3}x+229376b^{\frac{19}{2}}+537472b^{\frac{11}{2}}a^{4}x^{\frac{4}{3}}-739024b^{\frac{9}{2}}a^{5}x^{\frac{5}{3}}+1108536b^{\frac{7}{2}}a^{6}x^{2}-1939938b^{\frac{5}{2}}a^{7}x^{\frac{7}{3}}+4\right.}{688}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/688128*(b+a*x^(1/3))*(-413440*b^(13/2)*a^3*x+229376*b^(19/2)+537472*b^(1
1/2)*a^4*x^(4/3)-739024*b^(9/2)*a^5*x^(5/3)+1108536*b^(7/2)*a^6*x^2-1939938
*b^(5/2)*a^7*x^(7/3)+4849845*b^(3/2)*a^8*x^(8/3)+14549535*a^9*x^3*b^(1/2)-2
72384*b^(17/2)*a*x^(1/3)+330752*b^(15/2)*a^2*x^(2/3)-14549535*(b+a*x^(1/3))
^(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^9*x^3)/x^2/(b*x^(2/3)+a*x)^(3
/2)/b^(21/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)
```

Giac [A]
time = 1.78, size = 207, normalized size = 0.64

$$-\frac{692835\,a^9\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768\,\sqrt{-b}\,b^{10}}-\frac{6\,a^9}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{10}}-\frac{10420767\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^9-88937058\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9b+334408914\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^9b^2-724860666\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^9b^3+993296384\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^9b^4-884769030\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^9b^5+503730990\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^9b^6-169799070\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^9b^7+26738145\sqrt{ax^{\frac{1}{3}}+b}\,a^9b^8}{688128\,a^9b^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6*
a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2)*
a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^(13
/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x^(1/
3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 503730990
*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*b^7 +
26738145*sqrt(a*x^(1/3) + b)*a^9*b^8)/(a^9*b^10*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)), x)
```

3.202 $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=412

$$\frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}$$

```
[Out] -50702925/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(27
/2)+6/b/x^(11/3)/(b*x^(2/3)+a*x)^(1/2)-25/4*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(13
/3)+575/88*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^4-2415/352*a^2*(b*x^(2/3)+a*x)^(1/
2)/b^4/x^(11/3)+15295/2112*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(10/3)-260015/33
792*a^4*(b*x^(2/3)+a*x)^(1/2)/b^6/x^3+185725/22528*a^5*(b*x^(2/3)+a*x)^(1/2
)/b^7/x^(8/3)-2414425/270336*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(7/3)+482885/4
9152*a^7*(b*x^(2/3)+a*x)^(1/2)/b^9/x^2-1448655/131072*a^8*(b*x^(2/3)+a*x)^(
1/2)/b^10/x^(5/3)+3380195/262144*a^9*(b*x^(2/3)+a*x)^(1/2)/b^11/x^(4/3)-169
00975/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^12/x+50702925/2097152*a^11*(b*x^
(2/3)+a*x)^(1/2)/b^13/x^(2/3)
```

Rubi [A]
time = 0.54, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2048, 2050, 2054, 212}

$$-\frac{50702925a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{27/2}}+\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^9\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}}-\frac{1448655a^8\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}}+\frac{482885a^7\sqrt{ax+bx^{2/3}}}{49152b^9x^2}-\frac{2414425a^6\sqrt{ax+bx^{2/3}}}{270336b^8x^{7/3}}+\frac{185725a^5\sqrt{ax+bx^{2/3}}}{22528b^7x^{8/3}}-\frac{260015a^4\sqrt{ax+bx^{2/3}}}{33792b^6x^3}+\frac{15295a^3\sqrt{ax+bx^{2/3}}}{2112b^5x^{10/3}}-\frac{2415a^2\sqrt{ax+bx^{2/3}}}{352b^4x^{11/3}}+\frac{575a\sqrt{ax+bx^{2/3}}}{88b^3x^4}-\frac{25\sqrt{ax+bx^{2/3}}}{4b^2x^{13/3}}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^
(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(
2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b
^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725
*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/
3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152
*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (33
80195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqr
t[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x]
)/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/(2097152*b^(27/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} + \frac{25\int \frac{1}{x^{14/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} - \frac{(575a)\int \frac{1}{x^{13/3}\sqrt{bx^{2/3}+ax}}\,dx}{24b^2}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} + \frac{(4025a^2)\int \frac{}{x^4\sqrt{}}}{176}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}}}{352b^4x^{11/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.06, size = 48, normalized size = 0.12

$$\frac{6a^{12}\sqrt[3]{x}\ {}_2F_1\left(-\frac{1}{2},13;\frac{1}{2};1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{13}\sqrt{bx^{2/3}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13*
Sqrt[b*x^(2/3) + a*x])
```

Maple [A]
time = 0.34, size = 192, normalized size = 0.47

method	result
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-19660800b^{\frac{23}{2}}a\,x^{\frac{1}{3}}+22609920b^{\frac{21}{2}}a^{2}x^{\frac{2}{3}}-26378240b^{\frac{19}{2}}a^{3}x+31324160b^{\frac{17}{2}}a^{4}x^{\frac{4}{3}}-38036480b^{\frac{15}{2}}a^{5}x^{\frac{5}{3}}+475\right.}{}$
default	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(19660800b^{\frac{23}{2}}a\,x^{\frac{1}{3}}-1673196525\sqrt{b+a\,x^{\frac{1}{3}}}\ \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{12}x^{4}-17301504b^{\frac{25}{2}}-22609920\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/69206016*(b+a*x^(1/3))*(19660800*b^(23/2)*a*x^(1/3)-1673196525*(b+a*x^(1/
3))^(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^12*x^4-17301504*b^(25/2)-2
2609920*b^(21/2)*a^2*x^(2/3)+26378240*b^(19/2)*a^3*x-31324160*b^(17/2)*a^4*
x^(4/3)+38036480*b^(15/2)*a^5*x^(5/3)-47545600*b^(13/2)*a^6*x^2+61809280*b^
(11/2)*a^7*x^(7/3)-84987760*b^(9/2)*a^8*x^(8/3)+127481640*b^(7/2)*a^9*x^3-2
23092870*b^(5/2)*a^10*x^(10/3)+557732175*b^(3/2)*a^11*x^(11/3)+1673196525*a
^12*x^4*b^(1/2))/x^3/(b*x^(2/3)+a*x)^(3/2)/b^(27/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**4*(a*x + b*x**(2/3))**(3/2)), x)
```

Giac [A]
time = 2.41, size = 258, normalized size = 0.63

$$\frac{50702925\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{2097152\,\sqrt{-b}\,b^{13}}+\frac{6\,a^{12}}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{13}}+\frac{1257960429\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{12}-14537792973\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}b+76667241519\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b^2-243717614415\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{12}b^3+519393101810\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{12}b^4-780150847218\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{12}b^5+844265343246\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{12}b^6-659969685518\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{12}b^7+366679446705\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{12}b^8-138840292305\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{12}b^9+32660709939\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{12}b^{10}-3724872723\sqrt{ax^{\frac{1}{3}}+b}\,a^{12}b^{11}}{69206016\,a^{12}b^{13}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
[Out] 50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13)
+ 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) + b
)^(23/2)*a^12 - 14537792973*(a*x^(1/3) + b)^(21/2)*a^12*b + 76667241519*(a*
x^(1/3) + b)^(19/2)*a^12*b^2 - 243717614415*(a*x^(1/3) + b)^(17/2)*a^12*b^3
 + 519393101810*(a*x^(1/3) + b)^(15/2)*a^12*b^4 - 780150847218*(a*x^(1/3) +
 b)^(13/2)*a^12*b^5 + 844265343246*(a*x^(1/3) + b)^(11/2)*a^12*b^6 - 659969
685518*(a*x^(1/3) + b)^(9/2)*a^12*b^7 + 366679446705*(a*x^(1/3) + b)^(7/2)*
a^12*b^8 - 138840292305*(a*x^(1/3) + b)^(5/2)*a^12*b^9 + 32660709939*(a*x^(
1/3) + b)^(3/2)*a^12*b^10 - 3724872723*sqrt(a*x^(1/3) + b)*a^12*b^11)/(a^12
*b^13*x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)),x)
[Out] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)), x)
```

3.203 $\int x^2(ax^2+bx^3)\,dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

[Out] 1/5*a*x^5+1/6*b*x^6

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = $\{14\}$

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3),x]

[Out] (a*x^5)/5 + (b*x^6)/6

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3\right)\,dx&=\int \left(ax^4+bx^5\right)\,dx\\&=\frac{ax^5}{5}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3),x]

[Out] (a*x^5)/5 + (b*x^6)/6

Maple [A]
time = 0.04, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{x^5(5bx+6a)}{30}$	14
default	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6$	14
norman	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6$	14
risch	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/6*b*x^6
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

Fricas [A]
time = 2.51, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6
```

Giac [A]
time = 1.64, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^5\,\left(6\,a+5\,b\,x\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3),x)
[Out] (x^5*(6*a + 5*b*x))/30
```

3.204 $\int x(ax^2+bx^3)\,dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{14\}$

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3\right)\,dx&=\int \left(ax^3+bx^4\right)\,dx\\&=\frac{ax^4}{4}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{x^4(4bx+5a)}{20}$	14
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4+1/5*b*x^5
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Fricas [A]
time = 2.97, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

Giac [**A**]
time = 1.28, size = 13, normalized size = 0.76

$$\frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [**B**]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^4\,(5\,a+4\,b\,x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3),x)
```

```
[Out] (x^4*(5*a + 4*b*x))/20
```

3.205 $\int (ax^2 + bx^3) \, dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

[Out] 1/3*a*x^3+1/4*b*x^4

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[a*x^2 + b*x^3,x]

[Out] (a*x^3)/3 + (b*x^4)/4

Rubi steps

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[a*x^2 + b*x^3,x]

[Out] (a*x^3)/3 + (b*x^4)/4

Maple [A]
time = 0.02, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{x^3(3bx+4a)}{12}$	14
default	$\frac{1}{3}a\,x^3 + \frac{1}{4}b\,x^4$	14

norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A]
time = 3.56, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

Giac [A]
time = 1.46, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,\left(4\,a+3\,b\,x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x^2 + b*x^3,x)
```

```
[Out] (x^3*(4*a + 3*b*x))/12
```

3.206 $\int \frac{ax^2+bx^3}{x} dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/x,x]
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2+bx^3}{x} dx &= \int \left(ax+bx^2\right) dx \\ &= \frac{ax^2}{2} + \frac{bx^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/x,x]
[Out] (a*x^2)/2 + (b*x^3)/3
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{x^2(2bx+3a)}{6}$	14
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A]
time = 1.14, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x,x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

Giac [**A**]
time = 1.12, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [**B**]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\,\left(3\,a+2\,b\,x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/x,x)
```

```
[Out] (x^2*(3*a + 2*b*x))/6
```

3.207 $\int \frac{ax^2+bx^3}{x^2} dx$

Optimal. Leaf size=12

$$ax + \frac{bx^2}{2}$$

[Out] a*x+1/2*b*x^2

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{14\}$

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)/x^2,x]

[Out] a*x + (b*x^2)/2

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2 + bx^3}{x^2} dx &= \int (a + bx) dx \\ &= ax + \frac{bx^2}{2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)/x^2,x]

[Out] a*x + (b*x^2)/2

Maple [A]
time = 0.02, size = 11, normalized size = 0.92

method	result	size
gosper	$\frac{x(bx+2a)}{2}$	11
default	$ax+\frac{1}{2}b\,x^{2}$	11
risch	$ax+\frac{1}{2}b\,x^{2}$	11
norman	$\frac{a\,x^{2}+\frac{1}{2}b\,x^{3}}{x}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*b*x^2
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 2.59, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$ax+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A]
time = 1.47, size = 10, normalized size = 0.83

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/x^2,x)
```

```
[Out] a*x + (b*x^2)/2
```

3.208 $\int x^2(ax^2+bx^3)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3\right)^2\,dx&=\int x^6(a+bx)^2\,dx\\&=\int\left(a^2x^6+2abx^7+b^2x^8\right)\,dx\\&=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Maple [A]
time = 0.43, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{x^7(28b^2x^2+63abx+36a^2)}{252}$	25
default	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Fricas [A]
time = 1.28, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**2,x)
[Out] a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9
```

Giac [A]
time = 1.44, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^7}{7}+\frac{a\,b\,x^8}{4}+\frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^7)/7 + (b^2*x^9)/9 + (a*b*x^8)/4
```

3.209 $\int x(ax^2+bx^3)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {1598, 45}

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3\right)^2\,dx&=\int x^5(a+bx)^2\,dx\\&=\int\left(a^2x^5+2abx^6+b^2x^7\right)\,dx\\&=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6} + \frac{2}{7}abx^7 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Maple [A]
time = 0.34, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{x^6(21b^2x^2+48abx+28a^2)}{168}$	25
default	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{2}{7}\,abx^7 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Fricas [A]
time = 1.14, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{2}{7}\,abx^7 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8
```

Giac [A]
time = 1.44, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{2}{7}\,abx^7 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6} + \frac{2\,a\,b\,x^7}{7} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*x^8)/8 + (2*a*b*x^7)/7
```

3.210 $\int \left(ax^2+bx^3\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {1607, 45}

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3\right)^2\,dx &= \int x^4(a+bx)^2\,dx\\ &= \int \left(a^2x^4+2abx^5+b^2x^6\right)\,dx\\ &= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Maple [A]
time = 0.33, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{x^5(15b^2x^2+35abx+21a^2)}{105}$	25
default	$\frac{1}{5}a^2x^5 + \frac{1}{3}ab\,x^6 + \frac{1}{7}b^2x^7$	25
norman	$\frac{1}{5}a^2x^5 + \frac{1}{3}ab\,x^6 + \frac{1}{7}b^2x^7$	25
risch	$\frac{1}{5}a^2x^5 + \frac{1}{3}ab\,x^6 + \frac{1}{7}b^2x^7$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Fricas [A]
time = 1.32, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

Giac [A]
time = 1.42, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^7)/7 + (a*b*x^6)/3
```

3.211 $\int \frac{(ax^2+bx^3)^2}{x}\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 45}

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax^2+bx^3)^2}{x}\,dx &= \int x^3(a+bx)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x,x]
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Maple [A]
time = 0.34, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{x^4(10b^2x^2+24abx+15a^2)}{60}$	25
default	$\frac{1}{4}a^2x^4 + \frac{2}{5}ab\,x^5 + \frac{1}{6}b^2x^6$	25
norman	$\frac{1}{4}a^2x^4 + \frac{2}{5}ab\,x^5 + \frac{1}{6}b^2x^6$	25
risch	$\frac{1}{4}a^2x^4 + \frac{2}{5}ab\,x^5 + \frac{1}{6}b^2x^6$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Fricas [A]
time = 1.21, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

Giac [A]
time = 1.38, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2/x,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

3.212 $\int \frac{(ax^2+bx^3)^2}{x^2} dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax^2+bx^3)^2}{x^2}\,dx &= \int x^2(a+bx)^2\,dx\\ &= \int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Maple [A]
time = 0.33, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$	25
default	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
norman	$\frac{\frac{1}{3}a^2x^4+\frac{1}{5}b^2x^6+\frac{1}{2}ab\,x^5}{x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Fricas [A]
time = 1.15, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x**2,x)
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Giac [A]
time = 0.98, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2/x^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

3.213 $\int \frac{x^6}{ax^2+bx^3} \, dx$

Optimal. Leaf size=57

$$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 45}

$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{ax^2+bx^3} \, dx &= \int \frac{x^4}{a+bx} \, dx\\
&= \int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right) \, dx\\
&= -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.00

$$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Maple [A]
time = 0.39, size = 52, normalized size = 0.91

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{1}{3}a\,b^2x^3-\frac{1}{2}a^2b\,x^2+a^3x}{b^4}+\frac{a^4\ln(bx+a)}{b^5}$	52
risch	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52
norman	$\frac{\frac{x^5}{4b}-\frac{a\,x^4}{3b^2}+\frac{a^2x^3}{2b^3}-\frac{a^3x^2}{b^4}}{x}+\frac{a^4\ln(bx+a)}{b^5}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/4*b^3*x^4+1/3*a*b^2*x^3-1/2*a^2*b*x^2+a^3*x)+a^4*ln(b*x+a)/b^5
```

Maxima [A]
time = 0.30, size = 52, normalized size = 0.91

$$\frac{a^4\log(bx+a)}{b^5}+\frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

Fricas [A]
time = 1.16, size = 52, normalized size = 0.91

$$\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log(bx+a)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

Sympy [A]
time = 0.05, size = 49, normalized size = 0.86

$$\frac{a^4 \log{(a + bx)}}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

Giac [A]
time = 0.92, size = 53, normalized size = 0.93

$$\frac{a^4 \log{(|bx + a|)}}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

Mupad [B]
time = 5.09, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,b} + \frac{a^4\,\ln{(a + b\,x)}}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

3.214 $\int \frac{x^5}{ax^2+bx^3} \, dx$

Optimal. Leaf size=44

$$\frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3 \log(a+bx)}{b^4}$$

```
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 45}

$$-\frac{a^3 \log(a+bx)}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ax^2+bx^3} \, dx &= \int \frac{x^3}{a+bx} \, dx \\
&= \int \left(\frac{a^2}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} - \frac{a^3}{b^3(a+bx)}\right) dx \\
&= \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3 \log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$\frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3),x]
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Maple [A]
time = 0.35, size = 41, normalized size = 0.93

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x}{b^3}-\frac{a^3\ln(bx+a)}{b^4}$	41
risch	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41
norman	$\frac{\frac{a^2x^2}{b^3}+\frac{x^4}{3b}-\frac{a\,x^3}{2b^2}}{x}-\frac{a^3\ln(bx+a)}{b^4}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2+a^2*x)-a^3*ln(b*x+a)/b^4
```

Maxima [A]
time = 0.29, size = 42, normalized size = 0.95

$$-\frac{a^3\log\left(bx+a\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Fricas [A]
time = 1.43, size = 41, normalized size = 0.93

$$\frac{2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A]
time = 0.04, size = 37, normalized size = 0.84

$$-\frac{a^3\log\left(a+bx\right)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Giac [A]
time = 1.74, size = 43, normalized size = 0.98

$$-\frac{a^3\log\left(|bx+a|\right)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Mupad [B]
time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\,b}-\frac{a^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{a\,x^2}{2\,b^2}+\frac{a^2\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3),x)
```

```
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

3.215 $\int \frac{x^4}{ax^2+bx^3}\,dx$

Optimal. Leaf size=31

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 45}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax^2+bx^3}\,dx &= \int \frac{x^2}{a+bx}\,dx\\ &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3),x]
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A]
time = 0.35, size = 30, normalized size = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}b\,x^2+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
norman	$\frac{\frac{x^3}{2b}-\frac{a\,x^2}{b^2}}{x}+\frac{a^2\ln(bx+a)}{b^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*b*x^2+a*x)+a^2*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.27, size = 29, normalized size = 0.94

$$\frac{a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A]
time = 1.36, size = 29, normalized size = 0.94

$$\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A]
time = 0.04, size = 26, normalized size = 0.84

$$\frac{a^2\log\left(a+bx\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Giac [A]
time = 1.37, size = 30, normalized size = 0.97

$$\frac{a^2\log\left(|bx+a|\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B]
time = 0.04, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.216 $\int \frac{x^3}{ax^2+bx^3}\,dx$

Optimal. Leaf size=18

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax^2+bx^3}\,dx &= \int \frac{x}{a+bx}\,dx\\ &= \int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx\\ &= \frac{x}{b}-\frac{a\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3),x]
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A]
time = 0.35, size = 19, normalized size = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] x/b-a*ln(b*x+a)/b^2
```

Maxima [A]
time = 0.29, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A]
time = 1.35, size = 17, normalized size = 0.94

$$\frac{bx - a \log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.78

$$-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

Giac [A]
time = 1.03, size = 19, normalized size = 1.06

$$\frac{x}{b}-\frac{a\log(|bx+a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B]
time = 0.04, size = 18, normalized size = 1.00

$$-\frac{a\,\ln(a+b\,x)-b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3),x)
```

```
[Out] -(a*log(a + b*x) - b*x)/b^2
```

3.217 $\int \frac{x^2}{ax^2+bx^3} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] ln(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ax^2+bx^3}\,dx &= \int \frac{1}{a+bx}\,dx\\ &= \frac{\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] Log[a + b*x]/b
```

Maple [A]
time = 0.34, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A]
time = 0.28, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A]
time = 1.58, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A]
time = 2.61, size = 11, normalized size = 1.10

$$\frac{\log(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3),x)
```

```
[Out] log(a + b*x)/b
```

3.218 $\int \frac{x}{ax^2+bx^3}\,dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

[Out] ln(x)/a-ln(b*x+a)/a

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {1598, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x}{ax^2+bx^3}\,dx = \int \frac{1}{x(a+bx)}\,dx$$
$$= \frac{\int \frac{1}{x}\,dx}{a} - \frac{b\int \frac{1}{a+bx}\,dx}{a}$$
$$= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3),x]
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A]
time = 0.34, size = 19, normalized size = 1.06

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
risch	$-\frac{\ln(bx+a)}{a} + \frac{\ln(-x)}{a}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] ln(x)/a-ln(b*x+a)/a
```

Maxima [A]
time = 0.29, size = 18, normalized size = 1.00

$$-\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A]
time = 1.49, size = 16, normalized size = 0.89

$$-\frac{\log\left(bx+a\right)-\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.56

$$\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [A]
time = 1.90, size = 20, normalized size = 1.11

$$-\frac{\log\left(|bx+a|\right)}{a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B]
time = 5.12, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3),x)
```

```
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

3.219 $\int \frac{1}{ax^2+bx^3}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 46}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2+bx^3}\,dx &= \int \frac{1}{x^2(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-1),x]
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A]
time = 0.36, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(-bx-a)}{a^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A]
time = 0.27, size = 28, normalized size = 1.00

$$\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A]
time = 2.32, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A]
time = 0.08, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A]
time = 1.26, size = 30, normalized size = 1.07

$$\frac{b\log\left(\left|bx+a\right|\right)}{a^2}-\frac{b\log\left(\left|x\right|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.220 $\int \frac{1}{x(ax^2+bx^3)} dx$

Optimal. Leaf size=42

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 46}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax^2+bx^3\right)} dx &= \int \frac{1}{x^3(a+bx)} dx \\
&= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right) dx \\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.00

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)),x]
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Maple [A]
time = 0.34, size = 41, normalized size = 0.98

method	result	size
default	$-\frac{1}{2a\,x^2}+\frac{b}{a^2x}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
norman	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}-\frac{b^2\ln(bx+a)}{a^3}+\frac{b^2\ln(-x)}{a^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.95

$$-\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log\left(x\right)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Fricas [A]
time = 1.90, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A]
time = 0.09, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2),x)
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Giac [A]
time = 1.62, size = 45, normalized size = 1.07

$$-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B]
time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)),x)
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.221 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 46}

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx &= \int \frac{1}{x^4(a+bx)}\,dx\\
&= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Maple [A]
time = 0.37, size = 53, normalized size = 0.95

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b}{2a^{2}x^{2}}-\frac{b^{2}}{a^{3}x}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}$	53
norman	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}$	53
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(-bx-a\right)}{a^{4}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.91

$$\frac{b^3\log\left(bx+a\right)}{a^4}-\frac{b^3\log\left(x\right)}{a^4}-\frac{6\,b^2x^2-3\,abx+2\,a^2}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

Fricas [A]
time = 1.33, size = 54, normalized size = 0.96

$$\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log\left(x\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

Sympy [A]
time = 0.13, size = 44, normalized size = 0.79

$$\frac{-2a^2+3abx-6b^2x^2}{6a^3x^3}+\frac{b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

Giac [A]
time = 1.09, size = 56, normalized size = 1.00

$$\frac{b^3\log\left(|bx+a|\right)}{a^4}-\frac{b^3\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

Mupad [B]
time = 0.06, size = 48, normalized size = 0.86

$$\frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{a^3}{3}-\frac{a^2\,b\,x}{2}+a\,b^2\,x^2}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

3.222 $\int \frac{x^8}{(ax^2+bx^3)^2}\,dx$

Optimal. Leaf size=58

$$\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}$$

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(ax^2+bx^3)^2}\,dx &= \int \frac{x^4}{(a+bx)^2}\,dx\\ &= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\ &= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.93

$$\frac{9a^2bx - 3ab^2x^2 + b^3x^3 - \frac{3a^4}{a+bx} - 12a^3\log(a+bx)}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

Maple [A]
time = 0.37, size = 57, normalized size = 0.98

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x}{b^4}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
risch	$\frac{3a^2x}{b^4}-\frac{a\,x^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{x^7}{3b}-\frac{2a\,x^6}{3b^2}+\frac{2a^2x^5}{b^3}-\frac{4a^4x^3}{b^5}}{x^3(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x)-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 1.02

$$-\frac{a^4}{b^6x + ab^5} - \frac{4\,a^3\log\left(bx+a\right)}{b^5} + \frac{b^2x^3 - 3\,abx^2 + 9\,a^2x}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

Fricas [A]
time = 1.39, size = 73, normalized size = 1.26

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\,(a^3bx + a^4)\log\left(bx+a\right)}{3\,(b^6x + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A]
time = 0.08, size = 54, normalized size = 0.93

$$-\frac{a^4}{ab^5 + b^6x} - \frac{4a^3\log(a + bx)}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a*x**2)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

Giac [A]
time = 1.12, size = 62, normalized size = 1.07

$$-\frac{4\,a^3\log(|bx + a|)}{b^5} - \frac{a^4}{(bx + a)b^5} + \frac{b^4x^3 - 3\,ab^3x^2 + 9\,a^2b^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

Mupad [B]
time = 0.04, size = 62, normalized size = 1.07

$$\frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln(a + b\,x)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5 + a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

3.223 $\int \frac{x^7}{(ax^2+bx^3)^2} \, dx$

Optimal. Leaf size=46

$$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}$$

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 45}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax^2+bx^3)^2} \, dx &= \int \frac{x^3}{(a+bx)^2} \, dx \\
&= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx \\
&= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.93

$$\frac{-4abx+b^2x^2+\frac{2a^3}{a+bx}+6a^2\log(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

Maple [A]
time = 0.35, size = 46, normalized size = 1.00

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	46
norman	$\frac{\frac{3a^3x^3}{b^4}+\frac{x^6}{2b}-\frac{3a\,x^5}{2b^2}}{x^3(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/2*b*x^2+2*a*x)+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Maxima [A]
time = 0.29, size = 47, normalized size = 1.02

$$\frac{a^3}{b^5x+ab^4}+\frac{3\,a^2\log\left(bx+a\right)}{b^4}+\frac{bx^2-4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

Fricas [A]
time = 1.43, size = 62, normalized size = 1.35

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log\left(bx+a\right)}{2\,(b^5x+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

Sympy [A]
time = 0.07, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4+b^5x}+\frac{3a^2\log{(a+bx)}}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a*x**2)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

Giac [A]
time = 1.62, size = 48, normalized size = 1.04

$$\frac{3\,a^2\log\left(|bx+a|\right)}{b^4}+\frac{a^3}{(bx+a)b^4}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

Mupad [B]
time = 0.05, size = 50, normalized size = 1.09

$$\frac{x^2}{2\,b^2}+\frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4}+\frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

3.224 $\int \frac{x^6}{(ax^2+bx^3)^2} dx$

Optimal. Leaf size=33

$$\frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}$$

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 45}

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(ax^2+bx^3)^2}\,dx &= \int \frac{x^2}{(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx\\ &= \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{bx - \frac{a^2}{a+bx} - 2a\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^2,x]
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Maple [A]
time = 0.36, size = 34, normalized size = 1.03

method	result	size
default	$\frac{x}{b^2} - \frac{a^2}{b^3(bx+a)} - \frac{2a\ln(bx+a)}{b^3}$	34
risch	$\frac{x}{b^2} - \frac{a^2}{b^3(bx+a)} - \frac{2a\ln(bx+a)}{b^3}$	34
norman	$\frac{\frac{x^5}{b} - \frac{2a^2x^3}{b^3}}{x^3(bx+a)} - \frac{2a\ln(bx+a)}{b^3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.27, size = 36, normalized size = 1.09

$$-\frac{a^2}{b^4x + ab^3} + \frac{x}{b^2} - \frac{2\,a\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Fricas [A]
time = 1.47, size = 47, normalized size = 1.42

$$\frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log\left(bx+a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A]
time = 0.07, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3 + b^4x} - \frac{2a\log{(a + bx)}}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**2,x)
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Giac [A]
time = 1.67, size = 34, normalized size = 1.03

$$\frac{x}{b^2} - \frac{2\,a\log\left(|bx + a|\right)}{b^3} - \frac{a^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

Mupad [B]
time = 0.04, size = 36, normalized size = 1.09

$$\frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln\left(a + b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3)^2,x)
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

3.225 $\int \frac{x^5}{(ax^2+bx^3)^2}\,dx$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(ax^2+bx^3)^2}\,dx &= \int \frac{x}{(a+bx)^2}\,dx\\
&= \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx\\
&= \frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx} + \log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^2,x]
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

Maple [A]
time = 0.36, size = 24, normalized size = 1.04

method	result	size
default	$\frac{a}{b^2(bx+a)} + \frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)} + \frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)} + \frac{\ln(bx+a)}{b^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.13

$$\frac{a}{b^3x + ab^2} + \frac{\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Fricas [A]
time = 1.39, size = 28, normalized size = 1.22

$$\frac{(bx+a)\log(bx+a) + a}{b^3x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.87

$$\frac{a}{ab^2 + b^3x} + \frac{\log(a + bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**2,x)
```

```
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Giac [A]
time = 1.71, size = 24, normalized size = 1.04

$$\frac{\log(|bx + a|)}{b^2} + \frac{a}{(bx + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

Mupad [B]
time = 0.04, size = 23, normalized size = 1.00

$$\frac{\ln(a + b\,x)}{b^2} + \frac{a}{b^2\,(a + b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3)^2,x)
```

```
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

3.226 $\int \frac{x^4}{(ax^2+bx^3)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(ax^2+bx^3)^2} dx &= \int \frac{1}{(a+bx)^2} dx\\ &= -\frac{1}{b(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^2,x]
[Out] -(1/(b*(a + b*x)))
```

Maple [A]
time = 0.41, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/b/(b*x+a)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -1/(b^2*x + a*b)
```

Fricas [A]
time = 1.43, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -1/(b^2*x + a*b)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.83

$$-\frac{1}{ab + b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A]
time = 1.67, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

Mupad [B]
time = 5.17, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^2,x)
```

```
[Out] -1/(b*(a + b*x))
```

3.227 $\int \frac{x^3}{(ax^2+bx^3)^2} \, dx$

Optimal. Leaf size=29

$$\frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}$$

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 46}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3)^2,x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(ax^2+bx^3)^2} \, dx &= \int \frac{1}{x(a+bx)^2} \, dx\\ &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) \, dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}+\log(x)-\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^2,x]
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

Maple [A]
time = 0.36, size = 30, normalized size = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}-\frac{\ln(bx+a)}{a^2}+\frac{\ln(-x)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.97

$$\frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Fricas [A]
time = 1.08, size = 39, normalized size = 1.34

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A]
time = 0.09, size = 22, normalized size = 0.76

$$\frac{1}{a^2 + abx} + \frac{\log{(x)} - \log{\left(\frac{a}{b} + x \right)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**2,x)
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Giac [A]
time = 1.28, size = 31, normalized size = 1.07

$$-\frac{\log{(|bx + a|)}}{a^2} + \frac{\log{(|x|)}}{a^2} + \frac{1}{(bx + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.90

$$\frac{1}{a^2 + b\,x\,a} - \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^2,x)
[Out] 1/(a^2 + a*b*x) - (2*atanh((2*b*x)/a + 1))/a^2
```

3.228 $\int \frac{x^2}{(ax^2+bx^3)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 46}

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(ax^2+bx^3)^2}\,dx &= \int \frac{1}{x^2(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A]
time = 0.36, size = 43, normalized size = 1.02

method	result	size
default	$-\frac{1}{a^2 x}-\frac{b}{a^2(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}$	43
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{x(bx+a)}+\frac{2b\ln(-bx-a)}{a^3}-\frac{2b\ln(x)}{a^3}$	49
norman	$\frac{\frac{2b^2x^4}{a^3}-\frac{x^2}{a}}{x^3(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Maxima [A]
time = 0.27, size = 45, normalized size = 1.07

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A]
time = 1.77, size = 63, normalized size = 1.50

$$-\frac{2\,abx+a^2-2\,(b^2x^2+abx)\log\left(bx+a\right)+2\,(b^2x^2+abx)\log\left(x\right)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A]
time = 0.12, size = 37, normalized size = 0.88

$$\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Giac [A]
time = 1.11, size = 45, normalized size = 1.07

$$\frac{2\,b\log\left(|bx+a|\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}-\frac{2\,bx+a}{(bx^2+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

Mupad [B]
time = 5.34, size = 41, normalized size = 0.98

$$\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}-\frac{\frac{1}{a}+\frac{2\,b\,x}{a^2}}{b\,x^2+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^2,x)
```

```
[Out] (4*b*atanh((2*b*x)/a + 1))/a^3 - (1/a + (2*b*x)/a^2)/(a*x + b*x^2)
```

3.229 $\int \frac{x}{(ax^2+bx^3)^2} dx$

Optimal. Leaf size=58

$$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}$$

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 46}

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax^2+bx^3)^2} dx &= \int \frac{1}{x^3(a+bx)^2} dx \\
&= \int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right) dx \\
&= -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.91

$$\frac{a\left(-\frac{a}{x^2}+\frac{4b}{x}+\frac{2b^2}{a+bx}\right)+6b^2\log(x)-6b^2\log(a+bx)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

Maple [A]
time = 0.37, size = 57, normalized size = 0.98

method	result	size
default	$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	57
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)}+\frac{3b^2\ln(-x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	63
norman	$\frac{-\frac{3b^3x^4}{a^4}-\frac{x}{2a}+\frac{3b\,x^2}{2a^2}}{x^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Maxima [A]
time = 0.29, size = 64, normalized size = 1.10

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\,(a^3bx^3+a^4x^2)}-\frac{3\,b^2\log\,(bx+a)}{a^4}+\frac{3\,b^2\log\,(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

Fricas [A]
time = 1.24, size = 86, normalized size = 1.48

$$\frac{6\,ab^2x^2+3\,a^2bx-a^3-6\,(b^3x^3+ab^2x^2)\log\,(bx+a)+6\,(b^3x^3+ab^2x^2)\log\,(x)}{2\,(a^4bx^3+a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A]
time = 0.13, size = 54, normalized size = 0.93

$$\frac{-a^2+3abx+6b^2x^2}{2a^4x^2+2a^3bx^3}+\frac{3b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

Giac [A]
time = 1.77, size = 64, normalized size = 1.10

$$-\frac{3\,b^2\log\left(\left|bx+a\right|\right)}{a^4}+\frac{3\,b^2\log\left(\left|x\right|\right)}{a^4}+\frac{6\,ab^2x^2+3\,a^2bx-a^3}{2\,(bx+a)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

Mupad [B]
time = 5.31, size = 57, normalized size = 0.98

$$\frac{\frac{3\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^2}}{b\,x^3+a\,x^2}-\frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^2,x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

3.230 $\int \frac{1}{(ax^2+bx^3)^2} dx$

Optimal. Leaf size=69

$$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}$$

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Rubi [A]

time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 46}

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4
*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax^2+bx^3)^2}\,dx &= \int \frac{1}{x^4(a+bx)^2}\,dx\\
&= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\
&= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.96

$$-\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)}+12b^3\log(x)-12b^3\log(a+bx)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

Maple [A]
time = 0.37, size = 68, normalized size = 0.99

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	68
norman	$\frac{\frac{4b^4x^4}{a^5}-\frac{1}{3a}+\frac{2bx}{3a^2}-\frac{2b^2x^2}{a^3}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	72
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{2b^2x^2}{a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(-bx-a)}{a^5}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Maxima [A]
time = 0.27, size = 73, normalized size = 1.06

$$-\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\,(a^4bx^4+a^5x^3)}+\frac{4\,b^3\log\left(bx+a\right)}{a^5}-\frac{4\,b^3\log\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Fricas [A]
time = 1.58, size = 95, normalized size = 1.38

$$-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\,(b^4x^4+ab^3x^3)\log\left(bx+a\right)+12\,(b^4x^4+ab^3x^3)\log\left(x\right)}{3\,(a^5bx^4+a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

Sympy [A]
time = 0.15, size = 66, normalized size = 0.96

$$\frac{-a^3 + 2a^2bx - 6ab^2x^2 - 12b^3x^3}{3a^5x^3 + 3a^4bx^4} + \frac{4b^3\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Giac [A]
time = 1.09, size = 73, normalized size = 1.06

$$\frac{4\,b^3\log\left(|bx+a|\right)}{a^5} - \frac{4\,b^3\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x^3 + 6\,a^2b^2x^2 - 2\,a^3bx + a^4}{3\,(bx+a)a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 6
*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

Mupad [B]
time = 0.07, size = 69, normalized size = 1.00

$$\frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{3\,a} + \frac{2\,b^2\,x^2}{a^3} + \frac{4\,b^3\,x^3}{a^4} - \frac{2\,b\,x}{3\,a^2}}{b\,x^4 + a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^2,x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

3.231 $\int \frac{1}{x(ax^2+bx^3)^2} dx$

Optimal. Leaf size=84

$$-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}$$

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 46}

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] -1/4*1/(a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x
) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{x^5(a+bx)^2}\,dx\\
&= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\
&= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.94

$$\frac{\frac{a(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4)}{x^4(a+bx)}+60b^4\log(x)-60b^4\log(a+bx)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

Maple [A]
time = 0.38, size = 79, normalized size = 0.94

method	result	size
default	$-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	79
norman	$\frac{-\frac{5b^5x^5}{a^6}-\frac{1}{4a}+\frac{5bx}{12a^2}-\frac{5b^2x^2}{6a^3}+\frac{5b^3x^3}{2a^4}}{x^4(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	83
risch	$\frac{\frac{5b^4x^4}{a^5}+\frac{5b^3x^3}{2a^4}-\frac{5b^2x^2}{6a^3}+\frac{5bx}{12a^2}-\frac{1}{4a}}{x^4(bx+a)}-\frac{5b^4\ln(bx+a)}{a^6}+\frac{5b^4\ln(-x)}{a^6}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.02

$$\frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\,(a^5bx^5+a^6x^4)}-\frac{5\,b^4\log\left(bx+a\right)}{a^6}+\frac{5\,b^4\log\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

Fricas [A]
time = 1.68, size = 108, normalized size = 1.29

$$\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5-60\,(b^5x^5+ab^4x^4)\log\left(bx+a\right)+60\,(b^5x^5+ab^4x^4)\log\left(x\right)}{12\,(a^6bx^5+a^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

Sympy [A]
time = 0.19, size = 80, normalized size = 0.95

$$\frac{-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4}{12a^6x^4+12a^5bx^5}+\frac{5b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

Giac [A]
time = 1.22, size = 86, normalized size = 1.02

$$-\frac{5\,b^4\log\left(|bx+a|\right)}{a^6}+\frac{5\,b^4\log\left(|x|\right)}{a^6}+\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5}{12\,(bx+a)a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4 +
 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

Mupad [B]
time = 0.08, size = 79, normalized size = 0.94

$$\frac{\frac{5\,b^3\,x^3}{2\,a^4}-\frac{5\,b^2\,x^2}{6\,a^3}-\frac{1}{4\,a}+\frac{5\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{12\,a^2}}{b\,x^5+a\,x^4}-\frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

3.232 $\int x^2 \sqrt{ax^2 + bx^3} \, dx$

Optimal. Leaf size=105

$$\frac{2(ax^2+bx^3)^{3/2}}{9b} - \frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3} + \frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2} - \frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}$$

```
[Out] 2/9*(b*x^3+a*x^2)^(3/2)/b-32/315*a^3*(b*x^3+a*x^2)^(3/2)/b^4/x^3+16/105*a^2
*(b*x^3+a*x^2)^(3/2)/b^3/x^2-4/21*a*(b*x^3+a*x^2)^(3/2)/b^2/x
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$-\frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3} + \frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2} - \frac{4a(ax^2+bx^3)^{3/2}}{21b^2x} + \frac{2(ax^2+bx^3)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x
^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(
3/2))/(21*b^2*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{ax^2+bx^3}\,dx &= \frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{(2a)\int x\sqrt{ax^2+bx^3}\,dx}{3b}\\ &= \frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}+\frac{(8a^2)\int\sqrt{ax^2+bx^3}\,dx}{21b^2}\\ &= \frac{2(ax^2+bx^3)^{3/2}}{9b}+\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2}-\frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}-\frac{(16a^3)\int\frac{\sqrt{ax^2+bx^3}}{x}}{105b^3}\\ &= \frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3}+\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2}-\frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 53, normalized size = 0.50

$$\frac{2(x^2(a+bx))^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3)
)/(315*b^4*x^3)
```

Maple [**A**]
time = 0.36, size = 57, normalized size = 0.54

method	result	size
gosper	$-\frac{2(bx+a)\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
default	$-\frac{2(bx+a)\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	57
risch	$-\frac{2\sqrt{x^2\,(bx+a)}\,\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)}{315x\,b^4}$	61
trager	$-\frac{2\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(b*x+a)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)*(b*x^3+a*x^2)^(
1/2)/b^4/x
```

Maxima [A]
time = 0.30, size = 53, normalized size = 0.50

$$\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Fricas [A]
time = 1.20, size = 62, normalized size = 0.59

$$\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.41, size = 131, normalized size = 1.25

$$\frac{32\,a^{\frac{9}{2}}\operatorname{sgn}(x)}{315\,b^4}+\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a\operatorname{sgn}(x)}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)\operatorname{sgn}(x)}{b^3}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b^3 + (35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^3)/b
```

Mupad [B]
time = 5.49, size = 62, normalized size = 0.59

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2
 + 8*a^3*b*x))/(315*b^4*x)
```

3.233 $\int x\sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=80

$$\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3}-\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}$$

```
[Out] 16/105*a^2*(b*x^3+a*x^2)^(3/2)/b^3/x^3-8/35*a*(b*x^3+a*x^2)^(3/2)/b^2/x^2+2
/7*(b*x^3+a*x^2)^(3/2)/b/x
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2041, 2027, 2039}

$$\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3}-\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/
(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{ax^2+bx^3}\,dx &= \frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{(4a)\int\sqrt{ax^2+bx^3}\,dx}{7b}\\ &= -\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}+\frac{(8a^2)\int\frac{\sqrt{ax^2+bx^3}}{x}\,dx}{35b^2}\\ &= \frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3}-\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.52

$$\frac{2(x^2(a+bx))^{3/2}\,(8a^2-12abx+15b^2x^2)}{105b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3],x]
[Out] (2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

Maple [A]
time = 0.37, size = 46, normalized size = 0.58

method	result	size
gosper	$\frac{2(bx+a)(15b^2x^2-12abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	46
default	$\frac{2(bx+a)(15b^2x^2-12abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	46
risch	$\frac{2\sqrt{x^2\,(bx+a)}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105x\,b^3}$	50
trager	$\frac{2(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/105*(b*x+a)*(15*b^2*x^2-12*a*b*x+8*a^2)*(b*x^3+a*x^2)^(1/2)/b^3/x
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.52

$$\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Fricas [A]
time = 1.41, size = 51, normalized size = 0.64

$$\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(b
^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\,(a+bx)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x*sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.25, size = 108, normalized size = 1.35

$$-\frac{16\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,b^3}+\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\mathrm{sgn}(x)}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)\mathrm{sgn}(x)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] -16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/
2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x)/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x
+ a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b^2)/b
```

Mupad [B]
time = 5.48, size = 51, normalized size = 0.64

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3\right)}{105\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^(1/2),x)
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x))/(1
05*b^3*x)
```

3.234 $\int \sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=52

$$-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}+\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}$$

```
[Out] -4/15*a*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*(b*x^3+a*x^2)^(3/2)/b/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 2039}

$$\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*
x^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{ax^2+bx^3}\,dx &= \frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{(2a)\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx}{5b}\\ &= -\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}+\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.79

$$\frac{2\sqrt{x^2(a+bx)}\,(-2a^2+abx+3b^2x^2)}{15b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2*x)
```

Maple [A]
time = 0.38, size = 35, normalized size = 0.67

method	result	size
gosper	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	35
default	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	35
risch	$-\frac{2\sqrt{x^2\,(bx+a)}\,(-3b^2x^2-abx+2a^2)}{15x\,b^2}$	39
trager	$-\frac{2(-3b^2x^2-abx+2a^2)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15*(b*x+a)*(-3*b*x+2*a)*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Maxima [A]
time = 0.29, size = 30, normalized size = 0.58

$$\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Fricas [A]
time = 1.56, size = 39, normalized size = 0.75

$$\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2),x)
[Out] Integral(sqrt(a*x**2 + b*x**3), x)
```

Giac [A]
time = 1.67, size = 81, normalized size = 1.56

$$\frac{4\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^2} + \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\mathrm{sgn}(x)}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)\mathrm{sgn}(x)}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*s
gn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*sgn(x)/b)/b
```

Mupad [B]
time = 5.30, size = 39, normalized size = 0.75

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-2\,a^2+a\,b\,x+3\,b^2\,x^2\right)}{15\,b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2),x)
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

3.235 $\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx$

Optimal. Leaf size=25

$$\frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)/b/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{2039\}$

$$\frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x,x]
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.92

$$\frac{2(x^2(a+bx))^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
[Out] (2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

Maple [A]
time = 0.36, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{2\sqrt{x^2\,(bx+a)}\,(bx+a)}{3xb}$	25
gosper	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
default	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
trager	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x+a)*(b*x^3+a*x^2)^(1/2)/b/x
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.48

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A]
time = 1.35, size = 26, normalized size = 1.04

$$\frac{2\,\sqrt{bx^3+ax^2}\,(bx+a)}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.
time = 1.52, size = 50, normalized size = 2.00

$$-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b}+\frac{2\left(3\,\sqrt{bx+a}\,a\mathrm{sgn}(x)+\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)\mathrm{sgn}(x)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*a^(3/2)*sgn(x)/b + 2/3*(3*sqrt(b*x + a)*a*sgn(x) + ((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*sgn(x))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{b\,x^3+a\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x, x)
```

3.236 $\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2)+2*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2046, 2033, 212}

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2\sqrt{ax^2+bx^3}}{x} + a\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2\sqrt{ax^2+bx^3}}{x} - (2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)$$
$$= \frac{2\sqrt{ax^2+bx^3}}{x} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.04

$$\frac{2x\left(a+bx-\sqrt{a}\,\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqrt
[x^2*(a + b*x)]
```

Maple [A]
time = 0.37, size = 51, normalized size = 1.00

method	result	size
default	$\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\right)}{x\sqrt{bx+a}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*x^3+a*x^2)^(1/2)*(-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1/2
))/x/(b*x+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

[Out] integrate(sqrt(b*x^3 + a*x^2)/x^2, x)

Fricas [A]
time = 1.80, size = 111, normalized size = 2.18

$$\left[\frac{\sqrt{a}\,x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x},\ \frac{2\left(\sqrt{-a}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sq
rt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + sqrt(b*x^3 + a*x^2))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

Giac [A]
time = 1.26, size = 67, normalized size = 1.31

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\operatorname{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B]
time = 5.36, size = 73, normalized size = 1.43

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}}{x}+\frac{\sqrt{a}\ \mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{\frac{1}{x}}\ 1\mathrm{i}}{\sqrt{b}}\right)\sqrt{b\,x^3+a\,x^2}\ \left(\frac{1}{x}\right)^{3/2}2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^2,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2))/x + (a^(1/2)*asin((a^(1/2)*(1/x)^(1/2)*1i)/b^(1/2
))*(a*x^2 + b*x^3)^(1/2)*(1/x)^(3/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

3.237 $\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] -b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)-(b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2045, 2033, 212}

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/S
qrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{x^2}+\frac{1}{2}b\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\ &= -\frac{\sqrt{ax^2+bx^3}}{x^2}-b\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= -\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 64, normalized size = 1.23

$$-\frac{\sqrt{a+bx}\left(\sqrt{a}\,\sqrt{a+bx}+bx\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}\,\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -((Sqrt[a + b*x]*(Sqrt[a]*Sqrt[a + b*x] + b*x*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
]))/(Sqrt[a]*Sqrt[x^2*(a + b*x)]))
```

Maple [A]
time = 0.51, size = 56, normalized size = 1.08

method	result	size
default	$-\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx+\sqrt{bx+a}\,\sqrt{a}\right)}{x^2\sqrt{bx+a}\,\sqrt{a}}$	56
risch	$-\frac{\sqrt{x^2\,(bx+a)}}{x^2}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2\,(bx+a)}}{\sqrt{a}\,x\sqrt{bx+a}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^3+a*x^2)^(1/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x+(b*x+a)^(1/2)*a^(1
/2))/x^2/(b*x+a)^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

Fricas [A]
time = 1.42, size = 127, normalized size = 2.44

$$\left[\frac{\sqrt{a}\,bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\,a}{2\,ax^2},\ \frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}\,a}{ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 + a
*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(a+bx)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

Giac [A]
time = 0.96, size = 45, normalized size = 0.87

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{\sqrt{bx+a}\,b\mathrm{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - sqrt(b*x + a)*b*sgn(x
)/x)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{b\,x^3+a\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^3,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^3, x)
```

3.238 $\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-1/2*(b*x^3+a*x^2)^(1
/2)/x^3-1/4*b*(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2045, 2050, 2033, 212}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3]/x^3 - (b*Sqrt[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{2x^3}+\frac{1}{4}b\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{b^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}+\frac{b^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 81, normalized size = 0.96

$$\frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\,\sqrt{a+bx}\,(2a+bx)+b^2x^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(2*a + b*x)) + b^2*x^2*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(3/2)*x^3*Sqrt[a + b*x])
```

Maple [A]
time = 0.42, size = 73, normalized size = 0.87

method	result	size
risch	$-\frac{(bx+2a)\sqrt{x^2\,(bx+a)}}{4x^3a}+\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2\,(bx+a)}}{4a^{\frac{3}{2}}x\sqrt{bx+a}}$	69

default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left((bx+a)^{\frac{3}{2}} a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\,b^{2} x^{2}+\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{4 x^{3} \sqrt{bx+a}\,a^{\frac{5}{2}}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/4*(b*x^3+a*x^2)^(1/2)*((b*x+a)^(3/2)*a^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/
2))*a*b^2*x^2+(b*x+a)^(1/2)*a^(5/2))/x^3/(b*x+a)^(1/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

Fricas [A]
time = 1.74, size = 149, normalized size = 1.77

$$\left[\frac{\sqrt{a}\,b^{2}x^{3}\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\,\sqrt{a}}{x^{2}}\right)-2\,\sqrt{bx^{3}+ax^{2}}\,(abx+2\,a^{2})}{8\,a^{2}x^{3}},\,-\frac{\sqrt{-a}\,b^{2}x^{3}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^{3}+ax^{2}}\,(abx+2\,a^{2})}{4\,a^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] [1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^2*
x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(a*b*x
 + 2*a^2))/(a^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}\,(a+bx)}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**4,x)
[Out] Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

Giac [A]
time = 1.37, size = 72, normalized size = 0.86

$$-\frac{\frac{b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a} + \frac{(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x) + \sqrt{bx+a}\,ab^3\mathrm{sgn}(x)}{ab^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + ((b*x + a)^(
3/2)*b^3*sgn(x) + sqrt(b*x + a)*a*b^3*sgn(x))/(a*b^2*x^2))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^4, x)
```

3.239 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/3*(b*x^3+a*x^2)^(
1/2)/x^4-1/12*b*(b*x^3+a*x^2)^(1/2)/a/x^3+1/8*b^2*(b*x^3+a*x^2)^(1/2)/a^2/x
^2
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2033, 212}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^3]/x^4 - (b*Sqrt[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*Sq
rt[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/(8*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{3x^4}+\frac{1}{6}b\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}+\frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+b}}\right)}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 0.83

$$-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\,\sqrt{a+bx}\,(8a^2+2abx-3b^2x^2)+3b^3x^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{5/2}x^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] -1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2*
x^2) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2)*x^4*Sqrt[a + b*x
])
```

Maple [A]
time = 0.39, size = 89, normalized size = 0.79

method	result	size
risch	$-\frac{(-3b^2x^2+2abx+8a^2)\sqrt{x^2(bx+a)}}{24x^4a^2}-\frac{b^3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{5}{2}}x\sqrt{bx+a}}$	81
default	$\frac{\sqrt{b\,x^3+a\,x^2}\,\left(3(bx+a)^{\frac{5}{2}}a^{\frac{5}{2}}-8(bx+a)^{\frac{3}{2}}a^{\frac{7}{2}}-3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^3x^3-3\sqrt{bx+a}\,a^{\frac{9}{2}}\right)}{24x^4\sqrt{bx+a}\,a^{\frac{9}{2}}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(b*x^3+a*x^2)^(1/2)*(3*(b*x+a)^(5/2)*a^(5/2)-8*(b*x+a)^(3/2)*a^(7/2)-3
*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*b^3*x^3-3*(b*x+a)^(1/2)*a^(9/2))/x^4/(b
*x+a)^(1/2)/a^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

Fricas [A]
time = 1.33, size = 175, normalized size = 1.56

$$\left[\frac{3\sqrt{a}\,b^3x^4\log\left(\frac{bx^2+2\,ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^3x^4},\ \frac{3\sqrt{-a}\,b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^3+ax^2}}{24\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)
, 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (3*
a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(a+bx)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

Giac [A]
time = 1.49, size = 92, normalized size = 0.82

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}\,b^4\mathrm{sgn}(x)-8\,(bx+a)^{\frac{3}{2}}\,ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}\,a^2b^4\mathrm{sgn}(x)}{a^2b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(5/2)*b^4*sgn(x) - 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^
2*b^4*sgn(x))/(a^2*b^3*x^3))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^5, x)
```

3.240 $\int x^2(ax^2 + bx^3)^{3/2}\, dx$

Optimal. Leaf size=161

$$\frac{2(ax^2+bx^3)^{5/2}}{15b} - \frac{512a^5(ax^2+bx^3)^{5/2}}{45045b^6x^5} + \frac{256a^4(ax^2+bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3(ax^2+bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^2} - \frac{4a}{}$$

```
[Out] 2/15*(b*x^3+a*x^2)^(5/2)/b-512/45045*a^5*(b*x^3+a*x^2)^(5/2)/b^6/x^5+256/90
09*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^4-64/1287*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^3+
32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^2-4/39*a*(b*x^3+a*x^2)^(5/2)/b^2/x
```

Rubi [A]
time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$-\frac{512a^5(ax^2+bx^3)^{5/2}}{45045b^6x^5} + \frac{256a^4(ax^2+bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3(ax^2+bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^2} - \frac{4a(ax^2+bx^3)^{5/2}}{39b^2x} + \frac{2(ax^2+bx^3)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b
^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 +
 b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2
) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{(2a)\int x\left(ax^2+bx^3\right)^{3/2}\,dx}{3b}\\ &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{\left(16a^2\right)\int\left(ax^2+bx^3\right)^{3/2}\,dx}{39b^2}\\ &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}-\frac{\left(32a^3\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,d}{143b^3}\\ &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\\ &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/}}{429b^3x^2}\\ &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5}}{1287b^4x^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.50

$$\frac{2x(a+bx)^3\left(-256a^5+640a^4bx-1120a^3b^2x^2+1680a^2b^3x^3-2310ab^4x^4+3003b^5x^5\right)}{45045b^6\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3*
x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.37, size = 79, normalized size = 0.49

method	result	size
gosper	$-\frac{2(bx+a)\left(-3003x^5b^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3x^2b^2-640a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{45045b^6x^3}$	79
default	$-\frac{2(bx+a)\left(-3003x^5b^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3x^2b^2-640a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{45045b^6x^3}$	79

risch	$-\frac{2\sqrt{x^{2}\left(b x +a \right)}\, \left(-3003 b^{7} x^{7}-3696 a \,b^{6} x^{6}-63 a^{2} b^{5} x^{5}+70 a^{3} b^{4} x^{4}-80 a^{4} x^{3} b^{3}+96 a^{5} x^{2} b^{2}-128 a^{6} b x +256 a^{7}\right)}{45045 x \,b^{6}}$	94
trager	$-\frac{2 \left(-3003 b^{7} x^{7}-3696 a \,b^{6} x^{6}-63 a^{2} b^{5} x^{5}+70 a^{3} b^{4} x^{4}-80 a^{4} x^{3} b^{3}+96 a^{5} x^{2} b^{2}-128 a^{6} b x +256 a^{7}\right) \sqrt{b \,x^{3}+a \,x^{2}}}{45045 b^{6} x}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45045*(b*x+a)*(-3003*b^5*x^5+2310*a*b^4*x^4-1680*a^2*b^3*x^3+1120*a^3*b^
2*x^2-640*a^4*b*x+256*a^5)*(b*x^3+a*x^2)^(3/2)/b^6/x^3
```

Maxima [A]
time = 0.30, size = 86, normalized size = 0.53

$$\frac{2 \left(3003 \, b^{7} x^{7} + 3696 \, a b^{6} x^{6} + 63 \, a^{2} b^{5} x^{5} - 70 \, a^{3} b^{4} x^{4} + 80 \, a^{4} b^{3} x^{3} - 96 \, a^{5} b^{2} x^{2} + 128 \, a^{6} b x - 256 \, a^{7}\right)} \sqrt{b x + a}}{45045 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^6
```

Fricas [A]
time = 1.77, size = 95, normalized size = 0.59

$$\frac{2 \left(3003 \, b^{7} x^{7} + 3696 \, a b^{6} x^{6} + 63 \, a^{2} b^{5} x^{5} - 70 \, a^{3} b^{4} x^{4} + 80 \, a^{4} b^{3} x^{3} - 96 \, a^{5} b^{2} x^{2} + 128 \, a^{6} b x - 256 \, a^{7}\right)} \sqrt{b x^{3} + a x^{2}}}{45045 \, b^{6} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x^2
)/(b^6*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(x^{2} \left(a + b x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(137) = 274$.

time = 1.28, size = 282, normalized size = 1.75

$$\frac{512\,a^{\frac{15}{2}}\operatorname{sgn}(x)}{45045\,b^6}+\frac{2\left(\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a^2\operatorname{sgn}(x)}{b^5}+\frac{30\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\,\sqrt{bx+a}\,a^6\right)a\operatorname{sgn}(x)}{b^5}+\frac{7\left(429\,(bx+a)^{\frac{15}{2}}-3465\,(bx+a)^{\frac{13}{2}}a+12285\,(bx+a)^{\frac{11}{2}}a^2-25025\,(bx+a)^{\frac{9}{2}}a^3+32175\,(bx+a)^{\frac{7}{2}}a^4-27027\,(bx+a)^{\frac{5}{2}}a^5+15015\,(bx+a)^{\frac{3}{2}}a^6-6435\,\sqrt{bx+a}\,a^7\right)\operatorname{sgn}(x)}{b^5}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(65*(63*(b*x + a)^(11/2) - 385*(b*x
 + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(
b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^5 + 30*(231*(b*x +
 a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x
 + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 300
3*sqrt(b*x + a)*a^6)*a*sgn(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a
)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175
*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^
6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^5)/b
```

Mupad [B]

time = 5.24, size = 80, normalized size = 0.50

$$-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(256\,a^5-640\,a^4\,b\,x+1120\,a^3\,b^2\,x^2-1680\,a^2\,b^3\,x^3+2310\,a\,b^4\,x^4-3003\,b^5\,x^5\right)}{45045\,b^6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(256*a^5 - 3003*b^5*x^5 + 2310*a*b^4*
x^4 + 1120*a^3*b^2*x^2 - 1680*a^2*b^3*x^3 - 640*a^4*b*x))/(45045*b^6*x)
```

3.241 $\int x(ax^2+bx^3)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{256a^4(ax^2+bx^3)^{5/2}}{15015b^5x^5}-\frac{128a^3(ax^2+bx^3)^{5/2}}{3003b^4x^4}+\frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^3}-\frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2}+\frac{2(ax^2+bx^3)^{5/2}}{13bx}$$

```
[Out] 256/15015*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^5-128/3003*a^3*(b*x^3+a*x^2)^(5/2)/
b^4/x^4+32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^3-16/143*a*(b*x^3+a*x^2)^(5/2)
/b^2/x^2+2/13*(b*x^3+a*x^2)^(5/2)/b/x
```

Rubi [A]
time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2041, 2027, 2039}

$$\frac{256a^4(ax^2+bx^3)^{5/2}}{15015b^5x^5}-\frac{128a^3(ax^2+bx^3)^{5/2}}{3003b^4x^4}+\frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^3}-\frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2}+\frac{2(ax^2+bx^3)^{5/2}}{13bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^
(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*
a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(ax^2+bx^3\right)^{3/2}\,dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{(8a)\int\left(ax^2+bx^3\right)^{3/2}\,dx}{13b}\\
&= -\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}+\frac{(48a^2)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx}{143b^2}\\
&= \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{(64a^3)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,d}{429b^3}\\
&= -\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/}}{13bx}\\
&= \frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a(ax^2+bx^3}{143b^2x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.51

$$
\frac{2x(a+bx)^3\left(128a^4-320a^3bx+560a^2b^2x^2-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3 +
 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.37, size = 68, normalized size = 0.50

method	result	size
gosper	$\frac{2(bx+a)\left(1155b^4x^4-840a\,b^3x^3+560a^2b^2x^2-320a^3bx+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015b^5x^3}$	68
default	$\frac{2(bx+a)\left(1155b^4x^4-840a\,b^3x^3+560a^2b^2x^2-320a^3bx+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015b^5x^3}$	68
risch	$\frac{2\sqrt{x^2\left(bx+a\right)}\,\left(1155b^6x^6+1470a\,b^5x^5+35a^2b^4x^4-40a^3b^3x^3+48a^4x^2b^2-64a^5bx+128a^6\right)}{15015x\,b^5}$	83
trager	$\frac{2\left(1155b^6x^6+1470a\,b^5x^5+35a^2b^4x^4-40a^3b^3x^3+48a^4x^2b^2-64a^5bx+128a^6\right)\sqrt{b\,x^3+a\,x^2}}{15015b^5x}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15015*(b*x+a)*(1155*b^4*x^4-840*a*b^3*x^3+560*a^2*b^2*x^2-320*a^3*b*x+128
*a^4)*(b*x^3+a*x^2)^(3/2)/b^5/x^3
```

Maxima [**A**]
time = 0.30, size = 75, normalized size = 0.55

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}}{15015\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

Fricas [**A**]
time = 1.28, size = 84, normalized size = 0.62

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx^3+ax^2}}{15015\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x))**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(116) = 232.
time = 1.49, size = 246, normalized size = 1.81

$$-\frac{256\,a^{\frac{13}{2}}\mathrm{sgn}(x)}{15015\,b^5}+\frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a^2\mathrm{sgn}(x)}{b^4}+\frac{130\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a\mathrm{sgn}(x)}{b^4}+\frac{15\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\,\sqrt{bx+a}\,a^6\right)\mathrm{sgn}(x)}{b^4}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] -256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2*sgn(x)/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*sgn(x)/b^4 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*sgn(x)/b^4)/b

Mupad [B]
time = 5.24, size = 69, normalized size = 0.51

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(128\,a^4-320\,a^3\,b\,x+560\,a^2\,b^2\,x^2-840\,a\,b^3\,x^3+1155\,b^4\,x^4\right)}{15015\,b^5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x^2 + b*x^3)^(3/2),x)

[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(128*a^4 + 1155*b^4*x^4 - 840*a*b^3*x^3 + 560*a^2*b^2*x^2 - 320*a^3*b*x))/(15015*b^5*x)

3.242 $\int (ax^2+bx^3)^{3/2}\,dx$

Optimal. Leaf size=108

$$-\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5}+\frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4}-\frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3}+\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}$$

```
[Out] -32/1155*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^5+16/231*a^2*(b*x^3+a*x^2)^(5/2)/b^3
/x^4-4/33*a*(b*x^3+a*x^2)^(5/2)/b^2/x^3+2/11*(b*x^3+a*x^2)^(5/2)/b/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {2027, 2041, 2039}

$$-\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5}+\frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4}-\frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3}+\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5
/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 +
 b*x^3)^(5/2))/(11*b*x^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3\right)^{3/2}\,dx &= \frac{2(ax^2+bx^3)^{5/2}}{11bx^2}-\frac{(6a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx}{11b}\\ &= -\frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3}+\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}+\frac{(8a^2)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx}{33b^2}\\ &= \frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4}-\frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3}+\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}-\frac{(16a^3)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx}{231b^3}\\ &= -\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5}+\frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4}-\frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3}+\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.54

$$\frac{2x(a+bx)^3\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(1155
*b^4*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.36, size = 57, normalized size = 0.53

method	result	size
gosper	$-\frac{2(bx+a)\left(-105b^3x^3+70a\,b^2x^2-40a^2bx+16a^3\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{1155b^4x^3}$	57
default	$-\frac{2(bx+a)\left(-105b^3x^3+70a\,b^2x^2-40a^2bx+16a^3\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{1155b^4x^3}$	57
risch	$-\frac{2\sqrt{x^2\left(bx+a\right)}\,\left(-105x^5b^5-140a\,b^4x^4-5a^2b^3x^3+6a^3x^2b^2-8a^4bx+16a^5\right)}{1155x\,b^4}$	72
trager	$-\frac{2\left(-105x^5b^5-140a\,b^4x^4-5a^2b^3x^3+6a^3x^2b^2-8a^4bx+16a^5\right)\sqrt{b\,x^3+a\,x^2}}{1155b^4x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] -2/1155*(b*x+a)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)*(b*x^3+a*x^2)^(3/2)/b^4/x^3

Maxima [A]
time = 0.28, size = 64, normalized size = 0.59

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4*b*x - 16*a^5)*sqrt(b*x + a)/b^4

Fricas [A]
time = 1.46, size = 73, normalized size = 0.68

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{1155\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^2+bx^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)**(3/2),x)

[Out] Integral((a*x**2 + b*x**3)**(3/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(92) = 184.
time = 1.39, size = 210, normalized size = 1.94

$$\frac{32\,a^{\frac{11}{2}}\operatorname{sgn}(x)}{1155\,b^4}+\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2\operatorname{sgn}(x)}{b^3}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a\operatorname{sgn}(x)}{b^3}+\frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)\operatorname{sgn}(x)}{b^3}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")

```
[Out] 32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 + 2
2*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 4
20*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b*x +
 a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x +
a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(x)/b^3
)/b
```

Mupad [B]
time = 5.19, size = 58, normalized size = 0.54

$$-\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
- 40*a^2*b*x))/(1155*b^4*x)
```

3.243 $\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx$

Optimal. Leaf size=80

$$\frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5}-\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4}+\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

```
[Out] 16/315*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^5-8/63*a*(b*x^3+a*x^2)^(5/2)/b^2/x^4+2
/9*(b*x^3+a*x^2)^(5/2)/b/x^3
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$\frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5}-\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4}+\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/
(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{9bx^3} - \frac{(4a)\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx}{9b}$$
$$= -\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3} + \frac{(8a^2)\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx}{63b^2}$$
$$= \frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5} - \frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.59

$$\frac{2x(a+bx)^3(8a^2-20abx+35b^2x^2)}{315b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
[Out] (2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b*
x)])
```

Maple [A]
time = 0.36, size = 46, normalized size = 0.58

method	result	size
gosper	$\frac{2(bx+a)(35b^2x^2-20abx+8a^2)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{315b^3x^3}$	46
default	$\frac{2(bx+a)(35b^2x^2-20abx+8a^2)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{315b^3x^3}$	46
risch	$\frac{2\sqrt{x^2(bx+a)}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315x\,b^3}$	61
trager	$\frac{2(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)\sqrt{b\,x^3+a\,x^2}}{315b^3x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/315*(b*x+a)*(35*b^2*x^2-20*a*b*x+8*a^2)*(b*x^3+a*x^2)^(3/2)/b^3/x^3
```

Maxima [A]
time = 0.29, size = 53, normalized size = 0.66

$$\frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

Fricas [A]
time = 1.24, size = 62, normalized size = 0.78

$$\frac{2\left(35\,b^{4}x^{4}+50\,ab^{3}x^{3}+3\,a^{2}b^{2}x^{2}-4\,a^{3}bx+8\,a^{4}\right)\sqrt{bx^{3}+ax^{2}}}{315\,b^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}(a+bx)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(68) = 136.
time = 1.87, size = 173, normalized size = 2.16

$$-\frac{16\,a^{\frac{9}{2}}\mathrm{sgn}\left(x\right)}{315\,b^{3}}+\frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a^{2}\mathrm{sgn}\left(x\right)}{b^{2}}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a\mathrm{sgn}\left(x\right)}{b^{2}}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)\mathrm{sgn}\left(x\right)}{b^{2}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*sgn(x)/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(
b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/
b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^2)/b
```

Mupad [B]
time = 5.18, size = 47, normalized size = 0.59

$$\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(8\,a^2-20\,a\,b\,x+35\,b^2\,x^2\right)}{315\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*b*x))/(315*
b^3*x)
```

3.244 $\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=52

$$-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}+\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}$$

```
[Out] -4/35*a*(b*x^3+a*x^2)^(5/2)/b^2/x^5+2/7*(b*x^3+a*x^2)^(5/2)/b/x^4
```

Rubi [A]
time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*
x^4)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx &= \frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{(2a)\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx}{7b}\\ &= -\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}+\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.60

$$\frac{2(x^2(a+bx))^{5/2}(-2a+5bx)}{35b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
[Out] (2*(x^2*(a + b*x))^(5/2)*(-2*a + 5*b*x))/(35*b^2*x^5)
```

Maple [A]
time = 0.36, size = 35, normalized size = 0.67

method	result	size
gosper	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
default	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
risch	$-\frac{2\sqrt{x^{2}\left(bx+a\right)}\,\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)}{35x\,b^{2}}$	50
trager	$-\frac{2\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35b^{2}x}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.79

$$\frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Fricas [A]
time = 1.10, size = 50, normalized size = 0.96

$$\frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(44) =$ 88.
time = 1.34, size = 136, normalized size = 2.62

$$\frac{4\,a^{\frac{7}{2}}\operatorname{sgn}(x)}{35\,b^2}+\frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^2\operatorname{sgn}(x)}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\operatorname{sgn}(x)}{b}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)\operatorname{sgn}(x)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a
^2*sgn(x)/b + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*a*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x
+ a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b)/b
```

Mupad [B]
time = 5.17, size = 36, normalized size = 0.69

$$-\frac{2\,\left(2\,a-5\,b\,x\right)\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2}{35\,b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^2,x)
```

```
[Out] -(2*(2*a - 5*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(35*b^2*x)
```

3.245 $\int \frac{(ax^2+bx^3)^{3/2}}{x^3} dx$

Optimal. Leaf size=25

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

```
[Out] 2/5*(b*x^3+a*x^2)^(5/2)/b/x^5
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.92

$$\frac{2(x^2(a+bx))^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
[Out] (2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

Maple [A]
time = 0.36, size = 27, normalized size = 1.08

method	result	size
gosper	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
default	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
risch	$\frac{2\sqrt{x^{2}\left(bx+a\right)}\,\left(b^{2}x^{2}+2abx+a^{2}\right)}{5xb}$	36
trager	$\frac{2\left(b^{2}x^{2}+2abx+a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{5bx}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 2/5*(b*x+a)*(b*x^3+a*x^2)^(3/2)/b/x^3
```

Maxima [A]
time = 0.28, size = 28, normalized size = 1.12

$$\frac{2\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Fricas [A]
time = 2.05, size = 37, normalized size = 1.48

$$\frac{2\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{bx^{3}+ax^{2}}}{5\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}(a+bx)\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(21) = 42$.
time = 1.56, size = 89, normalized size = 3.56

$$-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b}+\frac{2\left(15\,\sqrt{bx+a}\,a^{2}\mathrm{sgn}(x)+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\mathrm{sgn}(x)+\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)\mathrm{sgn}(x)\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/5*a^(5/2)*sgn(x)/b + 2/15*(15*sqrt(b*x + a)*a^2*sgn(x) + 10*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a*sgn(x) + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*sgn(x))/b
```

Mupad [B]
time = 5.62, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2}{5\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^3,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(5*b*x)
```

3.246 $\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=74

$$\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)/x^3-2*a^(3/2)*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)+2*a*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2046, 2033, 212}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)+\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2(ax^2+bx^3)^{3/2}}{3x^3} + a\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$$
$$= \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3} + a^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3} - (2a^2)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)$$
$$= \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3} - 2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.92

$$\frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}\,(4a+bx) - 3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.37, size = 61, normalized size = 0.82

method	result	size
default	$\frac{2\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-3a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+(bx+a)^{\frac{3}{2}}+3a\sqrt{bx+a}\right)}{3x^3\,(bx+a)^{\frac{3}{2}}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)*(-3*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^
(3/2)+3*a*(b*x+a)^(1/2))/x^3/(b*x+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Fricas [A]
time = 2.75, size = 130, normalized size = 1.76

$$\left[\frac{3\,a^{\frac{3}{2}}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,(bx+4\,a)}{3\,x},\ \frac{2\,\left(3\,\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,(bx+4\,a)\right)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
+ 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b*x
^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

Giac [A]
time = 1.28, size = 85, normalized size = 1.15

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}\mathrm{sgn}(x)+2\,\sqrt{bx+a}\,a\mathrm{sgn}(x)-\frac{2\,\left(3\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\,\sqrt{-a}\,a^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)*
sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) + 4
*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^4, x)
```

3.247 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5} dx$

Optimal. Leaf size=73

$$\frac{3b\sqrt{ax^2+bx^3}}{x} - \frac{(ax^2+bx^3)^{3/2}}{x^4} - 3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] -(b*x^3+a*x^2)^(3/2)/x^4-3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2)
+3*b*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2046, 2033, 212}

$$\frac{3b\sqrt{ax^2+bx^3}}{x} - 3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right) - \frac{(ax^2+bx^3)^{3/2}}{x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTa
nh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx &= -\frac{(ax^2+bx^3)^{3/2}}{x^4}+\frac{1}{2}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx \\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{(ax^2+bx^3)^{3/2}}{x^4}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx \\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{(ax^2+bx^3)^{3/2}}{x^4}-(3ab)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right) \\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{(ax^2+bx^3)^{3/2}}{x^4}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.90

$$-\frac{\sqrt{a+bx}\left((a-2bx)\sqrt{a+bx}+3\sqrt{a}\,bx\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
[Out] -((Sqrt[a + b*x]*((a - 2*b*x)*Sqrt[a + b*x] + 3*Sqrt[a]*b*x*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]]))/Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.38, size = 72, normalized size = 0.99

method	result	size
risch	$-\frac{a\sqrt{x^2(bx+a)}}{x^2}+\frac{b\left(4\sqrt{bx+a}-6\sqrt{a}\ \text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{x^2(bx+a)}}{2x\sqrt{bx+a}}$	70
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(-2\sqrt{bx+a}\ bx\sqrt{a}+3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+\sqrt{bx+a}\ a^{\frac{3}{2}}\right)}{x^4(bx+a)^{\frac{3}{2}}\sqrt{a}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^3+a*x^2)^(3/2)*(-2*(b*x+a)^(1/2)*b*x*a^(1/2)+3*arctanh((b*x+a)^(1/2)/
a^(1/2))*a*b*x+(b*x+a)^(1/2)*a^(3/2))/x^4/(b*x+a)^(3/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Fricas [A]
time = 2.04, size = 136, normalized size = 1.86

$$\left[\frac{3\sqrt{a}\,bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,(2\,bx-a)}{2\,x^2},\ \frac{3\,\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,(2\,bx-a)}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

Giac [A]
time = 1.67, size = 62, normalized size = 0.85

$$\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,b^2\mathrm{sgn}(x)-\frac{\sqrt{bx+a}\,ab\mathrm{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*b
^2*sgn(x) - sqrt(b*x + a)*a*b*sgn(x)/x)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^5, x)
```

3.248 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6} dx$

Optimal. Leaf size=81

$$-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}$$

```
[Out] -1/2*(b*x^3+a*x^2)^(3/2)/x^5-3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))
/a^(1/2)-3/4*b*(b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2045, 2033, 212}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^(3/2)/(2*x^5) - (3*b^2
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}+\frac{1}{4}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}-\frac{1}{4}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 82, normalized size = 1.01

$$
-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\,\sqrt{a+bx}\,(2a+5bx)+3b^2x^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}\,x^3\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] -1/4*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(2*a + 5*b*x) + 3*b^2*x^2*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^3*Sqrt[a + b*x])
```

Maple [A]
time = 0.38, size = 74, normalized size = 0.91

method	result	size
risch	$-\frac{(5bx+2a)\sqrt{x^2\,(bx+a)}}{4x^3}-\frac{3b^2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2\,(bx+a)}}{4\sqrt{a}\,x\sqrt{bx+a}}$	67
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2+5(bx+a)^{\frac{3}{2}}\sqrt{a}-3\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{4x^5(bx+a)^{\frac{3}{2}}\sqrt{a}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x^3+a*x^2)^(3/2)*(3*arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2+5*(b*x+a
)^(3/2)*a^(1/2)-3*(b*x+a)^(1/2)*a^(3/2))/x^5/(b*x+a)^(3/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

Fricas [A]
time = 1.10, size = 154, normalized size = 1.90

$$\left[\frac{3\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\,(5\,abx+2\,a^2)}{8\,ax^3},\ \frac{3\,\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}\,(5\,abx+2\,a^2)}{4\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)*b
^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*(5*
a*b*x + 2*a^2))/(a*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

Giac [A]
time = 1.03, size = 70, normalized size = 0.86

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)-3\,\sqrt{bx+a}\,ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(3
/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3)^(3/2)/x^6,x)

[Out] int((a*x^2 + b*x^3)^(3/2)/x^6, x)

3.249 $\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=109

$$-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}$$

```
[Out] -1/3*(b*x^3+a*x^2)^(3/2)/x^6+1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))
/a^(3/2)-1/4*b*(b*x^3+a*x^2)^(1/2)/x^3-1/8*b^2*(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2045, 2050, 2033, 212}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] -1/4*(b*Sqrt[a*x^2 + b*x^3])/x^3 - (b^2*Sqrt[a*x^2 + b*x^3])/(8*a*x^2) - (a
*x^2 + b*x^3)^(3/2)/(3*x^6) + (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/(8*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx &= -\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{1}{2}b\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{1}{8}b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}-\frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{b^3\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2 \cdots}}\right.}{8a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 94, normalized size = 0.86

$$\frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\,\sqrt{a+bx}\,(8a^2+14abx+3b^2x^2)+3b^3x^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{3/2}x^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^2
)) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(24*a^(3/2)*x^4*Sqrt[a + b*
x])
```

Maple [**A**]
time = 0.37, size = 87, normalized size = 0.80

method	result	size
risch	$-\frac{(3b^2x^2+14abx+8a^2)\sqrt{x^2(bx+a)}}{24x^4a}+\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{3}{2}}x\sqrt{bx+a}}$	81
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(3(bx+a)^{\frac{5}{2}}a^{\frac{3}{2}}-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^3x^3+8(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}-3\sqrt{bx+a}\,a^{\frac{7}{2}}\right)}{24x^6(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(b*x^3+a*x^2)^(3/2)*(3*(b*x+a)^(5/2)*a^(3/2)-3*arctanh((b*x+a)^(1/2)/
a^(1/2))*a*b^3*x^3+8*(b*x+a)^(3/2)*a^(5/2)-3*(b*x+a)^(1/2)*a^(7/2))/x^6/(b*
x+a)^(3/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Fricas [A]
time = 1.81, size = 175, normalized size = 1.61

$$\left[\frac{3\,\sqrt{a}\,b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^2x^4},-\frac{3\,\sqrt{-a}\,b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{24\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4
), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(a+bx))^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

Giac [A]
time = 1.56, size = 92, normalized size = 0.84

$$-\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a}+\frac{3\,(bx+a)^{\frac{5}{2}}\,b^4\mathrm{sgn}(x)+8\,(bx+a)^{\frac{3}{2}}\,ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}\,a^2b^4\mathrm{sgn}(x)}{ab^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x +
 a)^(5/2)*b^4*sgn(x) + 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^2
*b^4*sgn(x))/(a*b^3*x^3))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^7, x)
```

3.250 $\int \frac{(ax^2+bx^3)^{3/2}}{x^8} dx$

Optimal. Leaf size=137

$$-\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}$$

```
[Out] -1/4*(b*x^3+a*x^2)^(3/2)/x^7-3/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)/a^(5/2)-1/8*b*(b*x^3+a*x^2)^(1/2)/x^4-1/32*b^2*(b*x^3+a*x^2)^(1/2)/a/x^3+
3/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2033, 212}

$$-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] -1/8*(b*Sqrt[a*x^2 + b*x^3])/x^4 - (b^2*Sqrt[a*x^2 + b*x^3])/(32*a*x^3) + (
3*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^2) - (a*x^2 + b*x^3)^(3/2)/(4*x^7) - (
3*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx &= -\frac{(ax^2+bx^3)^{3/2}}{4x^7}+\frac{1}{8}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}+\frac{1}{16}b^2\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{(3b^3)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{64a} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}+\frac{(3b^4)\int \frac{}{\sqrt{a}}}{12} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{(3b^4)\,\mathrm{Subst}}{} \\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{3b^4\tanh^{-1}}{}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 104, normalized size = 0.76

$$-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}\,(16a^3+24a^2bx+2ab^2x^2-3b^3x^3)+3b^4x^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{64a^{5/2}x^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] -1/64*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(16*a^3 + 24*a^2*b*x + 2*
a*b^2*x^2 - 3*b^3*x^3) + 3*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2
)*x^5*Sqrt[a + b*x])
```

Maple [A]
time = 0.51, size = 101, normalized size = 0.74

method	result	size
risch	$-\frac{\left(-3 b^{3} x^{3}+2 a \,b^{2} x^{2}+24 a^{2} b x +16 a^{3}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 x^{5} a^{2}}-\frac{3 b^{4} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x +a \right)}}{64 a^{\frac{5}{2}} x \sqrt{b x +a}}$	92
default	$\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(3 (b x +a)^{\frac{7}{2}} a^{\frac{5}{2}}-11 (b x +a)^{\frac{5}{2}} a^{\frac{7}{2}}-3 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{2} b^{4} x^{4}-11 (b x +a)^{\frac{3}{2}} a^{\frac{9}{2}}+3 \sqrt{b x +a}\, a^{\frac{11}{2}}\right)}{64 x^{7} (b x +a)^{\frac{3}{2}} a^{\frac{9}{2}}}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(b*x^3+a*x^2)^(3/2)*(3*(b*x+a)^(7/2)*a^(5/2)-11*(b*x+a)^(5/2)*a^(7/2)-
3*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*b^4*x^4-11*(b*x+a)^(3/2)*a^(9/2)+3*(b*
x+a)^(1/2)*a^(11/2))/x^7/(b*x+a)^(3/2)/a^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

Fricas [A]
time = 0.94, size = 197, normalized size = 1.44

$$\left[\frac{3 \sqrt{a} b^{4} x^{5} \log\left(\frac{b x^{2}+2 a x-2 \sqrt{b x^{3}+a x^{2}} \sqrt{a}}{x^{2}}\right)+2 \left(3 a b^{3} x^{3}-2 a^{2} b^{2} x^{2}-24 a^{3} b x-16 a^{4}\right) \sqrt{b x^{3}+a x^{2}}}{128 a^{3} x^{5}}, \frac{3 \sqrt{-a} b^{4} x^{5} \arctan\left(\frac{\sqrt{b x^{3}+a x^{2}} \sqrt{-a}}{a x}\right)+\left(3 a b^{3} x^{3}-2 a^{2} b^{2} x^{2}-24 a^{3} b x-16 a^{4}\right) \sqrt{b x^{3}+a x^{2}}}{64 a^{3} x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^3
+ a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)*sq
rt(-a)/(a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*
x^3 + a*x^2))/(a^3*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2} \left(a + b x\right)\right)^{\frac{3}{2}}}{x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

Giac [A]
time = 2.27, size = 109, normalized size = 0.80

$$\frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}\,a^3b^5\mathrm{sgn}(x)}{a^2b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3/2
)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^8,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^8, x)
```

3.251 $\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=165

$$-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}-\frac{(ax^2+bx^3)^{3/2}}{5x^8}+\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}$$

```
[Out] -1/5*(b*x^3+a*x^2)^(3/2)/x^8+3/128*b^5*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2
))/a^(7/2)-3/40*b*(b*x^3+a*x^2)^(1/2)/x^5-1/80*b^2*(b*x^3+a*x^2)^(1/2)/a/x^
4+1/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^3-3/128*b^4*(b*x^3+a*x^2)^(1/2)/a^3/x^
2
```

Rubi [A]
time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2033, 212}

$$\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{(ax^2+bx^3)^{3/2}}{5x^8}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(40*x^5) - (b^2*Sqrt[a*x^2 + b*x^3])/(80*a*x^4)
+ (b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*Sqrt[a*x^2 + b*x^3])/(128
*a^3*x^2) - (a*x^2 + b*x^3)^(3/2)/(5*x^8) + (3*b^5*ArcTanh[(Sqrt[a]*x)/Sqrt
[a*x^2 + b*x^3]])/(128*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
```

```
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = -\frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{1}{10}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{1}{80}(3b^2)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} - \frac{b^3\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{32a}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{(3b^4)\int \frac{}{x\sqrt{(}}}{12}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx}{5x^8}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx}{5x^8}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx}{5x^8}$$

Mathematica [A]
time = 0.15, size = 116, normalized size = 0.70

$$\frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}\,(128a^4+176a^3bx+8a^2b^2x^2-10ab^3x^3+15b^4x^4)+15b^5x^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{640a^{7/2}x^6\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(128*a^4 + 176*a^3*b*x + 8*a^
2*b^2*x^2 - 10*a*b^3*x^3 + 15*b^4*x^4)) + 15*b^5*x^5*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]]))/(640*a^(7/2)*x^6*Sqrt[a + b*x])
```

Maple [A]
time = 0.43, size = 113, normalized size = 0.68

method	result
risch	$-\frac{\left(15 b^{4} x^{4}-10 a\,b^{3} x^{3}+8 a^{2} b^{2} x^{2}+176 a^{3} b x+128 a^{4}\right) \sqrt{x^{2} \left(b x+a\right)}}{640 x^{6} a^{3}}+\frac{3 b^{5} \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(b x+a\right)}}{128 a^{\frac{7}{2}} x \sqrt{b x+a}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} \left(15 \left(b x+a\right)^{\frac{9}{2}} a^{\frac{7}{2}}-70 \left(b x+a\right)^{\frac{7}{2}} a^{\frac{9}{2}}+128 \left(b x+a\right)^{\frac{5}{2}} a^{\frac{11}{2}}-15 \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) a^{3} b^{5} x^{5}+70 \left(b x+a\right)^{\frac{3}{2}} a^{\frac{13}{2}}-15 \sqrt{bx}\right.}{640 x^{8} \left(b x+a\right)^{\frac{3}{2}} a^{\frac{13}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(b*x^3+a*x^2)^(3/2)*(15*(b*x+a)^(9/2)*a^(7/2)-70*(b*x+a)^(7/2)*a^(9/
2)+128*(b*x+a)^(5/2)*a^(11/2)-15*arctanh((b*x+a)^(1/2)/a^(1/2))*a^3*b^5*x^5
+70*(b*x+a)^(3/2)*a^(13/2)-15*(b*x+a)^(1/2)*a^(15/2))/x^8/(b*x+a)^(3/2)/a^(
13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)
```

Fricas [A]
time = 1.94, size = 219, normalized size = 1.33

$$\left[\frac{15\,\sqrt{a}\,b^{5}x^{6}\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\,\sqrt{a}}{x^{2}}\right)-2\,(15\,ab^{4}x^{4}-10\,a^{2}b^{3}x^{3}+8\,a^{3}b^{2}x^{2}+176\,a^{4}bx+128\,a^{5})\sqrt{bx^{3}+ax^{2}}}{1280\,a^{4}x^{6}},\,-\frac{15\,\sqrt{-a}\,b^{5}x^{6}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\,\sqrt{-a}}{ax}\right)+(15\,ab^{4}x^{4}-10\,a^{2}b^{3}x^{3}+8\,a^{3}b^{2}x^{2}+176\,a^{4}bx+128\,a^{5})\sqrt{bx^{3}+ax^{2}}}{640\,a^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x
+ 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*arct
```

```
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8
*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**9, x)
```

Giac [A]
time = 1.58, size = 126, normalized size = 0.76

$$-\frac{\frac{15\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^3}+\frac{15\,(bx+a)^{\frac{9}{2}}b^6\mathrm{sgn}(x)-70\,(bx+a)^{\frac{7}{2}}ab^6\mathrm{sgn}(x)+128\,(bx+a)^{\frac{5}{2}}a^2b^6\mathrm{sgn}(x)+70\,(bx+a)^{\frac{3}{2}}a^3b^6\mathrm{sgn}(x)-15\,\sqrt{bx+a}\,a^4b^6\mathrm{sgn}(x)}{a^3b^5x^5}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/640*(15*b^6*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*(
b*x + a)^(9/2)*b^6*sgn(x) - 70*(b*x + a)^(7/2)*a*b^6*sgn(x) + 128*(b*x + a)
^(5/2)*a^2*b^6*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b^6*sgn(x) - 15*sqrt(b*x + a
)*a^4*b^6*sgn(x))/(a^3*b^5*x^5))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^9,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^9, x)
```

3.252 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=103

$$\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

```
[Out] 16/35*a^2*(b*x^3+a*x^2)^(1/2)/b^3-32/35*a^3*(b*x^3+a*x^2)^(1/2)/b^4/x-12/35
*a*x*(b*x^3+a*x^2)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x^2)^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 1602}

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^
4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/
(7*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2x^2\sqrt{ax^2+bx^3}}{7b} - \frac{(6a)\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{7b}$$
$$= -\frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b} + \frac{(24a^2)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{35b^2}$$
$$= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b} - \frac{(16a^3)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{35b^3}$$
$$= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.51

$$\frac{2\sqrt{x^2(a+bx)}\,(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35
*b^4*x)
```

Maple [A]
time = 0.37, size = 55, normalized size = 0.53

method	result	size
trager	$-\frac{2(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{35b^4x}$	52
risch	$-\frac{2x(bx+a)(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)}{35\sqrt{x^2\,(bx+a)}\,b^4}$	53
gosper	$-\frac{2(bx+a)(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)x}{35b^4\sqrt{b\,x^3+a\,x^2}}$	55
default	$-\frac{2(bx+a)(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)x}{35b^4\sqrt{b\,x^3+a\,x^2}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/35*(b*x+a)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)*x/b^4/(b*x^3+a*x^2)
^(1/2)
```

Maxima [A]
time = 0.29, size = 53, normalized size = 0.51

$$\frac{2\left(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4\right)}{35\,\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

Fricas [A]
time = 1.23, size = 51, normalized size = 0.50

$$\frac{2\left(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^
4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{x^2\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.75, size = 64, normalized size = 0.62

$$\frac{32\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^4}+\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{35\,b^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/35*a^(7/2)*sgn(x)/b^4 + 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a +
 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/(b^4*sgn(x))
```

Mupad [B]
time = 5.19, size = 51, normalized size = 0.50

$$-\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3\right)}{35\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^(1/2),x)
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2 - 8*a^2*b*x))/(
35*b^4*x)
```

3.253 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=75

$$-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

```
[Out] -8/15*a*(b*x^3+a*x^2)^(1/2)/b^2+16/15*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x+2/5*x*(
b*x^3+a*x^2)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 1602}

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-8*a*Sqrt[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*Sqrt[a*x^2 + b*x^3])/(15*b^3*
x) + (2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2x\sqrt{ax^2+bx^3}}{5b} - \frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b}$$
$$= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2} + \frac{2x\sqrt{ax^2+bx^3}}{5b} + \frac{(8a^2)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{15b^2}$$
$$= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2} + \frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x} + \frac{2x\sqrt{ax^2+bx^3}}{5b}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.56

$$\frac{2\sqrt{x^2(a+bx)}\,(8a^2-4abx+3b^2x^2)}{15b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

Maple [A]
time = 0.36, size = 44, normalized size = 0.59

method	result	size
trager	$\frac{2(3b^2x^2-4abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{15b^3x}$	41
risch	$\frac{2x(bx+a)(3b^2x^2-4abx+8a^2)}{15\sqrt{x^2\,(bx+a)}\;b^3}$	42
gosper	$\frac{2(bx+a)(3b^2x^2-4abx+8a^2)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	44
default	$\frac{2(bx+a)(3b^2x^2-4abx+8a^2)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(b*x+a)*(3*b^2*x^2-4*a*b*x+8*a^2)*x/b^3/(b*x^3+a*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 42, normalized size = 0.56

$$\frac{2\,(3\,b^3x^3 - ab^2x^2 + 4\,a^2bx + 8\,a^3)}{15\,\sqrt{bx+a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

Fricas [A]
time = 2.17, size = 40, normalized size = 0.53

$$\frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.78, size = 52, normalized size = 0.69

$$-\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^3}+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)}{15\,b^3\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] -16/15*a^(5/2)*sgn(x)/b^3 + 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a
+ 15*sqrt(b*x + a)*a^2)/(b^3*sgn(x))
```

Mupad [B]
time = 5.20, size = 40, normalized size = 0.53

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^2-4\,a\,b\,x+3\,b^2\,x^2\right)}{15\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^(1/2),x)
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*b^3*x)
```

3.254 $\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

```
[Out] 2/3*(b*x^3+a*x^2)^(1/2)/b-4/3*a*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Rubi [A]

time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 1602}

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b}\\ &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.61

$$\frac{2(-2a+bx)\sqrt{x^2(a+bx)}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)
```

Maple [A]
time = 0.35, size = 33, normalized size = 0.67

method	result	size
trager	$-\frac{2(-bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{3b^{2}x}$	30
risch	$-\frac{2x(bx+a)(-bx+2a)}{3\sqrt{x^{2}\left(bx+a\right)}\,b^{2}}$	31
gosper	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33
default	$-\frac{2(bx+a)(-bx+2a)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x+a)*(-b*x+2*a)*x/b^2/(b*x^3+a*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.61

$$\frac{2\left(b^{2}x^{2}-abx-2\,a^{2}\right)}{3\,\sqrt{bx+a}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

Fricas [A]
time = 2.88, size = 28, normalized size = 0.57

$$\frac{2\,\sqrt{bx^{3}+ax^{2}}\,(bx-2\,a)}{3\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.14, size = 38, normalized size = 0.78

$$\frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^2}+\frac{2\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)}{3\,b^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*a^(3/2)*sgn(x)/b^2 + 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/(b^2*sgn
(x))
```

Mupad [B]
time = 5.16, size = 31, normalized size = 0.63

$$-\frac{\left(\frac{4\,a}{3\,b^2}-\frac{2\,x}{3\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] -(((4*a)/(3*b^2) - (2*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

3.255 $\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

```
[Out] 2*(b*x^3+a*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1602}

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{bx}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.91

$$\frac{2\sqrt{x^2(a+bx)}}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)])/(b*x)
```

Maple [A]
time = 0.36, size = 25, normalized size = 1.09

method	result	size
trager	$\frac{2\sqrt{b\,x^{3}+a\,x^{2}}}{bx}$	22
risch	$\frac{2x(bx+a)}{\sqrt{x^{2}\left(bx+a\right)}\,b}$	23
gosper	$\frac{2x(bx+a)}{b\sqrt{b\,x^{3}+a\,x^{2}}}$	25
default	$\frac{2x(bx+a)}{b\sqrt{b\,x^{3}+a\,x^{2}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x*(b*x+a)/b/(b*x^3+a*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.52

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A]
time = 1.35, size = 21, normalized size = 0.91

$$\frac{2\sqrt{bx^3+ax^2}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^2\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.37, size = 27, normalized size = 1.17

$$-\frac{2\sqrt{a}\,\mathrm{sgn}(x)}{b}+\frac{2\sqrt{bx+a}}{b\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*sgn(x)/b + 2*sqrt(b*x + a)/(b*sgn(x))
```

Mupad [B]
time = 5.14, size = 17, normalized size = 0.74

$$\frac{2\,|x|\,\sqrt{a+b\,x}}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*abs(x)*(a + b*x)^(1/2))/(b*x)
```

3.256 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2033, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.53

$$-\frac{2x\sqrt{a+bx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a + 
b*x)])
```

Maple [A]
time = 0.36, size = 39, normalized size = 1.30

method	result	size
default	$-\frac{2x\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{a}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b*x^3+a*x^2)^(1/2)*x*(b*x+a)^(1/2)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x^2), x)
```

Fricas [A]
time = 1.36, size = 74, normalized size = 2.47

$$\left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)}{\sqrt{a}}, \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(-
a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3), x)
```

Giac [A]
time = 1.25, size = 45, normalized size = 1.50

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(-
a))/(sqrt(-a)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{b\,x^3 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^3)^(1/2), x)
```

3.257 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
[Out] b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2033, 212}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{ax^2} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a}$$

$$= -\frac{\sqrt{ax^2+bx^3}}{ax^2} + \frac{b\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}$$

$$= -\frac{\sqrt{ax^2+bx^3}}{ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Mathematica [**A**]
time = 0.05, size = 60, normalized size = 1.11

$$\frac{-\sqrt{a}\,(a+bx)+bx\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-(Sqrt[a]*(a + b*x)) + b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(
a^(3/2)*Sqrt[x^2*(a + b*x)])
```

Maple [**A**]
time = 0.37, size = 55, normalized size = 1.02

method	result	size
default	$-\frac{\sqrt{bx+a}\,\left(\sqrt{bx+a}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{5}{2}}}$	55
risch	$-\frac{bx+a}{a\sqrt{x^{2}\left(bx+a\right)}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{a^{\frac{3}{2}}\sqrt{x^{2}\left(bx+a\right)}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1/2)*((b*x+a)^(1/2)*a^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/2))*a*b*x)
/(b*x^3+a*x^2)^(1/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

Fricas [A]
time = 1.99, size = 127, normalized size = 2.35

$$\left[\frac{\sqrt{a}\, bx^2 \log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right) - 2\,\sqrt{bx^3+ax^2}\,a}{2\,a^2x^2}, -\frac{\sqrt{-a}\, bx^2 \arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right) + \sqrt{bx^3+ax^2}\,a}{a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

Giac [A]
time = 1.31, size = 51, normalized size = 0.94

$$-\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{\sqrt{bx+a}\,b}{ax}}{b\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
(b*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

3.258 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}$$

```
[Out] -3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/2*(b*x^3+a*x^2)^(
1/2)/a/x^3+3/4*b*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2033, 212}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^2) -
(3*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}-\frac{(3b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}+\frac{(3b^2)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{(3b^2)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 83, normalized size = 0.95

$$\frac{\sqrt{a}\,(-2a^2+abx+3b^2x^2)-3b^2x^2\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (Sqrt[a]*(-2*a^2 + a*b*x + 3*b^2*x^2) - 3*b^2*x^2*Sqrt[a + b*x]*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.38, size = 77, normalized size = 0.89

method	result	size
risch	$-\frac{(bx+a)(-3bx+2a)}{4a^2x\sqrt{x^2\,(bx+a)}}-\frac{3b^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{4a^{\frac{5}{2}}\sqrt{x^2\,(bx+a)}}$	73
default	$-\frac{\sqrt{bx+a}\,\left(-3\sqrt{bx+a}\,a^{\frac{3}{2}}bx+3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^2x^2+2\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{4x\sqrt{b\,x^3+a\,x^2}\,a^{\frac{7}{2}}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x*(b*x+a)^(1/2)*(-3*(b*x+a)^(1/2)*a^(3/2)*b*x+3*arctanh((b*x+a)^(1/2)/
a^(1/2))*a*b^2*x^2+2*(b*x+a)^(1/2)*a^(5/2))/(b*x^3+a*x^2)^(1/2)/a^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

Fricas [A]
time = 1.29, size = 153, normalized size = 1.76

$$\left[\frac{3\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,(3\,abx-2\,a^2)}{8\,a^3x^3}, \frac{3\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,(3\,abx-2\,a^2)}{4\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(
3*a*b*x - 2*a^2))/(a^3*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

Giac [A]
time = 1.11, size = 73, normalized size = 0.84

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}\,ab^3}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/(b*sgn(x))
```

Mupad [B]
time = 5.41, size = 44, normalized size = 0.51

$$-\frac{2\,\sqrt{\frac{a}{b\,x}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a}{b\,x}\right)}{5\,x\,\sqrt{b\,x^3+a\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -a/(b*x)))/(5*x*(a*x^2 +
 b*x^3)^(1/2))
```

3.259 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}$$

```
[Out] 5/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)-1/3*(b*x^3+a*x^2)^(1
/2)/a/x^4+5/12*b*(b*x^3+a*x^2)^(1/2)/a^2/x^3-5/8*b^2*(b*x^3+a*x^2)^(1/2)/a^
3/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2033, 212}

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) + (5*b*Sqrt[a*x^2 + b*x^3])/(12*a^2*x^3) -
 (5*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[
a*x^2 + b*x^3]])/(8*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} - \frac{(5b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} + \frac{(5b^2)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^2} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} - \frac{(5b^3)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^3} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} + \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, -\ldots\right.}{8a^3} \\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 96, normalized size = 0.83

$$
\frac{-\sqrt{a}\,(8a^3-2a^2bx+5ab^2x^2+15b^3x^3)+15b^3x^3\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-(Sqrt[a]*(8*a^3 - 2*a^2*b*x + 5*a*b^2*x^2 + 15*b^3*x^3)) + 15*b^3*x^3*Sqr
t[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a + b*
x)])
```

Maple [A]
time = 0.39, size = 95, normalized size = 0.83

method	result	size
risch	$-\frac{(bx+a)(15b^2x^2-10abx+8a^2)}{24a^3x^2\sqrt{x^2(bx+a)}}+\frac{5b^3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{8a^{\frac{7}{2}}\sqrt{x^2(bx+a)}}$	84

default	$-\frac{\sqrt{bx+a}\,\left(15\sqrt{bx+a}\,a^{\frac{3}{2}}b^{2}x^{2}-15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{3}x^{3}-10\sqrt{bx+a}\,a^{\frac{5}{2}}bx+8\sqrt{bx+a}\,a^{\frac{7}{2}}\right)}{24x^{2}\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{9}{2}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/x^2*(b*x+a)^(1/2)*(15*(b*x+a)^(1/2)*a^(3/2)*b^2*x^2-15*arctanh((b*x+a
)^(1/2)/a^(1/2))*a*b^3*x^3-10*(b*x+a)^(1/2)*a^(5/2)*b*x+8*(b*x+a)^(1/2)*a^(
7/2))/(b*x^3+a*x^2)^(1/2)/a^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

Fricas [A]
time = 2.03, size = 175, normalized size = 1.52

$$\left[\frac{15\,\sqrt{a}\,b^{3}x^{4}\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\,\sqrt{a}}{x^{2}}\right)-2\,(15\,ab^{2}x^{2}-10\,a^{2}bx+8\,a^{3})\sqrt{bx^{3}+ax^{2}}}{48\,a^{4}x^{4}},-\frac{15\,\sqrt{-a}\,b^{3}x^{4}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\,\sqrt{-a}}{ax}\right)+(15\,ab^{2}x^{2}-10\,a^{2}bx+8\,a^{3})\sqrt{bx^{3}+ax^{2}}}{24\,a^{4}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x
^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))
+ (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{x^{2}\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

Giac [A]
time = 0.96, size = 88, normalized size = 0.77

$$-\frac{\frac{15\,b^{4}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}}+\frac{15\,(bx+a)^{\frac{5}{2}}\,b^{4}-40\,(bx+a)^{\frac{3}{2}}\,ab^{4}+33\,\sqrt{bx+a}\,a^{2}b^{4}}{a^{3}b^{3}x^{3}}}{24\,b\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/(b*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

3.260 $\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}$$

```
[Out] -2*x^4/b/(b*x^3+a*x^2)^(1/2)-16/5*a*(b*x^3+a*x^2)^(1/2)/b^3+32/5*a^2*(b*x^3
+a*x^2)^(1/2)/b^4/x+12/5*x*(b*x^3+a*x^2)^(1/2)/b^2
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 1602}

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) - (16*a*Sqrt[a*x^2 + b*x^3])/(5*b^3) + (32
*a^2*Sqrt[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*Sqrt[a*x^2 + b*x^3])/(5*b^2)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx &= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}+\frac{6\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{b}\\ &= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{(24a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b^2}\\ &= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}+\frac{(16a^2)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{5b^3}\\ &= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.51

$$\frac{2x(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*x
)])
```

Maple [A]
time = 0.37, size = 56, normalized size = 0.57

method	result	size
gosper	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
default	$\frac{2(bx+a)(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)x^3}{5b^4(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	56
trager	$\frac{2(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3)\sqrt{b\,x^3+a\,x^2}}{5(bx+a)b^4x}$	58

risch	$\frac{2(b^2x^2-3abx+11a^2)(bx+a)x}{5b^4\sqrt{x^2(bx+a)}}+\frac{2a^3x}{b^4\sqrt{x^2(bx+a)}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(3
/2)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.42

$$\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)}{5\,\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

Fricas [A]
time = 1.10, size = 60, normalized size = 0.61

$$\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)\sqrt{bx^3+ax^2}}{5\left(b^5x^2+ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*x
^2 + a*b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

Giac [A]
time = 1.65, size = 79, normalized size = 0.81

$$-\frac{32\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b^4}+\frac{2\,a^3}{\sqrt{bx+a}\,b^4\mathrm{sgn}\left(x\right)}+\frac{2\left(\left(bx+a\right)^{\frac{5}{2}}b^{16}-5\left(bx+a\right)^{\frac{3}{2}}ab^{16}+15\,\sqrt{bx+a}\,a^2b^{16}\right)}{5\,b^{20}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -32/5*a^(5/2)*sgn(x)/b^4 + 2*a^3/(sqrt(b*x + a)*b^4*sgn(x)) + 2/5*((b*x + a
)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/(b^20*
sgn(x))
```

Mupad [B]
time = 5.28, size = 57, normalized size = 0.58

$$\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3+8\,a^2\,b\,x-2\,a\,b^2\,x^2+b^3\,x^3\right)}{5\,b^4\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 + b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x))/(5*b
^4*x*(a + b*x))
```

3.261 $\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}$$

```
[Out] -2*x^3/b/(b*x^3+a*x^2)^(1/2)+8/3*(b*x^3+a*x^2)^(1/2)/b^2-16/3*a*(b*x^3+a*x^
2)^(1/2)/b^3/x
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 1602}

$$-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (8*Sqrt[a*x^2 + b*x^3])/(3*b^2) - (16*a*
Sqrt[a*x^2 + b*x^3])/(3*b^3*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx &= -\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{4\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{b}\\
&= -\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{(8a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b^2}\\
&= -\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 39, normalized size = 0.54

$$\frac{2x(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

Maple [**A**]
time = 0.37, size = 46, normalized size = 0.64

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^2x^2+4abx+8a^2\right)x^3}{3b^3\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}$	46
default	$-\frac{2(bx+a)\left(-b^2x^2+4abx+8a^2\right)x^3}{3b^3\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}$	46
trager	$-\frac{2\left(-b^2x^2+4abx+8a^2\right)\sqrt{b\,x^3+a\,x^2}}{3(bx+a)b^3x}$	48
risch	$-\frac{2(-bx+5a)(bx+a)x}{3b^3\sqrt{x^2\,(bx+a)}}-\frac{2a^2x}{b^3\sqrt{x^2\,(bx+a)}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

Maxima [A]
time = 0.31, size = 30, normalized size = 0.42

$$\frac{2\left(b^{2}x^{2}-4\,abx-8\,a^{2}\right)}{3\,\sqrt{bx+a}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

Fricas [A]
time = 1.86, size = 49, normalized size = 0.68

$$\frac{2\left(b^{2}x^{2}-4\,abx-8\,a^{2}\right)\sqrt{bx^{3}+ax^{2}}}{3\left(b^{4}x^{2}+ab^{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{5}}{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

Giac [A]
time = 1.13, size = 64, normalized size = 0.89

$$\frac{16\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^{3}}-\frac{2\,a^{2}}{\sqrt{bx+a}\,b^{3}\mathrm{sgn}\left(x\right)}+\frac{2\left(\left(bx+a\right)^{\frac{3}{2}}b^{6}-6\,\sqrt{bx+a}\,ab^{6}\right)}{3\,b^{9}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 16/3*a^(3/2)*sgn(x)/b^3 - 2*a^2/(sqrt(b*x + a)*b^3*sgn(x)) + 2/3*((b*x + a)
^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b^6)/(b^9*sgn(x))
```

Mupad [B]
time = 5.22, size = 47, normalized size = 0.65

$$-\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^2+4\,a\,b\,x-b^2\,x^2\right)}{3\,b^3\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3)^(3/2),x)
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*x*(a + b*x))
```

3.262 $\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}$$

```
[Out] -2*x^2/b/(b*x^3+a*x^2)^(1/2)+4*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 1602}

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3)^(3/2),x]
[Out] (-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{2\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{b}\\ &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.55

$$\frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.37, size = 34, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
default	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
trager	$\frac{2(bx+2a)\sqrt{b\,x^3+a\,x^2}}{(bx+a)b^2x}$	36
risch	$\frac{2(bx+a)x}{b^2\sqrt{x^2\,(bx+a)}}+\frac{2ax}{b^2\sqrt{x^2\,(bx+a)}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.40

$$\frac{2\,(bx+2\,a)}{\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] 2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

Fricas [A]
time = 1.45, size = 38, normalized size = 0.81

$$\frac{2\,\sqrt{bx^3+ax^2}\,(bx+2\,a)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.88, size = 48, normalized size = 1.02

$$\frac{2\left(\frac{\sqrt{bx+a}}{b\mathrm{sgn}(x)}+\frac{a}{\sqrt{bx+a}\,b\mathrm{sgn}(x)}\right)}{b}-\frac{4\sqrt{a}\,\mathrm{sgn}(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(b*x + a)/(b*sgn(x)) + a/(sqrt(b*x + a)*b*sgn(x)))/b - 4*sqrt(a)*sgn
(x)/b^2
```

Mupad [B]
time = 5.17, size = 35, normalized size = 0.74

$$\frac{2\left(2\,a+b\,x\right)\,\sqrt{b\,x^3+a\,x^2}}{b^2\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(2*a + b*x)*(a*x^2 + b*x^3)^(1/2))/(b^2*x*(a + b*x))
```

3.263 $\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx$

Optimal. Leaf size=21

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

[Out] -2*x/b/(b*x^3+a*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1602}

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3)^(3/2),x]

[Out] (-2*x)/(b*Sqrt[a*x^2 + b*x^3])

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx = -\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$-\frac{2x}{b\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]

```
[Out] (-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.36, size = 27, normalized size = 1.29

method	result	size
gosper	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
default	$-\frac{2(bx+a)x^3}{b(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	27
trager	$-\frac{2\sqrt{b\,x^3+a\,x^2}}{(bx+a)bx}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*(b*x+a)*x^3/b/(b*x^3+a*x^2)^(3/2)
```

Maxima [A]
time = 0.31, size = 12, normalized size = 0.57

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x + a)*b)
```

Fricas [A]
time = 1.67, size = 29, normalized size = 1.38

$$-\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

Giac [A]
time = 1.32, size = 27, normalized size = 1.29

$$\frac{2\,\mathrm{sgn}(x)}{\sqrt{a}\,b}-\frac{2}{\sqrt{bx+a}\,b\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2*sgn(x)/(sqrt(a)*b) - 2/(sqrt(b*x + a)*b*sgn(x))
```

Mupad [B]
time = 5.07, size = 28, normalized size = 1.33

$$-\frac{2\,\sqrt{b\,x^3+a\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^(3/2),x)
[Out] -(2*(a*x^2 + b*x^3)^(1/2))/(b*x*(a + b*x))
```

3.264 $\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x/a/(b*x^3+a*x^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2048, 2033, 212}

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rubi steps

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x}{a\sqrt{ax^2+bx^3}} + \frac{\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{a}$$
$$= \frac{2x}{a\sqrt{ax^2+bx^3}} - \frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}$$
$$= \frac{2x}{a\sqrt{ax^2+bx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 1.04

$$\frac{2x\left(\sqrt{a}-\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(Sqrt[a] - Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(3/2)*Sqr
t[x^2*(a + b*x)])
```

Maple [A]
time = 0.37, size = 54, normalized size = 1.04

method	result	size
default	$-\frac{2x^3(bx+a)\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\sqrt{bx+a}-a^{\frac{3}{2}}\right)}{(b\,x^3+a\,x^2)^{\frac{3}{2}}\,a^{\frac{5}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x^3*(b*x+a)*(arctanh((b*x+a)^(1/2)/a^(1/2))*a*(b*x+a)^(1/2)-a^(3/2))/(b*
x^3+a*x^2)^(3/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 1.47, size = 156, normalized size = 3.00

$$\left[\frac{(bx^2+ax)\sqrt{a}\log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\sqrt{bx^3+ax^2}a}{a^2bx^2+a^3x}, \frac{2\left((bx^2+ax)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}a\right)}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sqrt
(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a
^2*b*x^2 + a^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

Giac [A]
time = 1.31, size = 77, normalized size = 1.48

$$-\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^{\frac{3}{2}}}+\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(x)}+\frac{2}{\sqrt{bx+a}\,a\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(a)*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a))*sgn(x)/(sqrt(-a)*a^(3/2))
+ 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) + 2/(sqrt(b*x + a)*a
*sgn(x))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^(3/2),x)
[Out] int(x^2/(a*x^2 + b*x^3)^(3/2), x)
```

3.265 $\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2}{a\sqrt{ax^2+bx^3}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}$$

```
[Out] 3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)+2/a/(b*x^3+a*x^2)^(1/2)-
3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2048, 2050, 2033, 212}

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
 !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
```

```
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2}{a\sqrt{ax^2+bx^3}}+\frac{3\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2}{a\sqrt{ax^2+bx^3}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}-\frac{(3b)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a^2}\\
&= \frac{2}{a\sqrt{ax^2+bx^3}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{(3b)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{a^2}\\
&= \frac{2}{a\sqrt{ax^2+bx^3}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 62, normalized size = 0.83

$$\frac{-\sqrt{a}\,(a+3bx)+3bx\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*(a + 3*b*x)) + 3*b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
])/(a^(5/2)*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.40, size = 62, normalized size = 0.83

method	result	size
default	$\frac{x^{2}(bx+a)\left(3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-3bx\sqrt{a}-a^{\frac{3}{2}}\right)}{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{5}{2}}}$	62
risch	$-\frac{bx+a}{a^{2}\sqrt{x^{2}(bx+a)}}-\frac{b\left(-\frac{6\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{4}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{2a^{2}\sqrt{x^{2}(bx+a)}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(b*x+a)*(3*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-3*b*x*a^(1/
2)-a^(3/2))/(b*x^3+a*x^2)^(3/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 1.60, size = 189, normalized size = 2.52

$$\left[\frac{3\,(b^{2}x^{3}+abx^{2})\sqrt{a}\,\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\,\sqrt{a}}{x^{2}}\right)-2\,\sqrt{bx^{3}+ax^{2}}\,(3\,abx+a^{2})}{2\,(a^{3}bx^{3}+a^{4}x^{2})},-\frac{3\,(b^{2}x^{3}+abx^{2})\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^{3}+ax^{2}}\,(3\,abx+a^{2})}{a^{3}bx^{3}+a^{4}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x
^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4
*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^{2}\,(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x))**(3/2), x)
```

Giac [A]
time = 1.18, size = 72, normalized size = 0.96

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(x\right)}-\frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)a^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) - (3*(b*x + a)*b
- 2*a*b)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^3)^(3/2), x)
```

3.266 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)+2/a/x/(b*x^3+a*x^2
)^(1/2)-5/2*(b*x^3+a*x^2)^(1/2)/a^2/x^3+15/4*b*(b*x^3+a*x^2)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2031, 2050, 2033, 212}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*S
qrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
b*x^3]])/(4*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx &= \frac{2}{ax\sqrt{ax^2+bx^3}} + \frac{5\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} - \frac{(15b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a^2}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} + \frac{(15b^2)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^3}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{(15b^2)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, -\right.}{4a^3}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 84, normalized size = 0.76

$$
\frac{\sqrt{a}\,(-2a^2+5abx+15b^2x^2) - 15b^2x^2\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}x\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] (Sqrt[a]*(-2*a^2 + 5*a*b*x + 15*b^2*x^2) - 15*b^2*x^2*Sqrt[a + b*x]*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2)*x*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.40, size = 76, normalized size = 0.69

method	result	size
default	$-\frac{x(bx+a)\left(15\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-5a^{\frac{3}{2}}bx-15b^{2}x^{2}\sqrt{a}+2a^{\frac{5}{2}}\right)}{4(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{7}{2}}}$	76
risch	$-\frac{(bx+a)(-7bx+2a)}{4a^{3}x\sqrt{x^{2}(bx+a)}}+\frac{b^{2}\left(-\frac{30\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{16}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{8a^{3}\sqrt{x^{2}(bx+a)}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x*(b*x+a)*(15*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-5*a
^(3/2)*b*x-15*b^2*x^2*a^(1/2)+2*a^(5/2))/(b*x^3+a*x^2)^(3/2)/a^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(-3/2), x)
```

Fricas [A]
time = 1.52, size = 219, normalized size = 1.99

$$\left[\frac{15\,(b^{3}x^{4}+ab^{2}x^{3})\sqrt{a}\,\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\,\sqrt{a}}{x^{2}}\right)+2\,(15\,ab^{2}x^{2}+5\,a^{2}bx-2\,a^{3})\sqrt{bx^{3}+ax^{2}}}{8\,(a^{4}bx^{4}+a^{5}x^{3})},\ \frac{15\,(b^{3}x^{4}+ab^{2}x^{3})\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\,\sqrt{-a}}{ax}\right)+(15\,ab^{2}x^{2}+5\,a^{2}bx-2\,a^{3})\sqrt{bx^{3}+ax^{2}}}{4\,(a^{4}bx^{4}+a^{5}x^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*
x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sq
rt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^{2}+bx^{3})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(-3/2), x)
```

Giac [A]
time = 1.44, size = 92, normalized size = 0.84

$$\frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^3\mathrm{sgn}\left(x\right)}+\frac{2\,b^2}{\sqrt{bx+a}\,a^3\mathrm{sgn}\left(x\right)}+\frac{7\,(bx+a)^{\frac{3}{2}}b^2-9\,\sqrt{bx+a}\,ab^2}{4\,a^3b^2x^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) + 2*b^2/(sqrt
(b*x + a)*a^3*sgn(x)) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)
/(a^3*b^2*x^2*sgn(x))
```

Mupad [B]
time = 5.43, size = 42, normalized size = 0.38

$$-\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2 +
 b*x^3)^(3/2))
```

3.267 $\int \frac{1}{x(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}$$

```
[Out] 35/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(9/2)+2/a/x^2/(b*x^3+a*x^
2)^(1/2)-7/3*(b*x^3+a*x^2)^(1/2)/a^2/x^4+35/12*b*(b*x^3+a*x^2)^(1/2)/a^3/x^
3-35/8*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2048, 2050, 2033, 212}

$$\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b
*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^
2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
```

```
 !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax^2\sqrt{ax^2+bx^3}}+\frac{7\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{a}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}-\frac{(35b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a^2}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}+\frac{(35b^2)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^3}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}-\frac{(35b^3)}{}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{(35b^3)}{}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.11, size = 96, normalized size = 0.70

$$\frac{-\sqrt{a}\left(8a^3-14a^2bx+35ab^2x^2+105b^3x^3\right)+105b^3x^3\sqrt{a+bx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{9/2}x^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*(8*a^3 - 14*a^2*b*x + 35*a*b^2*x^2 + 105*b^3*x^3)) + 105*b^3*x^3
*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(9/2)*x^2*Sqrt[x^2*(a
+ b*x)])
```

Maple [A]
time = 0.46, size = 86, normalized size = 0.62

method	result	size
default	$\frac{(bx+a)\left(105\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}+14a^{\frac{5}{2}}bx-35a^{\frac{3}{2}}b^{2}x^{2}-105b^{3}x^{3}\sqrt{a}-8a^{\frac{7}{2}}\right)}{24\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{9}{2}}}$	86
risch	$-\frac{(bx+a)\left(57b^{2}x^{2}-22abx+8a^{2}\right)}{24a^{4}x^{2}\sqrt{x^{2}\left(bx+a\right)}}-\frac{b^{3}\left(-\frac{70\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{32}{\sqrt{bx+a}}\right)\sqrt{bx+a}\;x}{16a^{4}\sqrt{x^{2}\left(bx+a\right)}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(b*x+a)*(105*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b^3*x^3+14*a
^(5/2)*b*x-35*a^(3/2)*b^2*x^2-105*b^3*x^3*a^(1/2)-8*a^(7/2))/(b*x^3+a*x^2)^
(3/2)/a^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

Fricas [A]
time = 1.93, size = 241, normalized size = 1.75

$$\left[\frac{105\left(b^{4}x^{5}+ab^{3}x^{4}\right)\sqrt{a}\,\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\,\sqrt{a}}{x^{2}}\right)-2\left(105\,ab^{3}x^{3}+35\,a^{2}b^{2}x^{2}-14\,a^{3}bx+8\,a^{4}\right)\sqrt{bx^{3}+ax^{2}}}{48\left(a^{5}bx^{5}+a^{6}x^{4}\right)},\ -\frac{105\left(b^{4}x^{5}+ab^{3}x^{4}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\,\sqrt{-a}}{ax}\right)+\left(105\,ab^{3}x^{3}+35\,a^{2}b^{2}x^{2}-14\,a^{3}bx+8\,a^{4}\right)\sqrt{bx^{3}+ax^{2}}}{24\left(a^{5}bx^{5}+a^{6}x^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
+ a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8
*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 + a*b
```

```
^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (105*a*b^3*x^
3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 +
a^6*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

Giac [A]
time = 1.43, size = 107, normalized size = 0.78

$$-\frac{35\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^4\mathrm{sgn}\left(x\right)}-\frac{2\,b^3}{\sqrt{bx+a}\,a^4\mathrm{sgn}\left(x\right)}-\frac{57\,(bx+a)^{\frac{5}{2}}b^3-136\,(bx+a)^{\frac{3}{2}}ab^3+87\,\sqrt{bx+a}\,a^2b^3}{24\,a^4b^3x^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sgn(x)) - 2*b^3/(sqr
t(b*x + a)*a^4*sgn(x)) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/2)
*a*b^3 + 87*sqrt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

3.268 $\int \frac{1}{x^2(ax^2+bx^3)^{3/2}} \, dx$

Optimal. Leaf size=166

$$\frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{315b^4\tanh^{-1}}{6}$$

```
[Out] -315/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(11/2)+2/a/x^3/(b*x^3+
a*x^2)^(1/2)-9/4*(b*x^3+a*x^2)^(1/2)/a^2/x^5+21/8*b*(b*x^3+a*x^2)^(1/2)/a^3
/x^4-105/32*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^3+315/64*b^3*(b*x^3+a*x^2)^(1/2)/
a^5/x^2
```

Rubi [A]
time = 0.28, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2048, 2050, 2033, 212}

$$-\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b
*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x
^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a
]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
```

```
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (ax^2+bx^3)^{3/2}}\,dx = \frac{2}{ax^3\sqrt{ax^2+bx^3}} + \frac{9\int \frac{1}{x^4\sqrt{ax^2+bx^3}}\,dx}{a}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} - \frac{(63b)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{8a^2}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} + \frac{(105b^2)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,d}{16a^3}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} - \frac{(315}{}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b}{}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b}{}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b}{}$$

Mathematica [A]
time = 0.15, size = 106, normalized size = 0.64

$$\frac{\sqrt{a}\,(-16a^4+24a^3bx-42a^2b^2x^2+105ab^3x^3+315b^4x^4)-315b^4x^4\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{11/2}x^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-16*a^4 + 24*a^3*b*x - 42*a^2*b^2*x^2 + 105*a*b^3*x^3 + 315*b^4*x
^4) - 315*b^4*x^4*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(11/2
)*x^3*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.46, size = 100, normalized size = 0.60

method	result	size
default	$-\frac{(bx+a)\left(315\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}x^{4}-24a^{\frac{7}{2}}bx+42a^{\frac{5}{2}}b^{2}x^{2}-105a^{\frac{3}{2}}b^{3}x^{3}-315b^{4}x^{4}\sqrt{a}+16a^{\frac{9}{2}}\right)}{64x\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{11}{2}}}$	100
risch	$-\frac{(bx+a)\left(-187b^{3}x^{3}+82a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)}{64a^{5}x^{3}\sqrt{x^{2}\left(bx+a\right)}}+\frac{b^{4}\left(-\frac{630\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{256}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{128a^{5}\sqrt{x^{2}\left(bx+a\right)}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(b*x+a)*(315*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b^4*x^4-24*
a^(7/2)*b*x+42*a^(5/2)*b^2*x^2-105*a^(3/2)*b^3*x^3-315*b^4*x^4*a^(1/2)+16*a
^(9/2))/x/(b*x^3+a*x^2)^(3/2)/a^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Fricas [A]
time = 1.60, size = 263, normalized size = 1.58

$$\left[\frac{315\left(b^{5}x^{6}+ab^{4}x^{5}\right)\sqrt{a}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\left(315\,ab^{4}x^{4}+105\,a^{2}b^{3}x^{3}-42\,a^{3}b^{2}x^{2}+24\,a^{4}bx-16\,a^{5}\right)\sqrt{bx^{3}+ax^{2}}}{128\left(a^{6}bx^{6}+a^{7}x^{5}\right)},\ \frac{315\left(b^{5}x^{6}+ab^{4}x^{5}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{ax}\right)+\left(315\,ab^{4}x^{4}+105\,a^{2}b^{3}x^{3}-42\,a^{3}b^{2}x^{2}+24\,a^{4}bx-16\,a^{5}\right)\sqrt{bx^{3}+ax^{2}}}{64\left(a^{6}bx^{6}+a^{7}x^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x
^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/64*
(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x - 16*a
^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

Giac [A]
time = 1.13, size = 122, normalized size = 0.73

$$\frac{315\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}\,a^5\mathrm{sgn}\left(x\right)}+\frac{2\,b^4}{\sqrt{bx+a}\,a^5\mathrm{sgn}\left(x\right)}+\frac{187\left(bx+a\right)^{\frac{7}{2}}b^4-643\left(bx+a\right)^{\frac{5}{2}}ab^4+765\left(bx+a\right)^{\frac{3}{2}}a^2b^4-325\sqrt{bx+a}\,a^3b^4}{64\,a^5b^4x^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 315/64*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*sgn(x)) + 2*b^4/(sq
rt(b*x + a)*a^5*sgn(x)) + 1/64*(187*(b*x + a)^(7/2)*b^4 - 643*(b*x + a)^(5/
2)*a*b^4 + 765*(b*x + a)^(3/2)*a^2*b^4 - 325*sqrt(b*x + a)*a^3*b^4)/(a^5*b^
4*x^4*sgn(x))
```

Mupad [B]
time = 5.68, size = 44, normalized size = 0.27

$$-\frac{2\,{\left(\frac{a}{b\,x}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{2};\ \frac{13}{2};\ -\frac{a}{b\,x}\right)}{11\,x\,{\left(b\,x^3+a\,x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 11/2], 13/2, -a/(b*x)))/(11*x*(a*x^
2 + b*x^3)^(3/2))
```

3.269 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=125

$$\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}$$

```
[Out] -5/8*a^3*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)+1/3*x^(3/2)*(
b*x^3+a*x^2)^(1/2)/b+5/8*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x^(1/2)-5/12*a*x^(1/2)
*(b*x^3+a*x^2)^(1/2)/b^2
```

Rubi [A]
time = 0.26, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2049, 2054, 212}

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x
^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[
b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx}{6b}$$

$$= -\frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} + \frac{(5a^2)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2}$$

$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a^3)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{16b^3}$$

$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a^3)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.}{8b^3}$$

$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx}}\right.}{8b^{7/2}}$$

Mathematica [A]
time = 0.09, size = 102, normalized size = 0.82

$$\frac{\sqrt{b}\,x^{3/2}(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+15a^3x\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{24b^{7/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 15*a^3*x*
Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(24*b^(7/2)*Sqrt[x^2
*(a + b*x)])
```

Maple [A]
time = 0.39, size = 103, normalized size = 0.82

method	result	size

risch	$\frac{(8b^2x^2-10abx+15a^2)x^{\frac{3}{2}}(bx+a)}{24b^3\sqrt{x^2(bx+a)}}-\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x}\,\sqrt{x\,(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x^2(bx+a)}}$	100
default	$-\frac{\sqrt{x}\left(-16b^{\frac{9}{2}}x^4+4b^{\frac{7}{2}}a\,x^3-10b^{\frac{5}{2}}a^2x^2-30b^{\frac{3}{2}}a^3x+15\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)\sqrt{x\,(bx+a)}\,a^3b\right)}{48\sqrt{b\,x^3+a\,x^2}\,b^{\frac{9}{2}}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*x^(1/2)*(-16*b^(9/2)*x^4+4*b^(7/2)*a*x^3-10*b^(5/2)*a^2*x^2-30*b^(3/2
)*a^3*x+15*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*(x*(b*x+a)
)^(1/2)*a^3*b)/(b*x^3+a*x^2)^(1/2)/b^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

Fricas [A]
time = 1.53, size = 180, normalized size = 1.44

$$\left[\frac{15\,a^3\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx^3+ax^2}\,\sqrt{x}}{48\,b^4x},\ \frac{15\,a^3\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx^3+ax^2}\,\sqrt{x}}{24\,b^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*
sqrt(x))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqr
t(x))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/
(b*x^(3/2))) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqrt
(x))/(b^4*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.82, size = 83, normalized size = 0.66

$$-\frac{5\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{16\,b^{\frac{7}{2}}}+\frac{\sqrt{bx+a}\,\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}+\frac{15\,a^3\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}}{24\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 1/24*(sqrt(b*x + a)*(2*x*(4*x/b - 5*
a/b^2) + 15*a^2/b^3)*sqrt(x) + 15*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x +
 a)))/b^(7/2))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.270 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=95

$$-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)-3/4*a*(b*x^3+a
*x^2)^(1/2)/b^2/x^(1/2)+1/2*x^(1/2)*(b*x^3+a*x^2)^(1/2)/b
```

Rubi [A]
time = 0.20, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2049, 2054, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-3*a*Sqrt[a*x^2 + b*x^3])/(4*b^2*Sqrt[x]) + (Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) + (3*a^2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(4*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} - \frac{(3a)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{4b}$$

$$= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} + \frac{(3a^2)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2}$$

$$= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^2}$$

$$= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

Mathematica [A]
time = 0.07, size = 91, normalized size = 0.96

$$\frac{\sqrt{b}\,x^{3/2}(-3a^2-abx+2b^2x^2)-3a^2x\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
[Out] (Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) - 3*a^2*x*Sqrt[a + b*x]*Log[-
(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(5/2)*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.38, size = 92, normalized size = 0.97

method	result	size
risch	$-\frac{(-2bx+3a)x^{\frac{3}{2}}(bx+a)}{4b^2\sqrt{x^2(bx+a)}}+\frac{3a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x^2(bx+a)}}$	89
default	$\frac{\sqrt{x}\left(4b^{\frac{7}{2}}x^3-2b^{\frac{5}{2}}a\,x^2-6b^{\frac{3}{2}}a^2x+3\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b\right)}{8\sqrt{b\,x^3+a\,x^2}\,b^{\frac{7}{2}}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^(1/2)*(4*b^(7/2)*x^3-2*b^(5/2)*a*x^2-6*b^(3/2)*a^2*x+3*(x*(b*x+a))^(1
/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^2*b)/(b*x^3+a*x
^2)^(1/2)/b^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

Fricas [A]
time = 1.33, size = 159, normalized size = 1.67

$$\left[\frac{3\,a^{2}\sqrt{b}\,x\log\left(\frac{2\,bx^{2}+ax+2\,\sqrt{bx^{3}+ax^{2}}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\,\sqrt{bx^{3}+ax^{2}}\,(2\,b^{2}x-3\,ab)\sqrt{x}}{8\,b^{3}x},-\frac{3\,a^{2}\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)-\sqrt{bx^{3}+ax^{2}}\,(2\,b^{2}x-3\,ab)\sqrt{x}}{4\,b^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sq
rt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x), -1/4*
(3*a^2*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) - sqrt(b
*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^{2}\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.96, size = 71, normalized size = 0.75

$$\frac{3\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{8\,b^{\frac{5}{2}}}+\frac{\sqrt{bx+a}\,\sqrt{x}\,\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}}{4\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*a^2*log(abs(a))*sgn(x)/b^(5/2) + 1/4*(sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*
a/b^2) - 3*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.271 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

```
[Out] -a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)+(b*x^3+a*x^2)^(1/2)
/b/x^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2049, 2054, 212}

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
 b*x^3]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx &= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{2b}\\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b}\\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 1.18

$$\frac{\sqrt{b}\,x^{3/2}(a+bx)+ax\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(a + b*x) + a*x*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqr
t[a + b*x]])/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

Maple [A]
time = 0.39, size = 79, normalized size = 1.32

method	result	size
risch	$\frac{x^{\frac{3}{2}}(bx+a)}{b\sqrt{x^2(bx+a)}} - \frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x^2(bx+a)}}$	78
default	$\frac{\sqrt{x}\left(2b^{\frac{5}{2}}x^2+2b^{\frac{3}{2}}ax-a\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^3+a\,x^2}\,b^{\frac{5}{2}}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)*(2*b^(5/2)*x^2+2*b^(3/2)*a*x-a*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x
^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*b)/(b*x^3+a*x^2)^(1/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

Fricas [A]
time = 2.90, size = 131, normalized size = 2.18

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}\,b\sqrt{x}}{2\,b^2x},\ \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\sqrt{bx^3+ax^2}\,b\sqrt{x}}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x
))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) + sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2
*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.63, size = 55, normalized size = 0.92

$$-\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{2\,b^{\frac{3}{2}}}+\frac{\frac{a\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}\,\sqrt{x}}{b}}{\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(abs(a))*sgn(x)/b^(3/2) + (a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
+ a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x)/b)/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.272 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=34

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2054, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.56

$$-\frac{2x\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*x*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[
x^2*(a + b*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.
time = 0.37, size = 58, normalized size = 1.71

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{x\,(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)}{\sqrt{b\,x^3+a\,x^2}\,\sqrt{b}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)
*b^(1/2)+2*b*x+a)/b^(1/2))/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)
```

Fricas [A]
time = 2.84, size = 77, normalized size = 2.26

$$\left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -2
*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2)))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)
```

Giac [A]
time = 2.39, size = 37, normalized size = 1.09

$$\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{\sqrt{b}} - \frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}\,+\sqrt{bx+a}\,\right|\right)}{\sqrt{b}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(a))*sgn(x)/sqrt(b) - 2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/(
sqrt(b)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.273 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

[Out] -2*(b*x^3+a*x^2)^(1/2)/a/x^(3/2)

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2039}

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.92

$$-\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))
```

Maple [A]
time = 0.36, size = 27, normalized size = 1.08

method	result	size
risch	$-\frac{2\sqrt{x}\,(bx+a)}{\sqrt{x^2\,(bx+a)}\,a}$	25
gosper	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27
default	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*x^(1/2)*(b*x+a)/a/(b*x^3+a*x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)
```

Fricas [A]
time = 2.86, size = 21, normalized size = 0.84

$$-\frac{2\sqrt{bx^3+ax^2}}{ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^3 + a*x^2)/(a*x^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x}\,\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)
```

Giac [A]
time = 1.81, size = 34, normalized size = 1.36

$$\frac{4\sqrt{b}}{\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.274 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}$$

```
[Out] -2/3*(b*x^3+a*x^2)^(1/2)/a/x^(5/2)+4/3*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2041, 2039}

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x
^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}-\frac{(2b)\int \frac{1}{\sqrt{x}\,\sqrt{ax^2+bx^3}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.55

$$-\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))
```

Maple [A]
time = 0.37, size = 33, normalized size = 0.59

method	result	size
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^2\,(bx+a)}\,\sqrt{x}\,a^2}$	31
gosper	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^2\sqrt{b\,x^3+a\,x^2}}$	33
default	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^2\sqrt{b\,x^3+a\,x^2}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x+a)*(-2*b*x+a)/x^(1/2)/a^2/(b*x^3+a*x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)
```

Fricas [A]
time = 1.73, size = 29, normalized size = 0.52

$$\frac{2\sqrt{bx^3+ax^2}\,(2\,bx-a)}{3\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)
```

Giac [A]
time = 1.54, size = 54, normalized size = 0.96

$$\frac{2\left(\frac{2\,(bx+a)b^3}{a^2}-\frac{3\,b^3}{a}\right)\sqrt{bx+a}\,b}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}|b|\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 2/3*(2*(b*x + a)*b^3/a^2 - 3*b^3/a)*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(3
/2)*abs(b)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{3/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.275 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}$$

```
[Out] -2/5*(b*x^3+a*x^2)^(1/2)/a/x^(7/2)+8/15*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(5/2)-1
6/15*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2041, 2039}

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) + (8*b*Sqrt[a*x^2 + b*x^3])/(15*a^2*
x^(5/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(15*a^3*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}} - \frac{(4b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{5a}$$
$$= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}} + \frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}} + \frac{(8b^2)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{15a^2}$$
$$= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}} + \frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 0.51

$$-\frac{2\sqrt{x^2(a+bx)}\,(3a^2-4abx+8b^2x^2)}{15a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))
```

Maple [A]
time = 0.38, size = 46, normalized size = 0.53

method	result	size
risch	$-\frac{2(bx+a)(8b^2x^2-4abx+3a^2)}{15\sqrt{x^2(bx+a)}\,x^{\frac{3}{2}}a^3}$	44
gosper	$-\frac{2(bx+a)(8b^2x^2-4abx+3a^2)}{15x^{\frac{3}{2}}a^3\sqrt{b\,x^3+a\,x^2}}$	46
default	$-\frac{2(bx+a)(8b^2x^2-4abx+3a^2)}{15x^{\frac{3}{2}}a^3\sqrt{b\,x^3+a\,x^2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(3/2)/a^3/(b*x^3+a*x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)
```

Fricas [A]
time = 2.14, size = 40, normalized size = 0.47

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx^3+ax^2}}{15\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)
```

Giac [A]
time = 1.54, size = 81, normalized size = 0.94

$$\frac{32\left(10\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5*sg
n(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{5/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.276 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}$$

```
[Out] -2/7*(b*x^3+a*x^2)^(1/2)/a/x^(9/2)+12/35*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(7/2)-
16/35*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(5/2)+32/35*b^3*(b*x^3+a*x^2)^(1/2)/a^4
/x^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2041, 2039}

$$\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) + (12*b*Sqrt[a*x^2 + b*x^3])/(35*a^2
*x^(7/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(35*a^3*x^(5/2)) + (32*b^3*Sqrt[a*
x^2 + b*x^3])/(35*a^4*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} - \frac{(6b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx}{7a} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} + \frac{(24b^2)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{35a^2} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} - \frac{(16b^3)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}}{35a^3} \\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} + \frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.47

$$\frac{2\sqrt{x^2(a+bx)}\,(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^(9/2))
```

Maple [A]
time = 0.36, size = 57, normalized size = 0.49

method	result	size
risch	$-\frac{2(bx+a)\left(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3\right)}{35\sqrt{x^2\,(bx+a)}\;x^{\frac{5}{2}}\,a^4}$	55
gosper	$-\frac{2(bx+a)\left(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3\right)}{35x^{\frac{5}{2}}\,a^4\sqrt{b\,x^3+a\,x^2}}$	57
default	$-\frac{2(bx+a)\left(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3\right)}{35x^{\frac{5}{2}}\,a^4\sqrt{b\,x^3+a\,x^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(5/2)/a^4/(b*x^3+
a*x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)
```

Fricas [A]
time = 1.84, size = 51, normalized size = 0.44

$$\frac{2\left(16\,b^{3}x^{3}-8\,ab^{2}x^{2}+6\,a^{2}bx-5\,a^{3}\right)\sqrt{bx^{3}+ax^{2}}}{35\,a^{4}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a^
4*x^(9/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{7}{2}}\sqrt{x^{2}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)
```

Giac [A]
time = 2.17, size = 107, normalized size = 0.92

$$\frac{64\left(35\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{6}-21\,a\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{4}+7\,a^{2}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a^{3}\right)b^{\frac{7}{2}}}{35\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{7}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{7/2}\,\sqrt{b\,x^{3}+a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.277 $\int x^{1-3n}(ax^2+bx^3)^n\,dx$

Optimal. Leaf size=61

$$\frac{x^{2-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

```
[Out] x^(2-3*n)*(b*x^3+a*x^2)^n*hypergeom([-n, 2-n],[3-n],-b*x/a)/(2-n)/((1+b*x/a
)^n)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2057, 68, 66}

$$\frac{x^{2-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] (x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx = \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{1-n}(a+bx)^n\,dx$$
$$= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{1-n}\left(1+\frac{bx}{a}\right)^n\,dx$$
$$= \frac{x^{2-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

Mathematica [**A**]
time = 0.03, size = 59, normalized size = 0.97

$$\frac{x^{2-3n}\left(x^2(a+bx)\right)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] (x^(2 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^{1-3n}\left(b\,x^3+a\,x^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
[Out] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{1-3n}\left(x^2(a+bx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1-3*n)*(b*x**3+a*x**2)**n,x)
[Out] Integral(x**(1 - 3*n)*(x**2*(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{1-3\,n}\,\left(b\,x^3+a\,x^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x)
[Out] int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n, x)
```

3.278 $\int x^{-3n}(ax^2+bx^3)^n\,dx$

Optimal. Leaf size=48

$$\frac{x^{-1-3n}(ax^2+bx^3)^{1+n}\ {}_2F_1\left(1,2;2-n;-\frac{bx}{a}\right)}{a(1-n)}$$

```
[Out] x^(-1-3*n)*(b*x^3+a*x^2)^(1+n)*hypergeom([1, 2],[2-n],-b*x/a)/a/(1-n)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2057, 68, 66}

$$\frac{x^{1-3n}\left(\frac{bx}{a}+1\right)^{-n}(ax^2+bx^3)^n\ {}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

```
[Out] (x^(1 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx = \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-n}(a+bx)^n\,dx$$
$$= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-n}\left(1+\frac{bx}{a}\right)^n\,dx$$
$$= \frac{x^{1-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}$$

Mathematica [A]
time = 0.02, size = 59, normalized size = 1.23

$$\frac{x^{1-3n}\left(x^2(a+bx)\right)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^n/x^(3*n),x]
[Out] (x^(1 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\,x^2\right)^n x^{-3n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
[Out] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="fricas")
[Out] integral((b*x^3 + a*x^2)^n/x^(3*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n}\left(x^2(a+bx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**n/(x**(3*n)),x)
[Out] Integral((x**2*(a + b*x))**n/x**(3*n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\,x^2\right)^n}{x^{3\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n),x)
[Out] int((a*x^2 + b*x^3)^n/x^(3*n), x)
```

3.279 $\int x^{-1-3n}(ax^2+bx^3)^n\,dx$

Optimal. Leaf size=54

$$-\frac{x^{-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

```
[Out] -(b*x^3+a*x^2)^n*hypergeom([-n, -n],[1-n],-b*x/a)/n/(x^(3*n))/((1+b*x/a)^n)
```

Rubi [A]

time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2057, 68, 66}

$$-\frac{x^{-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx=\left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}(a+bx)^n\,dx$$
$$=\left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}\left(1+\frac{bx}{a}\right)^n\,dx$$
$$=-\frac{x^{-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.96

$$-\frac{x^{-3n}\left(x^2(a+bx)\right)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] -(((x^2*(a + b*x))^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^{-1-3n}\left(b\,x^3+a\,x^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
[Out] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n-1}\left(x^{2}(a+bx)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(b*x**3+a*x**2)**n,x)
[Out] Integral(x**(-3*n - 1)*(x**2*(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\,x^2\right)}^n}{x^{3\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 1),x)
[Out] int((a*x^2 + b*x^3)^n/x^(3*n + 1), x)
```

3.280 $\int x^{-2-3n}(ax^2+bx^3)^n\,dx$

Optimal. Leaf size=32

$$-\frac{x^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a(1+n)}$$

```
[Out] -(b*x^3+a*x^2)^(1+n)/a/(1+n)/(x^(3+3*n))
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2039}

$$-\frac{x^{-3(n+1)}(ax^2+bx^3)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-2-3n}(ax^2+bx^3)^n\,dx = -\frac{x^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a(1+n)}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.94

$$-\frac{x^{-3(1+n)}(x^2(a+bx))^{1+n}}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] -((x^2*(a + b*x))^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

Maple [A]
time = 0.40, size = 36, normalized size = 1.12

method	result	size
gosper	$-\frac{x^{-1-3n} (bx+a) \left(b\,x^{3}+a\,x^{2}\right)^{n}}{a(1+n)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
[Out] -x^(-1-3*n)*(b*x+a)/a/(1+n)*(b*x^3+a*x^2)^n
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

Fricas [A]
time = 1.59, size = 38, normalized size = 1.19

$$-\frac{(bx^2+ax)(bx^3+ax^2)^n x^{-3\,n-2}}{an+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
[Out] -(b*x^2 + a*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 2)/(a*n + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n-2} \left(x^2(a+bx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-3*n)*(b*x**3+a*x**2)**n,x)
[Out] Integral(x**(-3*n - 2)*(x**2*(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

Mupad [B]
time = 5.28, size = 54, normalized size = 1.69

$$-\left(b\,x^3+a\,x^2\right)^n\,\left(\frac{x}{x^{3\,n+2}\,\left(n+1\right)}+\frac{b\,x^2}{a\,x^{3\,n+2}\,\left(n+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 2),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*(x/(x^(3*n + 2)*(n + 1)) + (b*x^2)/(a*x^(3*n + 2)*(n + 1
)))
```

3.281 $\int x^{-3-3n}(ax^2+bx^3)^n\,dx$

Optimal. Leaf size=70

$$-\frac{x^{-4-3n}(ax^2+bx^3)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a^2(1+n)(2+n)}$$

```
[Out] -x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)/a/(2+n)+b*(b*x^3+a*x^2)^(1+n)/a^2/(1+n)/(2+
n)/(x^(3+3*n))
```

Rubi [A]

time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\frac{bx^{-3(n+1)}(ax^2+bx^3)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}(ax^2+bx^3)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)^
(1 + n))/(a^2*(1 + n)*(2 + n)*x^(3*(1 + n)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-3-3n}(ax^2+bx^3)^n\,dx&=-\frac{x^{-4-3n}(ax^2+bx^3)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-3n}(ax^2+bx^3)^n\,dx}{a(2+n)}\\&=-\frac{x^{-4-3n}(ax^2+bx^3)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.63

$$-\frac{x^{-4-3n}(a+an-bx)\left(x^2(a+bx)\right)^{1+n}}{a^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a + a*n - b*x)*(x^2*(a + b*x))^(1 + n))/(a^2*(1 + n)*(2 +
n)))
```

Maple [A]
time = 0.40, size = 50, normalized size = 0.71

method	result	size
gosper	$-\frac{\left(b\,x^3+a\,x^2\right)^n x^{-2-3n}(an-bx+a)(bx+a)}{(2+n)(1+n)a^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^3+a*x^2)^n*x^(-2-3*n)*(a*n-b*x+a)*(b*x+a)/(2+n)/(1+n)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

Fricas [A]
time = 2.06, size = 70, normalized size = 1.00

$$-\frac{(abnx^2-b^2x^3+(a^2n+a^2)x)(bx^3+ax^2)^n x^{-3\,n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 3)/(a^
2*n^2 + 3*a^2*n + 2*a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n-3}\left(x^2(a+bx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 3)*(x**2*(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

Mupad [B]
time = 5.28, size = 98, normalized size = 1.40

$$-\left(b\,x^3+a\,x^2\right)^n\,\left(\frac{x\,\left(n+1\right)}{x^{3\,n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 3),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*((x*(n + 1))/(x^(3*n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(
a^2*x^(3*n + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(3*n + 3)*(3*n + n^2 + 2)
))
```

3.282 $\int x^{-4-3n}(ax^2+bx^3)^n\,dx$

Optimal. Leaf size=116

$$-\frac{x^{-5-3n}(ax^2+bx^3)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}(ax^2+bx^3)^{1+n}}{a^2(2+n)(3+n)}-\frac{2b^2x^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a^3(1+n)(2+n)(3+n)}$$

```
[Out] -x^(-5-3*n)*(b*x^3+a*x^2)^(1+n)/a/(3+n)+2*b*x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)/
a^2/(2+n)/(3+n)-2*b^2*(b*x^3+a*x^2)^(1+n)/a^3/(2+n)/(n^2+4*n+3)/(x^(3+3*n))
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2041, 2039}

$$-\frac{2b^2x^{-3(n+1)}(ax^2+bx^3)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}(ax^2+bx^3)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}(ax^2+bx^3)^{n+1}}{a(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-5 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(3 + n))) + (2*b*x^(-4 - 3*n)*(
a*x^2 + b*x^3)^(1 + n))/(a^2*(2 + n)*(3 + n)) - (2*b^2*(a*x^2 + b*x^3)^(1 +
 n))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx = -\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)} - \frac{(2b)\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx}{a(3+n)}$$

$$= -\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)} + \frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)} + \frac{(2b^2)\int x^{-2-3n}\left(ax^2+bx\right.}{a^2(2+n)(3+n)}$$

$$= -\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)} + \frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)} - \frac{2b^2x^{-3(1+n)}\left(ax^2+bx^3\right)^1}{a^3(1+n)(2+n)(3+r}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.62

$$-\frac{x^{-3(1+n)}(a+bx)\left(x^2(a+bx)\right)^n\left(a^2(2+3n+n^2)-2ab(1+n)x+2b^2x^2\right)}{a^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a + b*x)*(x^2*(a + b*x))^n*(a^2*(2 + 3*n + n^2) - 2*a*b*(1 + n)*x + 2*b
^2*x^2))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n))))
```

Maple [A]
time = 0.40, size = 84, normalized size = 0.72

method	result	size
gosper	$-\frac{(bx+a)x^{-3-3n}\left(a^2n^2-2abnx+2b^2x^2+3a^2n-2abx+2a^2\right)\left(b\,x^3+a\,x^2\right)^n}{(3+n)(2+n)(1+n)a^3}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)*x^(-3-3*n)*(a^2*n^2-2*a*b*n*x+2*b^2*x^2+3*a^2*n-2*a*b*x+2*a^2)*(b*
x^3+a*x^2)^n/(3+n)/(2+n)/(1+n)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

Fricas [A]
time = 2.29, size = 111, normalized size = 0.96

$$\frac{\left(2\,ab^{2}nx^{3}-2\,b^{3}x^{4}-\left(a^{2}bn^{2}+a^{2}bn\right)x^{2}-\left(a^{3}n^{2}+3\,a^{3}n+2\,a^{3}\right)x\right)\left(bx^{3}+ax^{2}\right)^{n}x^{-3\,n-4}}{a^{3}n^{3}+6\,a^{3}n^{2}+11\,a^{3}n+6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 - 2*b^3*x^4 - (a^2*b*n^2 + a^2*b*n)*x^2 - (a^3*n^2 + 3*a^3*n
 + 2*a^3)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 4)/(a^3*n^3 + 6*a^3*n^2 + 11*a^3*n
 + 6*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n-4}\left(x^{2}\left(a+bx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 4)*(x**2*(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

Mupad [B]
time = 5.36, size = 157, normalized size = 1.35

$$-\left(b\,x^{3}+a\,x^{2}\right)^{n}\,\left(\frac{x\,\left(n^{2}+3\,n+2\right)}{x^{3\,n+4}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}+\frac{2\,b^{3}\,x^{4}}{a^{3}\,x^{3\,n+4}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}-\frac{2\,b^{2}\,n\,x^{3}}{a^{2}\,x^{3\,n+4}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}+\frac{b\,n\,x^{2}\,\left(n+1\right)}{a\,x^{3\,n+4}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 4),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*((x*(3*n + n^2 + 2))/(x^(3*n + 4)*(11*n + 6*n^2 + n^3 +
6)) + (2*b^3*x^4)/(a^3*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*n*x^3
)/(a^2*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (b*n*x^2*(n + 1))/(a*x^(3*n
+ 4)*(11*n + 6*n^2 + n^3 + 6)))
```

3.283 $\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

[Out] 1/6*x^6/a/(b*x^3+a)^2

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 270}

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a*x^2 + b*x^5)^3,x]

[Out] x^6/(6*a*(a + b*x^3)^2)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(ax^2+bx^5\right)^3}\,dx &= \int \frac{x^5}{\left(a+bx^3\right)^3}\,dx\\ &= \frac{x^6}{6a\left(a+bx^3\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2bx^3}{6b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] -1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

Maple [A]
time = 0.37, size = 31, normalized size = 1.63

method	result	size
gosper	$-\frac{2b\,x^{3}+a}{6(b\,x^{3}+a)^{2}b^{2}}$	23
risch	$\frac{-\frac{x^{3}}{3b}-\frac{a}{6b^{2}}}{(b\,x^{3}+a)^{2}}$	26
default	$-\frac{1}{3b^{2}(b\,x^{3}+a)}+\frac{a}{6b^{2}(b\,x^{3}+a)^{2}}$	31
norman	$\frac{-\frac{x^{8}}{3b}-\frac{a\,x^{5}}{6b^{2}}}{x^{5}(b\,x^{3}+a)^{2}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^5+a*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^2/(b*x^3+a)+1/6*a/b^2/(b*x^3+a)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 0.28, size = 36, normalized size = 1.89

$$-\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 1.98, size = 36, normalized size = 1.89

$$-\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.16, size = 36, normalized size = 1.89

$$\frac{-a - 2bx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**5+a*x**2)**3,x)
```

```
[Out] (-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Giac [A]
time = 0.94, size = 22, normalized size = 1.16

$$-\frac{2\,bx^3 + a}{6\,(bx^3 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

Mupad [B]
time = 5.11, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{6\,b^2} + \frac{x^3}{3\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a*x^2 + b*x^5)^3,x)
```

```
[Out] -(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.284 $\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=80

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

```
[Out] 16/45*a^2*(b*x^5+a*x^2)^(1/2)/b^3/x-8/45*a*x^2*(b*x^5+a*x^2)^(1/2)/b^2+2/15
*x^5*(b*x^5+a*x^2)^(1/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 1602}

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45
*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x^5\sqrt{ax^2+bx^5}}{15b} - \frac{(4a)\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx}{5b}$$
$$= -\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b} + \frac{(8a^2)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{15b^2}$$
$$= \frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x} - \frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.58

$$\frac{2\sqrt{x^2(a+bx^3)}\,(8a^2-4abx^3+3b^2x^6)}{45b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

Maple [A]
time = 0.37, size = 48, normalized size = 0.60

method	result	size
trager	$\frac{2(3b^2x^6-4ab\,x^3+8a^2)\sqrt{b\,x^5+a\,x^2}}{45b^3x}$	43
gosper	$\frac{2(b\,x^3+a)(3b^2x^6-4ab\,x^3+8a^2)x}{45b^3\sqrt{b\,x^5+a\,x^2}}$	48
default	$\frac{2(b\,x^3+a)(3b^2x^6-4ab\,x^3+8a^2)x}{45b^3\sqrt{b\,x^5+a\,x^2}}$	48
risch	$\frac{2x(b\,x^3+a)(3b^2x^6-4ab\,x^3+8a^2)}{45\sqrt{x^2(b\,x^3+a)}\,b^3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/45*(b*x^3+a)*(3*b^2*x^6-4*a*b*x^3+8*a^2)*x/b^3/(b*x^5+a*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.58

$$\frac{2\,(3\,b^3x^9-ab^2x^6+4\,a^2bx^3+8\,a^3)}{45\,\sqrt{bx^3+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)
```

Fricas [A]
time = 1.23, size = 42, normalized size = 0.52

$$\frac{2\,(3\,b^2x^6 - 4\,abx^3 + 8\,a^2)\sqrt{bx^5 + ax^2}}{45\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{x^2\,(a + bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**9/sqrt(x**2*(a + b*x**3)), x)
```

Giac [A]
time = 1.86, size = 65, normalized size = 0.81

$$-\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{45\,b^3} + \frac{2\,\sqrt{bx^3 + a}\;a^2}{3\,b^3\mathrm{sgn}\,(x)} + \frac{2\left(3\,(bx^3 + a)^{\frac{5}{2}} - 10\,(bx^3 + a)^{\frac{3}{2}}a\right)}{45\,b^3\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -16/45*a^(5/2)*sgn(x)/b^3 + 2/3*sqrt(b*x^3 + a)*a^2/(b^3*sgn(x)) + 2/45*(3*
(b*x^3 + a)^(5/2) - 10*(b*x^3 + a)^(3/2)*a)/(b^3*sgn(x))
```

Mupad [B]
time = 5.18, size = 42, normalized size = 0.52

$$\frac{2\,\sqrt{b\,x^5 + a\,x^2}\;(8\,a^2 - 4\,a\,b\,x^3 + 3\,b^2\,x^6)}{45\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a*x^2 + b*x^5)^(1/2),x)
[Out] (2*(a*x^2 + b*x^5)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3*x)
```

3.285 $\int \frac{x^6}{\sqrt{ax^2+bx^5}} \, dx$

Optimal. Leaf size=52

$$-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}+\frac{2x^2\sqrt{ax^2+bx^5}}{9b}$$

```
[Out] -4/9*a*(b*x^5+a*x^2)^(1/2)/b^2/x+2/9*x^2*(b*x^5+a*x^2)^(1/2)/b
```

Rubi [A]

time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 1602}

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt{ax^2+bx^5}} \, dx &= \frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{(2a)\int \frac{x^3}{\sqrt{ax^2+bx^5}} \, dx}{3b}\\ &= -\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}+\frac{2x^2\sqrt{ax^2+bx^5}}{9b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.65

$$\frac{2(-2a+bx^3)\sqrt{x^2(a+bx^3)}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)
```

Maple [A]
time = 0.37, size = 37, normalized size = 0.71

method	result	size
trager	$-\frac{2(-b\,x^3+2a)\sqrt{b\,x^5+a\,x^2}}{9b^2x}$	32
gosper	$-\frac{2(b\,x^3+a)(-b\,x^3+2a)x}{9b^2\sqrt{b\,x^5+a\,x^2}}$	37
default	$-\frac{2(b\,x^3+a)(-b\,x^3+2a)x}{9b^2\sqrt{b\,x^5+a\,x^2}}$	37
risch	$-\frac{2x(b\,x^3+a)(-b\,x^3+2a)}{9\sqrt{x^2(b\,x^3+a)}\,b^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(b*x^3+a)*(-b*x^3+2*a)*x/b^2/(b*x^5+a*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 34, normalized size = 0.65

$$\frac{2\,(b^2x^6-abx^3-2\,a^2)}{9\,\sqrt{bx^3+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)
```

Fricas [A]
time = 3.07, size = 30, normalized size = 0.58

$$\frac{2\,\sqrt{bx^5+ax^2}\,(bx^3-2\,a)}{9\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**6/sqrt(x**2*(a + b*x**3)), x)
```

Giac [A]
time = 1.95, size = 48, normalized size = 0.92

$$\frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b^2}+\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b^2\mathrm{sgn}\,(x)}-\frac{2\,\sqrt{bx^3+a}\,a}{3\,b^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/9*a^(3/2)*sgn(x)/b^2 + 2/9*(b*x^3 + a)^(3/2)/(b^2*sgn(x)) - 2/3*sqrt(b*x^
3 + a)*a/(b^2*sgn(x))
```

Mupad [B]
time = 5.33, size = 33, normalized size = 0.63

$$-\frac{\sqrt{b\,x^5+a\,x^2}\,\left(\frac{4\,a}{9\,b^2}-\frac{2\,x^3}{9\,b}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] -((a*x^2 + b*x^5)^(1/2)*((4*a)/(9*b^2) - (2*x^3)/(9*b)))/x
```

3.286 $\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

```
[Out] 2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1602}

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)
```

Maple [A]
time = 0.39, size = 27, normalized size = 1.08

method	result	size
trager	$\frac{2\sqrt{b\,x^{5}+a\,x^{2}}}{3bx}$	22
gosper	$\frac{2x\left(b\,x^{3}+a\right)}{3b\sqrt{b\,x^{5}+a\,x^{2}}}$	27
default	$\frac{2x\left(b\,x^{3}+a\right)}{3b\sqrt{b\,x^{5}+a\,x^{2}}}$	27
risch	$\frac{2x\left(b\,x^{3}+a\right)}{3\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,b}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x*(b*x^3+a)/b/(b*x^5+a*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.56

$$\frac{2\,\sqrt{bx^3+a}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^3 + a)/b
```

Fricas [A]
time = 1.52, size = 21, normalized size = 0.84

$$\frac{2\,\sqrt{bx^5+ax^2}}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^5 + a*x^2)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x**3)), x)
```

Giac [A]
time = 1.77, size = 29, normalized size = 1.16

$$-\frac{2\sqrt{a}\,\operatorname{sgn}(x)}{3\,b}+\frac{2\sqrt{bx^3+a}}{3\,b\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a)*sgn(x)/b + 2/3*sqrt(b*x^3 + a)/(b*sgn(x))
```

Mupad [B]
time = 5.19, size = 21, normalized size = 0.84

$$\frac{2\sqrt{b\,x^5+a\,x^2}}{3\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^5)^(1/2))/(3*b*x)
```

3.287 $\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=32

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2033, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.69

$$-\frac{2x\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2(a+bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^2
*(a + b*x^3)])
```

Maple [A]
time = 0.39, size = 43, normalized size = 1.34

method	result	size
default	$-\frac{2x\sqrt{b\,x^3+a}\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{b\,x^5+a\,x^2}\,\sqrt{a}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^5 + a*x^2), x)
```

Fricas [A]
time = 1.33, size = 75, normalized size = 2.34

$$\left[\frac{\log\left(\frac{bx^4+2\,ax-2\,\sqrt{bx^5+ax^2}\,\sqrt{a}}{x^4}\right)}{3\,\sqrt{a}}, \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{-a}}{ax}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3*
sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**5), x)
```

Giac [**A**]
time = 1.73, size = 47, normalized size = 1.47

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{3\sqrt{-a}} + \frac{2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)/
sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{b\,x^5 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^5)^(1/2), x)
```

3.288 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}$$

```
[Out] 1/3*b*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(3/2)-1/3*(b*x^5+a*x^2)^(1/2
)/a/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2050, 2033, 212}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^5]/(a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
5]])/(3*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{3ax^4} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx}{2a}$$
$$= -\frac{\sqrt{ax^2+bx^5}}{3ax^4} + \frac{b\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)}{3a}$$
$$= -\frac{\sqrt{ax^2+bx^5}}{3ax^4} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 1.29

$$\frac{-\sqrt{a}\,(a+bx^3)+bx^3\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}x^2\sqrt{x^2\,(a+bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-(Sqrt[a]*(a + b*x^3)) + b*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2)*x^2*Sqrt[x^2*(a + b*x^3)])
```

Maple [A]
time = 0.43, size = 66, normalized size = 1.12

method	result	size
default	$\frac{\sqrt{b\,x^3+a}\,\left(b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)a\,x^3-\sqrt{b\,x^3+a}\,a^{\frac{3}{2}}\right)}{3x^2\sqrt{b\,x^5+a\,x^2}\,a^{\frac{5}{2}}}$	66
risch	$-\frac{b\,x^3+a}{3a\,x^2\sqrt{x^2\,(b\,x^3+a)}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)\sqrt{b\,x^3+a}\,x}{3a^{\frac{3}{2}}\sqrt{x^2\,(b\,x^3+a)}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/x^2*(b*x^3+a)^(1/2)*(b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a*x^3-(b*x^3+a)
^(1/2)*a^(3/2))/(b*x^5+a*x^2)^(1/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)
```

Fricas [A]
time = 1.78, size = 127, normalized size = 2.15

$$\left[\frac{\sqrt{a}\,bx^4\log\left(\frac{bx^4+2\,ax+2\,\sqrt{bx^5+ax^2}\,\sqrt{a}}{x^4}\right)-2\,\sqrt{bx^5+ax^2}\,a}{6\,a^2x^4},\,-\frac{\sqrt{-a}\,bx^4\arctan\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^5+ax^2}\,a}{3\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4
) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(b*
x^5 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)
```

Giac [A]
time = 2.05, size = 55, normalized size = 0.93

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx^3+a}\,b}{ax^3}}{3\,b\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^3 + a)*b
/(a*x^3))/(b*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)), x)
```

3.289 $\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=238

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}\right.\right.}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 2/5*(b*x^5+a*x^2)^(1/2)/b-4/15*a*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2049, 2057, 224}

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{5b} \\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{\left(2ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b\sqrt{ax^2+bx^5}} \\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\Big(\mathrm{s}}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 68, normalized size = 0.29

$$\frac{2x^2\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{5b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(5*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [A]
time = 0.43, size = 248, normalized size = 1.04

method	result
default	$\frac{2x\left(ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}-2bx-\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bx\right.}{\left(-a\,b^2\right)}}\right.}{15\sqrt{b\,x^5}}$
	$4ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}$
risch	$\frac{2x^2\left(b\,x^3+a\right)}{5b\sqrt{x^2\left(b\,x^3+a\right)}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*x*(I*a*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2
)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^
(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+3*b^2*x^4+3*a*b*x)/(b*x^5+a*x^2)^(1/
2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 37, normalized size = 0.16

$$-\frac{2\left(2\,a\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\sqrt{bx^5+ax^2}\,b\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*a*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*x^5 + a*x^2)*b
)/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^4/(a*x^2 + b*x^5)^(1/2), x)
```

3.290 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=212

$$\frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

```
[Out] 2/3*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/b^(1/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2057, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 52, normalized size = 0.25

$$\frac{x^2\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sq
rt[x^2*(a + b*x^3)]
```

Maple [A]
time = 0.37, size = 231, normalized size = 1.09

method	result
default	$-\frac{i x\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}-2bx-(-a\,b^{2})^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+(-a\,b^{2})^{\frac{1}{3}}\right)}{(-a\,b^{2})^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}+2bx+\cdots\right.}{(-a\,b^{2})^{\frac{1}{3}}}}}{3\sqrt{b\,x^{5}+a\,x^{2}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/(b*x^5+a*x^2)^(1/2)*x*3^(1/2)/b*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*
b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2
)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b
^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 14, normalized size = 0.07

$$\frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassPInverse(0, -4*a/b, x)/sqrt(b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^{2}\,(a+bx^{3})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^5)^(1/2), x)
```

3.291 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=243

$$-\frac{\sqrt{ax^2+bx^5}}{2ax^3}-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\ }}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\ }}\right.\right.}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -1/2*(b*x^5+a*x^2)^(1/2)/a/x^3-1/6*b^(2/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/(b*x^5+a*x^2)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2057, 224}

$$-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^5]/(a*x^3) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*x*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{b\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{4a}$$

$$= -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4a\sqrt{ax^2+bx^5}}$$

$$= -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\ \cdots}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\ \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 55, normalized size = 0.23

$$-\frac{\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/
(x*Sqrt[x^2*(a + b*x^3)])
```

Maple [A]
time = 0.39, size = 248, normalized size = 1.02

method	result
default	$\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}-2bx-\left(-a b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a b^2\right)^{\frac{1}{3}}\right)}{\left(-a b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}+2bx+\left(-a \cdots\right.\right.}{\left(-a b^2\right)^{\frac{1}{3}}}}\cdots}{12x\sqrt{b x^5+\cdots}}$
	$i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\cdots}}\cdots$
risch	$-\frac{b x^3+a}{2ax\sqrt{x^2\left(b x^3+a\right)}}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/x*(I*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^
(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1
/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2-6*b*x^3-6*a)/(b*x^5+a*x^2)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 38, normalized size = 0.16

$$-\frac{\sqrt{b}\,x^3 \text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{bx^5+ax^2}}{2\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) + sqrt(b*x^5 + a*x^2))/
(a*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

Mupad [B]
time = 5.71, size = 44, normalized size = 0.18

$$-\frac{2\,\sqrt{\frac{a}{b\,x^3}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^3}\right)}{7\,x\,\sqrt{b\,x^5+a\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] -(2*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 7/6], 13/6, -a/(b*x^3)))/(7*x*(a*
x^2 + b*x^5)^(1/2))
```

3.292 $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=514

$$-\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}+\frac{2x\sqrt{ax^2+bx^5}}{7b}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{7b^{5/3}\sqrt{\cdots}}$$

```
[Out] -8/7*a*x*(b*x^3+a)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1
/2)+2/7*x*(b*x^5+a*x^2)^(1/2)/b-8/21*a^(4/3)*x*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/7*3^(1/4)*a^(4/3)*x*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(
b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2049, 2057, 309, 224, 1891}

$$-\frac{8\sqrt{2}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}+\frac{2x\sqrt{ax^2+bx^5}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-8*a*x*(a + b*x^3))/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*
x^2 + b*x^5]) + (2*x*Sqrt[a*x^2 + b*x^5])/(7*b) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - (8*Sqrt[2]*a^(4/3)*x*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
```

```
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(5/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{7b}$$

$$= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b\sqrt{ax^2+bx^5}}$$

$$= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}\sqrt{ax^2+bx^5}} - \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/3}\right.}{7b^{4/3}}$$

$$= -\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} + \frac{2x\sqrt{ax^2+bx^5}}{7b} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 68, normalized size = 0.13

$$\frac{2x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{7b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
-((b*x^3)/a)]))/(7*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [A]
time = 0.40, size = 676, normalized size = 1.32

method	result

risch	$\frac{2x^3(b\,x^3+a)}{7b\sqrt{x^2\,(b\,x^3+a)}}+\frac{8ia\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}$
default	$-\frac{2x\left(3i(-a\,b^2)^{\frac{2}{3}}\sqrt{3}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}-2bx-(-a\,b^2)^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+(-a\,b^2)^{\frac{1}{3}}\right)}{(-a\,b^2)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}+\right.}{(-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*x*(3*I*(-a*b^2)^(2/3)*3^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^
2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2
^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*a-2*I*(-a*b^2)^(2/3)*3^(1/2)*(-I*(I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)
*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*Ellip
ticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*a-
3*b^3*x^5+3*(-a*b^2)^(2/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/
(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*3^
(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I
*3^(1/2)/(I*3^(1/2)-3))^(1/2))*a-3*a*b^2*x^2)/(b*x^5+a*x^2)^(1/2)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 45, normalized size = 0.09

$$\frac{2\left(\sqrt{bx^5+ax^2}\,bx+4\,a\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*(sqrt(b*x^5 + a*x^2)*b*x + 4*a*sqrt(b)*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^5/(a*x^2 + b*x^5)^(1/2), x)
```

3.293 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=484

$$\frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}}\cdots}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}}\cdots}$$

```
[Out] 2*x*(b*x^3+a)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)+2
/3*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/
3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2057, 309, 224, 1891}

$$\frac{2\sqrt{2}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*(a + b*x^3))/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (
2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
```

```
 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}\,\sqrt{ax^2+bx^5}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\,x\sqrt{a+bx^3}\right)\int \frac{}{\sqrt{a+}}}{\sqrt[3]{b}\,\sqrt{ax^2+bx^5}}$$

$$= \frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 55, normalized size = 0.11

$$\frac{x^3\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[x^2*(a + b*x^3)])
```

Maple [A]
time = 0.46, size = 394, normalized size = 0.81

method	result
default	$-\frac{ix\sqrt{3}\,(-a\,b^2)^{\frac{2}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}-2bx-(-a\,b^2)^{\frac{1}{3}}\right)\sqrt{3}}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+(-a\,b^2)^{\frac{1}{3}}\right)}{(-a\,b^2)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}+2bx+\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*x*3^(1/2)*(-a*b^2)^(2/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(
1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(I*3^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(
-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^
(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))-3*EllipticE(1/6*3^(1/2)*2^(1
/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1
/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+2*EllipticF(1/6*3^(1/2)
*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^
2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2)))/(b*x^5+a*x^2)^(1/
2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 22, normalized size = 0.05

$$-\frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))/sqrt(b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^2/(a*x^2 + b*x^5)^(1/2), x)
```

3.294 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=510

$$\frac{\sqrt[3]{b}\,x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.}}}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\cdots}{\left(\left(1+\cdots\right.\right.}}}$$

```
[Out] b^(1/3)*x*(b*x^3+a)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)-(
b*x^5+a*x^2)^(1/2)/a/x^2+1/3*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^5+a*
x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2050, 2057, 309, 224, 1891}

$$\frac{\sqrt{2}\,\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{\sqrt[3]{b}\,x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (b^(1/3)*x*(a + b*x^3))/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - Sqrt[a*x^2 + b*x^5]/(a*x^2) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)
*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (Sqrt[2]*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
```

```
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b
*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{ax^2} + \frac{b\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{2a}$$

$$= -\frac{\sqrt{ax^2+bx^5}}{ax^2} + \frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}}$$

$$= -\frac{\sqrt{ax^2+bx^5}}{ax^2} + \frac{\left(b^{2/3}x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b\right.}{a}$$

$$= \frac{\sqrt[3]{b}\,x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt{ax^2+bx^5}}{ax^2} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,x\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 50, normalized size = 0.10

$$-\frac{\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/Sqr
t[x^2*(a + b*x^3)])
```

Maple [A]
time = 0.38, size = 673, normalized size = 1.32

method	result

risch	$-\frac{b\,x^{3}+a}{a\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}}{}$
default	$\frac{3i\sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(3*I*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b
^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1
/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1
/3))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/
2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x-2*I*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))
/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(
-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^
(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x+3*(
-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^
(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*
EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2
))*(-a*b^2)^(2/3)*x-12*b^2*x^3-12*a*b)/(b*x^5+a*x^2)^(1/2)/a/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 46, normalized size = 0.09

$$-\frac{\sqrt{b}\,x^2\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\sqrt{bx^5+ax^2}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + sqrt(b*x^5 + a*x^2))/(a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**2*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{x\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^5)^(1/2)), x)
```

3.295 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=265

$$-\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}+\frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\cdots\right.\right.}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\;\sqrt{a\cdots}}$$

```
[Out] 1/5*x^(5/2)*(b*x^5+a*x^2)^(1/2)/b-7/20*a*(b*x^5+a*x^2)^(1/2)/b^2/x^(1/2)+7/
120*a^(5/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/b^2/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2049, 2057, 335, 231}

$$\frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-7*a*Sqrt[a*x^2 + b*x^5])/(20*b^2*Sqrt[x]) + (x^(5/2)*Sqrt[a*x^2 + b*x^5])
/(5*b) + (7*a^(5/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x)
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} - \frac{(7a)\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx}{10b}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{(7a^2)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{40b^2}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{40b^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \ldots\right.}{20b^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x \ldots}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\ldots\right.\right.}}}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\ldots}{\left(\sqrt[3]{a}+\ldots\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 86, normalized size = 0.32

$$\frac{x^{3/2}\left(-7a^2-3abx^3+4b^2x^6+7a^2\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{20b^2\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.48, size = 2017, normalized size = 7.61

method	result

risch	$-\frac{\left(-4 b\,x^{3}+7 a\right) x^{\frac{3}{2}} \left(b\,x^{3}+a\right)}{20 b^{2} \sqrt{x^{2} \left(b\,x^{3}+a\right)}}+\frac{7 a^{2} \left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{\frac{\left(-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\ \left(x-\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/20/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3/(-a*b^2)^(1/3)*(14*I*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*3^(1/2)*a^2*b^2*x^2-28*I*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*
3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a^2*b
*x-4*I*(x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*b^2*x^3+14*I*(-a*b^2)^(2/3)*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*3^(1/2)*a^2-14*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b^2*x^2+28*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a
```

```
^2*b*x+12*x^3*(x*(b*x^3+a))^(1/2)*b^2*(-a*b^2)^(1/3)*(1/b^2*x*(-b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)-14*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2+7*
I*(x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*3^(1/2)*a*b-21*a*(x*(b*x^3+a))^(1/2)*b*(-a*b^2)^(1/3)*
(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2))/(x*(b*x^3+a))^(
1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(9/2)/(b*x^3 + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(13/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.296 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=525

$$-\frac{5\left(1+\sqrt{3}\right)ax^{3/2}(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}+\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}+\frac{5\sqrt[4]{3}\,a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\sqrt[3]{a}+\cdots\right.}}}{8b^{5/3}\sqrt{\frac{\cdots}{\left(\cdots\right.}}}$$

```
[Out] -5/8*a*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)/(b*x^5+a*x^2)^(1/2)+1/4*x^(3/2)*(b*x^5+a*x^2)^(1/2)/b+5/8*3^(1/4)*a^(4/3)
*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*
x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)+5/48*a^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*
x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2
^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2049, 2057, 335, 314, 231, 1895}

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{5\sqrt[4]{3}\,a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{5\left(1+\sqrt{3}\right)ax^{3/2}(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}+\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x^(3/2)*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) + (x^(3/2)*Sqrt[a*x^2 + b*x^5])/(4*b) + (
5*3^(1/4)*a^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcC
```

```
os[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)]
, (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (5*(1 - Sqrt[3])*a
^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3)
 + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt
[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
```

```
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{8b}$$

$$= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{\left(5ax\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax^2+bx^5}}$$

$$= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{\left(5ax\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{4b\sqrt{ax^2+bx^5}}$$

$$= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} + \frac{\left(5ax\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax^2+bx^5}} + \frac{(5}{}$$

$$= -\frac{5\left(1+\sqrt{3}\right)ax^{3/2}(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} + \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} + \frac{5\sqrt[4]{3}\,a^{4/3}x^{3/2}\left(\sqrt[3]{}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 70, normalized size = 0.13

$$\frac{x^{7/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(7/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/
6, -((b*x^3)/a)]))/(4*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.47, size = 2586, normalized size = 4.93

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2586

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3*(-5*I*(-a*b^2)^(1/3)*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*3^(1/2)*a*b*x^2-5*I*(-a*b^2)^(2/3)*3^(1/2)*a*x-10*(-a*b^2)^(1/3)*(-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*a*b*x^2+15*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x^2+5*I*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
```

```
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*3^(1/2)*a^2*b+20*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+
I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*x-30*(-a*b^2)^(2/3)*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*a*x+10*I*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I
*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*x-5*I*3^(1/2)*a*b^2
*x^3-5*I*(-a*b^2)^(1/3)*3^(1/2)*a*b*x^2+10*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b-15*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*a^2*b+I*(x*(b*x^3+a))^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3)))^(1/2)*3^(1/2)*b^2*x^2-3*(x*(b*x^3+a))^(1/2)*(1/b^2*x*(-b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^2+15*a*b^2*x^3+15*(-a*b^2)^(1/
3)*a*b*x^2+15*(-a*b^2)^(2/3)*a*x)/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*
x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(7/2)/(b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(11/2)/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(11/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.297 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

```
[Out] -1/3*a*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(3/2)+1/3*x^(1/2)*(b*
x^5+a*x^2)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2049, 2054, 212}

$$\frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x
^2 + b*x^5]])/(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b} - \frac{a\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx}{2b}$$

$$= \frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b}$$

$$= \frac{\sqrt{x}\,\sqrt{ax^2+bx^5}}{3b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

Mathematica [A]
time = 0.02, size = 81, normalized size = 1.25

$$\frac{\sqrt{b}\,x^{5/2}(a+bx^3) - ax\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{3b^{3/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/
(Sqrt[b]*x^(3/2))])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.58, size = 3347, normalized size = 51.49

method	result	size
risch	Expression too large to display	1035
default	Expression too large to display	3347

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3*(6*I*3^(1/2)*(-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
```

```
^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*
b^2*x^2-6*I*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*x^2-1
2*I*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x+12*I*3^(1/2)*(-a*b^2)^(1/3)*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3
^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x+6*I*3^(1/2)*(-a*b^2)^(2/3
)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*a-6*I*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*
3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*a-6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*x^2+6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
cPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3
^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*a*b^2*x^2+12*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF(
```

```
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)
+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x-12*(-a*b^2)^(1
/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x+I*(x*(b*x^3+a))^(1/2)
*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x-6
*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*a+6*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)
```

Fricas [A]
time = 2.03, size = 148, normalized size = 2.28

$$\left[\frac{a\sqrt{b}\,\log\left(-8\,b^{2}x^{6} - 8\,abx^{3} + 4\,\sqrt{bx^{5} + ax^{2}}\,(2\,bx^{3} + a)\sqrt{b}\,\sqrt{x} - a^{2}\right) + 4\,\sqrt{bx^{5} + ax^{2}}\,b\sqrt{x}}{12\,b^{2}}, \frac{a\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^{5} + ax^{2}}\,\sqrt{-b}\,\sqrt{x}}{2\,bx^{3} + a}\right) + 2\,\sqrt{bx^{5} + ax^{2}}\,b\sqrt{x}}{6\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x^
3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6*(
a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a)) + 2
*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{x^{2}\left(a + bx^{3}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(9/2)/sqrt(x**2*(a + b*x**3)), x)
```

Giac [A]
time = 0.68, size = 52, normalized size = 0.80

$$\frac{\sqrt{bx^3+a}\,x^{\frac{3}{2}}}{3\,b\mathrm{sgn}\,(x)}+\frac{a\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3 + a)*x^(3/2)/(b*sgn(x)) + 1/3*a*log(abs(-sqrt(b)*x^(3/2) + s
qrt(b*x^3 + a)))/(b^(3/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{9/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.298 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=237

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\left(\right.\right.}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

```
[Out] 1/2*(b*x^5+a*x^2)^(1/2)/b/x^(1/2)-1/12*a^(2/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2049, 2057, 335, 231}

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 +
 b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{4b}$$

$$= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax^2+bx^5}}$$

$$= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax^2+bx^5}}$$

$$= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\cdots\right.\right.}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.30

$$\frac{x^{3/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6
, -((b*x^3)/a)]))/(2*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.59, size = 1793, normalized size = 7.57

method	result

risch	$\frac{x^{\frac{3}{2}}\left(b\,x^{3}+a\right)}{2b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{a\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2/(-a*b^2)^(1/3)*(2*I*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*3^(1/2)*a*b^2*x^2-4*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*a*b*x+2*I*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a-2*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*x^2+4*(-a*b^2)^
(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*a*b*x-2*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
```

```
-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a+I*(x*(b*x^3+a))^
(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(
1/3)*3^(1/2)*b-3*(x*(b*x^3+a))^(1/2)*b*(-a*b^2)^(1/3)*(1/b^2*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2))/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1
/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(3/2)/(b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2 (a + bx^3)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(7/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.299 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=492

$$\frac{\left(1+\sqrt{3}\right)x^{3/2}(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}}}$$

```
[Out] x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^
5+a*x^2)^(1/2)-3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*
x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x^(3/2)*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*
x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{\left(1+\sqrt{3}\right)x^{3/2}(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] ((1 + Sqrt[3])*x^(3/2)*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
```

```
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a
^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*
x^5]) - ((1 - Sqrt[3])*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{\left(2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}}$$

$$= -\frac{\left(x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax^2+bx^5}} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}x\sqrt{a}\right.}{}$$

$$= \frac{\left(1+\sqrt{3}\right)x^{3/2}(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-}{\left(\sqrt[3]{a}-\right.}}}{b^{2/3}\sqrt{\frac{-}{(}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.12

$$\frac{2x^{7/2}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/
a)])/(5*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.51, size = 2374, normalized size = 4.83

method	result	size
default	Expression too large to display	2374

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2*(-I*EllipticE((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/
3)*3^(1/2)*b*x^2+2*I*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*x+3*EllipticE((-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*(-a*b^2)^(1/3)*b*x^2+I*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*a*b-2*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^
(1/3)*b*x^2-6*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(2/3)*x+4*(-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
```

```
pticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)
*x-I*3^(1/2)*b^2*x^3-3*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*a*b+2*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b-I*(-a*b^2)^(1/
3)*3^(1/2)*b*x^2-I*(-a*b^2)^(2/3)*3^(1/2)*x+3*b^2*x^3+3*(-a*b^2)^(1/3)*b*x^
2+3*(-a*b^2)^(2/3)*x)/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^3 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^{2}\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{5/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.300 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2054, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.64

$$\frac{2x\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{3\sqrt{b}\,\sqrt{x^2(a+bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))])/(3*Sqrt[b]
*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.52, size = 480, normalized size = 13.33

method	result
default	$-\frac{4x^{\frac{3}{2}}(bx^3+a)\left(-1+i\sqrt{3}\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)xb}{\left(-1+i\sqrt{3}\right)\left(-bx+(-ab^2)^{\frac{1}{3}}\right)}}\left(-bx+(-ab^2)^{\frac{1}{3}}\right)^2\sqrt{\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}+2bx+(-ab^2)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+(-ab^2)^{\frac{1}{3}}\right)}}\sqrt{}}{\sqrt{b\,x^5+a\,x^2}\,b^2\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*x^(3/2)*(b*x^3+a)*(-1+I*3^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*(-b*x+(-a*b^2)^(1/3))^2*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*(EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-
EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2)))/(b*x^5+a*x^2)^(1/2)/b^2/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)
-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)
```

Fricas [A]
time = 2.76, size = 101, normalized size = 2.81

$$\left[\frac{\log\left(-8\,b^{2}x^{6}-8\,abx^{3}-4\,\sqrt{bx^{5}+ax^{2}}\,(2\,bx^{3}+a)\sqrt{b}\,\sqrt{x}-a^{2}\right)}{6\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^{5}+ax^{2}}\,\sqrt{-b}\,\sqrt{x}}{2\,bx^{3}+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt(
b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(
-b)*sqrt(x)/(2*b*x^3 + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{x^{2}\,(a+bx^{3})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)
```

Giac [A]
time = 0.53, size = 40, normalized size = 1.11

$$\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{3\,\sqrt{b}}-\frac{2\,\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^{3}+a}\right|\right)}{3\,\sqrt{b}\,\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*log(abs(a))*sgn(x)/sqrt(b) - 2/3*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3
+ a)))/(sqrt(b)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x^{3/2}}{\sqrt{b\,x^{5}+a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.301 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=203

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 1/3*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^(1/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2057, 335, 231}

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1
- Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4
])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}} \\
&= \frac{\left(2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}} \\
&= \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)\right.}{\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 55, normalized size = 0.27

$$\frac{2x^{3/2}\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a
)])/Sqrt[x^2*(a + b*x^3)]
```

Maple [C] Result contains complex when optimal does not.
time = 0.52, size = 437, normalized size = 2.15

method	result
default	$-\frac{4x^{\frac{3}{2}}\left(b\,x^3+a\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)xb}{\left(-1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\ \sqrt{\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}+2bx+\left(-a\,b^2\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\ \sqrt{\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}-2bx-\left(-a\,b^2\right)^{\frac{1}{3}}}{\left(-1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}}{\sqrt{b\,x^5+a\,x^2}\ \left(-a\,b^2\right)^{\frac{1}{3}}b\sqrt{x\left(b\,x^3+a\right)}\ \left(i\sqrt{3}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/(-a*b^2)^(1/3)/b*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(I*
3^(1/2)*b^2*x^2-2*I*(-a*b^2)^(1/3)*3^(1/2)*b*x+I*(-a*b^2)^(2/3)*3^(1/2)-b^2
*x^2+2*(-a*b^2)^(1/3)*b*x-(-a*b^2)^(2/3))/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)
/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 16, normalized size = 0.08

$$-\frac{2\,\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(1/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.302 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=519

$$\frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\cdots}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\cdots\right.\right.}}}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\cdots\right.}{\left(\sqrt[3]{a}+\left(1\cdots\right.\right.}}}$$

[Out] 2*b^(1/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(
b*x^5+a*x^2)^(1/2)-2*(b*x^5+a*x^2)^(1/2)/a/x^(3/2)-2*3^(1/4)*b^(1/3)*x^(3/2
)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)
x(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^5+a*x
^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)-1/3*b^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*
(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.37, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2050, 2057, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]

[Out] (2*(1 + Sqrt[3])*b^(1/3)*x^(3/2)*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) - (2*3
^(1/4)*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[

```
(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (
2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*
x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 -
 Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4]
)/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{(2b)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{a}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{\left(2bx\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{\left(4bx\sqrt{a+bx^3}\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}-\frac{\left(2\sqrt[3]{b}\,x\sqrt{a+bx^3}\right)\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax^2+bx^5}}\\
&= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}\right.\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 55, normalized size = 0.11

$$-\frac{2\sqrt{x}\,\sqrt{1+\frac{b x^{3}}{a}}\;{}_{2}F_{1}\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{b x^{3}}{a}\right)}{\sqrt{x^{2}\left(a+b x^{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)
/a)])/Sqrt[x^2*(a + b*x^3)]
```

Maple [C] Result contains complex when optimal does not.
time = 0.53, size = 2860, normalized size = 5.51

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2860

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b*x^5+a*x^2)^(1/2)*x^(1/2)/b*(I*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^3-2*I*(-a*b^2)^(1/3)*3^(1/2)*(x*(b*x^3+a)
)^(1/2)*b*x^2-4*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*
3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*(b*x^3+a))^(1/2)*b*x^2+6*(-
a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(x*(b*x^3+a))^(1/2)*b*x^2+4*I*(-a*b^2)^(2/3)*3^(
1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*(x*(b*x^3+a))^(1/2)*x+8*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
```

```
*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*
(b*x^3+a))^(1/2)*x-12*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*(b*x^3+a))^(1/2)*x+2*
I*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(x*(b*x^3+a))^(1/2)*a*b-2*I*(-a*b^2)^(2/3)*3^(1/2)*
(x*(b*x^3+a))^(1/2)*x-2*I*(-a*b^2)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*(b*x^3+a)
)^(1/2)*b*x^2+4*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*(b*x^3+a))^(1/2)*a*b-6*(-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*(b
*x^3+a))^(1/2)*a*b-2*I*3^(1/2)*(x*(b*x^3+a))^(1/2)*b^2*x^3-3*(1/b^2*x*(-b*x
+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^3+6*(x*(b*x^3+a))^(1/2)*
b^2*x^3+6*(-a*b^2)^(1/3)*(x*(b*x^3+a))^(1/2)*b*x^2+I*3^(1/2)*(1/b^2*x*(-b*x
+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b+6*(-a*b^2)^(2/3)*(x*(b*x^3
+a))^(1/2)*x-3*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a
*b)/a/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 24, normalized size = 0.05

$$\frac{2\,\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.303 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

```
[Out] -2/3*(b*x^5+a*x^2)^(1/2)/a/x^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2039\}$

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))
```

Maple [A]
time = 0.45, size = 29, normalized size = 1.07

method	result	size
gosper	$-\frac{2(b\,x^3+a)}{3\sqrt{x}\,a\sqrt{b\,x^5+a\,x^2}}$	29
default	$-\frac{2(b\,x^3+a)}{3\sqrt{x}\,a\sqrt{b\,x^5+a\,x^2}}$	29
risch	$-\frac{2(b\,x^3+a)}{3\sqrt{x^2\,(b\,x^3+a)}\,\sqrt{x}\,a}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/x^(1/2)*(b*x^3+a)/a/(b*x^5+a*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.96

$$-\frac{2\,(bx^4+ax)}{3\,\sqrt{bx^3+a}\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

Fricas [A]
time = 2.71, size = 21, normalized size = 0.78

$$-\frac{2\,\sqrt{bx^5+ax^2}}{3\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x^5 + a*x^2)/(a*x^(5/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{3}{2}}\,\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)
```

Giac [A]
time = 0.50, size = 28, normalized size = 1.04

$$-\frac{2\left(\frac{\sqrt{b+\frac{a}{x^3}}}{a}-\frac{\sqrt{b}}{a}\right)}{3\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/3*(sqrt(b + a/x^3)/a - sqrt(b)/a)/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{3/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.304 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=235

$$-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}$$
$$\overline{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -2/5*(b*x^5+a*x^2)^(1/2)/a/x^(7/2)-2/15*b*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2050, 2057, 335, 231}

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^
(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{(2b)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{5a}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{\left(2bx\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{\left(4bx\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{5a\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-}\right.}{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.24

$$-\frac{2\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{5x^{3/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.52, size = 1795, normalized size = 7.64

method	result

risch	$-\frac{2\left(b\,x^{3}+a\right)}{5 a\,x^{\frac{3}{2}}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{4 b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^3+a)*(-4*I*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^
(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^2*x^5+8*I*(-a*b^2)^(1/3)*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*b*x^4-4*I*(-a*b^2)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3+4*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*b^2*x^5-8*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*x^4+4*(-a*b^2)^(2/3)*(-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a
```

```
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*x^3+I*(x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*(1/b^2*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)-3*(x*(b*x^3+a))^(1/2)*(-a*b^2)^(
1/3)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2))/(b*x^5+a*x
^2)^(1/2)/x^(3/2)/(-a*b^2)^(1/3)/a/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2
*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 48, normalized size = 0.20

$$\frac{2\left(2\sqrt{a}\,bx^4\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}\,a\sqrt{x}\right)}{5\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(2*sqrt(a)*b*x^4*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5 + a*x
^2)*a*sqrt(x))/(a^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{5}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.305 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=555

$$-\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}+\frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}}+\frac{8\sqrt[4]{3}\,b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

[Out] -8/7*b^(4/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)-2/7*(b*x^5+a*x^2)^(1/2)/a/x^(9/2)+8/7*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(3/2)+8/7*3^(1/4)*b^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+4/21*b^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.43, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2050, 2057, 335, 314, 231, 1895}

$$\frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\sqrt[4]{3}\,a^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{8\sqrt[4]{3}\,b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}+\frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-8*(1 + Sqrt[3])*b^(4/3)*x^(3/2)*(a + b*x^3))/(7*a^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(7*a*x^(9/2)) + (8*b*Sqrt[a*x^2 + b*x^5])/(7*a^2*x^(3/2)) + (8*3^(1/4)*b^(4/3)*x^(3/2)*(

```
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])
*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*a^(5
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x^2 + b*x^5]) + (4*(1 - Sqrt[3])*b^(4/3)*x^(3/2)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*3^(1/4)*a^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} - \frac{(4b)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx}{7a}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{(8b^2)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{7a^2}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(8b^2x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(16b^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,\right.}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} + \frac{\left(8b^{4/3}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}}{\sqrt{a+bx}}\right.}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 57, normalized size = 0.10

$$-\frac{2\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};-\frac{bx^3}{a}\right)}{7x^{5/2}\sqrt{x^2\,(a+bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((b*x^3)/a)])/(
7*x^(5/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.61, size = 3048, normalized size = 5.49

method	result	size
risch	Expression too large to display	1125
default	Expression too large to display	3048

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7/(b*x^5+a*x^2)^(1/2)/x^(5/2)*(3*I*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b*x^3+16*I*(-a*b^2)^(2/3)*3^(1/2)*(x*(b*x^3+
a))^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*x^4-16*(-a*b^2)^(1/3)*(x*(b*x^3+a))^(1/2)*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*b*x^5+24*(-a*b^2)^(1/3)*(x*(b*x^3+a))^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*x^5+8*I*3^
(1/2)*(x*(b*x^3+a))^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2
```

```
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/
2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x^3+32*(-a*b^2)^(2/3)*(x*(b*x^3
+a))^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*x^4-48*(-a*b^2)^(2/3)*(x*(b*x^3+a))^(1/2)*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*x^4-8*I*(-a*b^2)^(1/3)*3^(1/2)*(x*(b*x^3+a))^(1/2)*(-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*x^
5-8*I*3^(1/2)*(x*(b*x^3+a))^(1/2)*b^2*x^6+4*I*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^6+16*(x*(b*x^3+a))^(1/2)*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*a*b*x^3-24*(x*(b*x^3+a))^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b*x^3-8*I*(-a*b^2)^(
2/3)*3^(1/2)*(x*(b*x^3+a))^(1/2)*x^4+24*(x*(b*x^3+a))^(1/2)*b^2*x^6-12*(1/b
^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b^2*x^6+24*(-a*b^2)^
(1/3)*(x*(b*x^3+a))^(1/2)*b*x^5-I*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3)))^(1/2)*a^2+24*(-a*b^2)^(2/3)*(x*(b*x^3+a))^(1/2)*x^4-9*(
1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b*x^3-8*I*(-a*b
^2)^(1/3)*3^(1/2)*(x*(b*x^3+a))^(1/2)*b*x^5+3*(1/b^2*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 55, normalized size = 0.10

$$-\frac{2\left(4\sqrt{a}\,bx^5\mathrm{weierstrassZeta}\left(0,-\frac{4b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\sqrt{bx^5+ax^2}\,a\sqrt{x}\right)}{7\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/7*(4*sqrt(a)*b*x^5*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*
b/a, 1/x)) + sqrt(b*x^5 + a*x^2)*a*sqrt(x))/(a^2*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{7/2}\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.306 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}$$

```
[Out] -2/9*(b*x^5+a*x^2)^(1/2)/a/x^(11/2)+4/9*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2*
x^(5/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.62

$$-\frac{2(a-2bx^3)\sqrt{x^2(a+bx^3)}}{9a^2x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
[Out] (-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*a^2*x^(11/2))
```

Maple [A]
time = 0.43, size = 37, normalized size = 0.66

method	result	size
gosper	$-\frac{2(b\,x^3+a)(-2b\,x^3+a)}{9x^{\frac{7}{2}}a^2\sqrt{b\,x^5+a\,x^2}}$	37
default	$-\frac{2(b\,x^3+a)(-2b\,x^3+a)}{9x^{\frac{7}{2}}a^2\sqrt{b\,x^5+a\,x^2}}$	37
risch	$-\frac{2(b\,x^3+a)(-2b\,x^3+a)}{9\sqrt{x^2\,(b\,x^3+a)}\,x^{\frac{7}{2}}a^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 38, normalized size = 0.68

$$\frac{2\,(2\,b^2x^7+abx^4-a^2x)}{9\,\sqrt{bx^3+a}\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

Fricas [A]
time = 2.30, size = 31, normalized size = 0.55

$$\frac{2\,\sqrt{bx^5+ax^2}\,(2\,bx^3-a)}{9\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(2*b*x^3 - a)/(a^2*x^(11/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(9/2)*sqrt(x**2*(a + b*x**3))), x)
```

Giac [A]
time = 0.54, size = 43, normalized size = 0.77

$$-\frac{2\left(\frac{\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{a^2}-\frac{3\sqrt{b+\frac{a}{x^3}}\,b}{a^2}+\frac{2b^{\frac{3}{2}}}{a^2}\right)}{9\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/9*((b + a/x^3)^(3/2)/a^2 - 3*sqrt(b + a/x^3)*b/a^2 + 2*b^(3/2)/a^2)/sgn(
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{9/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.307 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=265

$$-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\cdots\right.\right.}{55\sqrt[4]{3}\,a^{7/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a\cdots}}$$

```
[Out] -2/11*(b*x^5+a*x^2)^(1/2)/a/x^(13/2)+16/55*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(7/2
)+16/165*b^2*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/a^(7/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2050, 2057, 335, 231}

$$\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}\,a^{7/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(11*a*x^(13/2)) + (16*b*Sqrt[a*x^2 + b*x^5])/(55*a
^2*x^(7/2)) + (16*b^2*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[
ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)], (2 + Sqrt[3])/4])/(55*3^(1/4)*a^(7/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} - \frac{(8b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx}{11a}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{(16b^2)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{55a^2}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{\left(16b^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{55a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{\left(32b^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,d\right.}{55a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{\cdots}}{\left(\sqrt[3]{a}+\left(1+\cdots\right.\right.}}}{55\sqrt[4]{3}\,a^{7/3}\sqrt{\frac{\cdots}{\left(\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.22

$$-\frac{2\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{11}{6},\frac{1}{2};-\frac{5}{6};-\frac{bx^3}{a}\right)}{11x^{9/2}\sqrt{x^2(a+bx^3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)])/
(11*x^(9/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.50, size = 2009, normalized size = 7.58

method	result

risch	$-\frac{2(b x^3+a)(-8b x^3+5a)}{55a^2 x^{\frac{9}{2}} \sqrt{x^2 (b x^3+a)}}+\frac{32b^3\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)}}\ \left(x \cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/55/(b*x^5+a*x^2)^(1/2)/x^(9/2)*(b*x^3+a)/(-a*b^2)^(1/3)/a^2*(32*I*3^(1/2
)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*b^3*x^8-64*I*(-a*b^2)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^2*x^7+3
2*I*(-a*b^2)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*x^6-32*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3*x^8+64*
(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*b^2*x^7-32*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
```

```
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*x^6-
8*I*(x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*b*x^3+24*b*(x*(b*x^3+a))^(1/2)*x^3*(-a*b^2)^
(1/3)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)+5*I*(x*(b*
x^3+a))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*a-15*(x*(b*x^3+a))^(1/2)*a*(-a*b^2)^(1/3)*(1/b^2*x*(-b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2))/(x*(b*x^3+a))^(1/2)/(I*3^(1
/2)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 62, normalized size = 0.23

$$-\frac{2\left(16\sqrt{a}\,b^2x^7\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}\,(8\,abx^3-5\,a^2)\sqrt{x}\right)}{55\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/55*(16*sqrt(a)*b^2*x^7*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5
+ a*x^2)*(8*a*b*x^3 - 5*a^2)*sqrt(x))/(a^3*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{11}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(11/2)*sqrt(x**2*(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{11/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.308 $\int \frac{x}{ax^3+bx^4} \, dx$

Optimal. Leaf size=28

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 46}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax^3+bx^4}\,dx &= \int \frac{1}{x^2(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^3 + b*x^4),x]
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A]
time = 0.35, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(-bx-a)}{a^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A]
time = 0.29, size = 28, normalized size = 1.00

$$\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A]
time = 1.57, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A]
time = 0.07, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A]
time = 0.50, size = 30, normalized size = 1.07

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B]
time = 5.21, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3 + b*x^4),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.309 $\int \frac{1}{ax^3+bx^4} dx$

Optimal. Leaf size=42

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 46}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^4)^(-1),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax^3+bx^4}\,dx &= \int \frac{1}{x^3(a+bx)}\,dx\\
&= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right) dx\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.00

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^4)^(-1),x]
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Maple [A]
time = 0.36, size = 41, normalized size = 0.98

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b}{a^{2}x}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	41
norman	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	41
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}-\frac{b^{2}\ln(bx+a)}{a^{3}}+\frac{b^{2}\ln(-x)}{a^{3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Maxima [A]
time = 0.29, size = 40, normalized size = 0.95

$$-\frac{b^2\log(bx+a)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Fricas [A]
time = 2.15, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log(bx+a)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A]
time = 0.08, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Giac [A]
time = 0.47, size = 45, normalized size = 1.07

$$-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B]
time = 5.72, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^4),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.310 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=112

$$-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}+\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}$$

```
[Out] -5/8*a^3*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(7/2)-5/12*a*(b*x^4+a*x
^3)^(1/2)/b^2+5/8*a^2*(b*x^4+a*x^3)^(1/2)/b^3/x+1/3*x*(b*x^4+a*x^3)^(1/2)/b
```

Rubi [A]
time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2049, 2054, 212}

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-5*a*Sqrt[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*Sqrt[a*x^3 + b*x^4])/(8*b^3*x)
 + (x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3
+ b*x^4]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx &= \frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{(5a)\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx}{6b}\\ &= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}+\frac{(5a^2)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{8b^2}\\ &= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}+\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{(5a^3)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{16b^3}\\ &= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}+\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{(5a^3)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{}{\sqrt{}}\right)}{8b^3}\\ &= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}+\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 104, normalized size = 0.93

$$\frac{\sqrt{b}\,x^2(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+15a^3x^{3/2}\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{24b^{7/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^3 + b*x^4],x]
[Out] (Sqrt[b]*x^2*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 15*a^3*x^(3/2
)*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(24*b^(7/2)*Sqrt[x
^3*(a + b*x)])
```

Maple [A]
time = 0.43, size = 120, normalized size = 1.07

method	result
risch	$\frac{(8b^2x^2-10abx+15a^2)x^2(bx+a)}{24b^3\sqrt{x^3(bx+a)}}-\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)x\sqrt{x\,(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x^3\,(bx+a)}}$

default	$\frac{x\sqrt{x\left(bx+a\right)}\,\left(16x^{2}\sqrt{b\,x^{2}+ax}\,b^{\frac{7}{2}}-20\sqrt{b\,x^{2}+ax}\,b^{\frac{5}{2}}ax+30\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}a^{2}-15\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}}{2\sqrt{b}}\right.\right.}{48\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/48*x*(x*(b*x+a))^(1/2)*(16*x^2*(b*x^2+a*x)^(1/2)*b^(7/2)-20*(b*x^2+a*x)^(1/2)*b^(5/2)*a*x+30*(b*x^2+a*x)^(1/2)*b^(3/2)*a^2-15*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^3*b)/(b*x^4+a*x^3)^(1/2)/b^(9/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)

Fricas [A]
time = 1.87, size = 171, normalized size = 1.53

$$\left[\frac{15\,a^{3}\sqrt{b}\,x\log\left(\frac{2\,bx^{2}+ax-2\,\sqrt{bx^{4}+ax^{3}}\,\sqrt{b}}{x}\right)+2\,(8\,b^{3}x^{2}-10\,ab^{2}x+15\,a^{2}b)\sqrt{bx^{4}+ax^{3}}}{48\,b^{4}x},\ \frac{15\,a^{3}\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^{4}+ax^{3}}\,\sqrt{-b}}{bx^{2}}\right)+(8\,b^{3}x^{2}-10\,ab^{2}x+15\,a^{2}b)\sqrt{bx^{4}+ax^{3}}}{24\,b^{4}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{4}}{\sqrt{x^{3}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x**4+a*x**3)**(1/2),x)

[Out] Integral(x**4/sqrt(x**3*(a + b*x)), x)

Giac [A]
time = 0.76, size = 106, normalized size = 0.95

$$\frac{1}{24}\sqrt{bx^2+ax}\left(2x\left(\frac{4x}{b\mathrm{sgn}\,(x)}-\frac{5a}{b^2\mathrm{sgn}\,(x)}\right)+\frac{15a^2}{b^3\mathrm{sgn}\,(x)}\right)-\frac{5a^3\log\left(|a|\right)\mathrm{sgn}(x)}{16b^{\frac{7}{2}}}+\frac{5a^3\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{16b^{\frac{7}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x^2 + a*x)*(2*x*(4*x/(b*sgn(x)) - 5*a/(b^2*sgn(x))) + 15*a^2/(b
^3*sgn(x))) - 5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 5/16*a^3*log(abs(-2*(sq
rt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/(b^(7/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a*x^3 + b*x^4)^(1/2), x)
```

3.311 $\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(5/2)+1/2*(b*x^4+a*x^3)^
(1/2)/b-3/4*a*(b*x^4+a*x^3)^(1/2)/b^2/x
```

Rubi [A]

time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2049, 2054, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{(3a)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{4b}\\ &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{(3a^2)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{8b^2}\\ &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^2}\\ &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 93, normalized size = 1.08

$$\frac{\sqrt{b}\,x^2(-3a^2-abx+2b^2x^2)-3a^2x^{3/2}\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4b^{5/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^3 + b*x^4],x]
[Out] (Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) - 3*a^2*x^(3/2)*Sqrt[a + b*x]*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(5/2)*Sqrt[x^3*(a + b*x)])
```

Maple [A]
time = 0.38, size = 98, normalized size = 1.14

method	result	size
risch	$-\frac{(-2bx+3a)x^2(bx+a)}{4b^2\sqrt{x^3(bx+a)}}+\frac{3a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)x\sqrt{x\,(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x^3\,(bx+a)}}$	87
default	$\frac{x\sqrt{x\,(bx+a)}\,\left(4x\sqrt{b\,x^2+ax}\,b^{\frac{5}{2}}-6b^{\frac{3}{2}}\sqrt{b\,x^2+ax}\,a+3\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b\right)}{8\sqrt{b\,x^4+a\,x^3}\,b^{\frac{7}{2}}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(x*(b*x+a))^(1/2)*(4*x*(b*x^2+a*x)^(1/2)*b^(5/2)-6*b^(3/2)*(b*x^2+a*x
)^(1/2)*a+3*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^2*b)/(b
*x^4+a*x^3)^(1/2)/b^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

Fricas [A]
time = 1.87, size = 150, normalized size = 1.74

$$\left[\frac{3\,a^2\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}\,(2\,b^2x-3\,ab)}{8\,b^3x},-\frac{3\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)-\sqrt{bx^4+ax^3}\,(2\,b^2x-3\,ab)}{4\,b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x
) + 2*sqrt(b*x^4 + a*x^3)*(2*b^2*x - 3*a*b))/(b^3*x), -1/4*(3*a^2*sqrt(-b)*
x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) - sqrt(b*x^4 + a*x^3)*(2*b^2
*x - 3*a*b))/(b^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

Giac [A]
time = 0.67, size = 90, normalized size = 1.05

$$\frac{1}{4}\sqrt{bx^2+ax}\left(\frac{2\,x}{b\mathrm{sgn}\,(x)}-\frac{3\,a}{b^2\mathrm{sgn}\,(x)}\right)+\frac{3\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{8\,b^{\frac{5}{2}}}-\frac{3\,a^2\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{8\,b^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x/(b*sgn(x)) - 3*a/(b^2*sgn(x))) + 3/8*a^2*log(abs
(a))*sgn(x)/b^(5/2) - 3/8*a^2*log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sq
rt(b) - a))/(b^(5/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x^3 + b*x^4)^(1/2), x)
```

3.312 $\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

```
[Out] -a*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(3/2)+(b*x^4+a*x^3)^(1/2)/b/x
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2049, 2054, 212}

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/
b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{2b}$$
$$= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{b}$$
$$= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

Mathematica [**A**]
time = 0.01, size = 73, normalized size = 1.30

$$\frac{\sqrt{b}\,x^2(a+bx)+ax^{3/2}\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x) + a*x^(3/2)*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[a + b*x]])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

Maple [**A**]
time = 0.37, size = 78, normalized size = 1.39

method	result	size
risch	$\frac{x^2(bx+a)}{b\sqrt{x^3\,(bx+a)}} - \frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)x\sqrt{x\,(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x^3\,(bx+a)}}$	76
default	$\frac{x\sqrt{x\,(bx+a)}\left(2\sqrt{b\,x^2+ax}\,b^{\frac{3}{2}}-a\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^4+a\,x^3}\,b^{\frac{5}{2}}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x*(b*x+a))^(1/2)*(2*(b*x^2+a*x)^(1/2)*b^(3/2)-a*ln(1/2*(2*(b*x^2+a*x
)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*b)/(b*x^4+a*x^3)^(1/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x^3), x)
```

Fricas [A]
time = 1.52, size = 122, normalized size = 2.18

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}\,b}{2\,b^2x}, \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)+\sqrt{bx^4+ax^3}\,b}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) +
2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*
sqrt(-b)/(b*x^2)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**3*(a + b*x)), x)
```

Giac [A]
time = 0.61, size = 71, normalized size = 1.27

$$-\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{2\,b^{\frac{3}{2}}}+\frac{a\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{2\,b^{\frac{3}{2}}\operatorname{sgn}(x)}+\frac{\sqrt{bx^2+ax}}{b\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(abs(a))*sgn(x)/b^(3/2) + 1/2*a*log(abs(-2*(sqrt(b)*x - sqrt(b*x^
2 + a*x))*sqrt(b) - a))/(b^(3/2)*sgn(x)) + sqrt(b*x^2 + a*x)/(b*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^3 + b*x^4)^(1/2),x)
[Out] int(x^2/(a*x^3 + b*x^4)^(1/2), x)
```

3.313 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2054, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.78

$$-\frac{2x^{3/2}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*x^(3/2)*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x^3*(a + b*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.
time = 0.35, size = 56, normalized size = 1.75

method	result	size
default	$\frac{x\sqrt{x(bx+a)}\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)}{\sqrt{b\,x^4+a\,x^3}\,\sqrt{b}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^4+a*x^3)^(1/2)*x*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/
2)+2*b*x+a)/b^(1/2))/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x^3), x)
```

Fricas [A]
time = 2.43, size = 74, normalized size = 2.31

$$\left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-b
)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2))/b]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**3*(a + b*x)), x)
```

Giac [**A**]
time = 0.72, size = 49, normalized size = 1.53

$$\frac{\log\left(|a|\right)\mathrm{sgn}(x)}{\sqrt{b}}-\frac{\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{\sqrt{b}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(a))*sgn(x)/sqrt(b) - log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqr
t(b) - a))/(sqrt(b)*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x/(a*x^3 + b*x^4)^(1/2), x)
```

3.314 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

```
[Out] -2*(b*x^4+a*x^3)^(1/2)/a/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2025}

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)])/(a*x^2)
```

Maple [A]
time = 0.36, size = 39, normalized size = 1.70

method	result	size
trager	$-\frac{2\sqrt{b\,x^{4}+a\,x^{3}}}{a\,x^{2}}$	22
risch	$-\frac{2x(bx+a)}{\sqrt{x^{3}\left(bx+a\right)}\,a}$	23
gosper	$-\frac{2x(bx+a)}{a\sqrt{b\,x^{4}+a\,x^{3}}}$	25
default	$-\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b\,x^{2}+ax}}{\sqrt{b\,x^{4}+a\,x^{3}}\,a}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/(b*x^4+a*x^3)^(1/2)*(x*(b*x+a))^(1/2)/a*(b*x^2+a*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*x^4 + a*x^3), x)
```

Fricas [A]
time = 1.98, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{bx^4+ax^3}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^4 + a*x^3)/(a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ax^3+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/sqrt(a*x**3 + b*x**4), x)
```

Giac [A]
time = 0.63, size = 27, normalized size = 1.17

$$\frac{2}{\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))
```

Mupad [B]
time = 5.14, size = 21, normalized size = 0.91

$$-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^4)^(1/2),x)
[Out] -(2*(a*x^3 + b*x^4)^(1/2))/(a*x^2)
```

3.315 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=52

$$-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}$$

```
[Out] -2/3*(b*x^4+a*x^3)^(1/2)/a/x^3+4/3*b*(b*x^4+a*x^3)^(1/2)/a^2/x^2
```

Rubi [A]

time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2025}

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}-\frac{(2b)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.60

$$-\frac{2(a-2bx)(a+bx)}{3a^2\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x)*(a + b*x))/(3*a^2*Sqrt[x^3*(a + b*x)])
```

Maple [A]
time = 0.34, size = 48, normalized size = 0.92

method	result	size
trager	$-\frac{2(-2bx+a)\sqrt{b\,x^4+a\,x^3}}{3a^2x^3}$	28
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^3\,(bx+a)}\;a^2}$	28
gosper	$-\frac{2(bx+a)(-2bx+a)}{3a^2\sqrt{b\,x^4+a\,x^3}}$	30
default	$-\frac{2\sqrt{x\,(bx+a)}\,\sqrt{b\,x^2+ax}\,(-2bx+a)}{3x\sqrt{b\,x^4+a\,x^3}\;a^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/x*(x*(b*x+a))^(1/2)*(b*x^2+a*x)^(1/2)*(-2*b*x+a)/(b*x^4+a*x^3)^(1/2)/a
^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)
```

Fricas [A]
time = 1.66, size = 29, normalized size = 0.56

$$\frac{2\sqrt{bx^4+ax^3}\,(2\,bx-a)}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^3\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**3*(a + b*x))), x)
```

Giac [**A**]
time = 0.59, size = 53, normalized size = 1.02

$$\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)}{3\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)^3\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)/((sqrt(b)*x - sqrt(b*x^
2 + a*x))^3*sgn(x))
```

Mupad [**B**]
time = 5.06, size = 42, normalized size = 0.81

$$-\frac{2\,a\,\sqrt{b\,x^4+a\,x^3}-4\,b\,x\,\sqrt{b\,x^4+a\,x^3}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] -(2*a*(a*x^3 + b*x^4)^(1/2) - 4*b*x*(a*x^3 + b*x^4)^(1/2))/(3*a^2*x^3)
```

3.316 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}$$

```
[Out] -2/5*(b*x^4+a*x^3)^(1/2)/a/x^4+8/15*b*(b*x^4+a*x^3)^(1/2)/a^2/x^3-16/15*b^2
*(b*x^4+a*x^3)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2025}

$$-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*Sqrt[a*x^3 + b*x^4])/(15*a^2*x^3)
 - (16*b^2*Sqrt[a*x^3 + b*x^4])/(15*a^3*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{5ax^4} - \frac{(4b)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{5a}$$
$$= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4} + \frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3} + \frac{(8b^2)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{15a^2}$$
$$= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4} + \frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3} - \frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.52

$$-\frac{2\sqrt{x^3(a+bx)}\,(3a^2-4abx+8b^2x^2)}{15a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)
```

Maple [A]
time = 0.36, size = 61, normalized size = 0.76

method	result	size
trager	$-\frac{2(8b^2x^2-4abx+3a^2)\sqrt{b\,x^4+a\,x^3}}{15a^3x^4}$	41
risch	$-\frac{2(bx+a)(8b^2x^2-4abx+3a^2)}{15x\sqrt{x^3\,(bx+a)}\,a^3}$	44
gosper	$-\frac{2(bx+a)(8b^2x^2-4abx+3a^2)}{15x\,a^3\sqrt{b\,x^4+a\,x^3}}$	46
default	$-\frac{2\sqrt{x\,(bx+a)}\,\sqrt{b\,x^2+ax}\,(8b^2x^2-4abx+3a^2)}{15x^2\sqrt{b\,x^4+a\,x^3}\,a^3}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/x^2*(x*(b*x+a))^(1/2)*(b*x^2+a*x)^(1/2)*(8*b^2*x^2-4*a*b*x+3*a^2)/(b*
x^4+a*x^3)^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)
```

Fricas [A]
time = 1.97, size = 40, normalized size = 0.50

$$-\frac{2\sqrt{bx^4+ax^3}\,(8\,b^2x^2-4\,abx+3\,a^2)}{15\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)
```

Giac [A]
time = 0.90, size = 82, normalized size = 1.02

$$\frac{2\left(20\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)^2b+15\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)a\sqrt{b}\,+3\,a^2\right)}{15\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)^5\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(20*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b + 15*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))*a*sqrt(b) + 3*a^2)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^5*sgn(x))
```

Mupad [B]
time = 5.14, size = 40, normalized size = 0.50

$$-\frac{2\sqrt{b\,x^4+a\,x^3}\,\left(3\,a^2-4\,a\,b\,x+8\,b^2\,x^2\right)}{15\,a^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x^3 + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^2 - 4*a*b*x))/(15*a^3*x^4)
```

3.317 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}$$

```
[Out] -2/7*(b*x^4+a*x^3)^(1/2)/a/x^5+12/35*b*(b*x^4+a*x^3)^(1/2)/a^2/x^4-16/35*b^
2*(b*x^4+a*x^3)^(1/2)/a^3/x^3+32/35*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2025}

$$\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*Sqrt[a*x^3 + b*x^4])/(35*a^2*x^4
) - (16*b^2*Sqrt[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*Sqrt[a*x^3 + b*x^4]
)/(35*a^4*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{7ax^5} - \frac{(6b)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{7a}$$
$$= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5} + \frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4} + \frac{(24b^2)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{35a^2}$$
$$= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5} + \frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4} - \frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3} - \frac{(16b^3)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{35a^3}$$
$$= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5} + \frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4} - \frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3} + \frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.49

$$\frac{2\sqrt{x^3(a+bx)}\,(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^5)
```

Maple [A]
time = 0.37, size = 72, normalized size = 0.67

method	result	size
trager	$-\frac{2(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)\sqrt{b\,x^4+a\,x^3}}{35a^4x^5}$	52
risch	$-\frac{2(bx+a)(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^2\sqrt{x^3\,(bx+a)}\,a^4}$	55
gosper	$-\frac{2(bx+a)(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^2a^4\sqrt{b\,x^4+a\,x^3}}$	57
default	$-\frac{2\sqrt{x\,(bx+a)}\,\sqrt{b\,x^2+ax}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^3\sqrt{b\,x^4+a\,x^3}\,a^4}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/x^3*(x*(b*x+a))^(1/2)*(b*x^2+a*x)^(1/2)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^
2*b*x+5*a^3)/(b*x^4+a*x^3)^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)
```

Fricas [A]
time = 1.94, size = 51, normalized size = 0.47

$$\frac{2\,(16\,b^3x^3 - 8\,ab^2x^2 + 6\,a^2bx - 5\,a^3)\sqrt{bx^4 + ax^3}}{35\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a^
4*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^3\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)
```

Giac [A]
time = 0.57, size = 111, normalized size = 1.03

$$\frac{2\left(70\left(\sqrt{b}\,x - \sqrt{bx^2 + ax}\right)^3 b^{\frac{3}{2}} + 84\left(\sqrt{b}\,x - \sqrt{bx^2 + ax}\right)^2 ab + 35\left(\sqrt{b}\,x - \sqrt{bx^2 + ax}\right)a^2\sqrt{b} + 5\,a^3\right)}{35\left(\sqrt{b}\,x - \sqrt{bx^2 + ax}\right)^7 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 2/35*(70*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2) + 84*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^2*a*b + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b) + 5*a^3
)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^7*sgn(x))
```

Mupad [B]
time = 5.14, size = 92, normalized size = 0.85

$$\frac{12\,b\,\sqrt{b\,x^4+a\,x^3}}{35\,a^2\,x^4}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{7\,a\,x^5}-\frac{16\,b^2\,\sqrt{b\,x^4+a\,x^3}}{35\,a^3\,x^3}+\frac{32\,b^3\,\sqrt{b\,x^4+a\,x^3}}{35\,a^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (12*b*(a*x^3 + b*x^4)^(1/2))/(35*a^2*x^4) - (2*(a*x^3 + b*x^4)^(1/2))/(7*a*
x^5) - (16*b^2*(a*x^3 + b*x^4)^(1/2))/(35*a^3*x^3) + (32*b^3*(a*x^3 + b*x^4
)^(1/2))/(35*a^4*x^2)
```

3.318 $\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}$$

```
[Out] -2/9*(b*x^4+a*x^3)^(1/2)/a/x^6+16/63*b*(b*x^4+a*x^3)^(1/2)/a^2/x^5-32/105*b
^2*(b*x^4+a*x^3)^(1/2)/a^3/x^4+128/315*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^3-256/
315*b^4*(b*x^4+a*x^3)^(1/2)/a^5/x^2
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 2025}

$$-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*Sqrt[a*x^3 + b*x^4])/(63*a^2*x^5
) - (32*b^2*Sqrt[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*Sqrt[a*x^3 + b*x^
4])/(315*a^4*x^3) - (256*b^4*Sqrt[a*x^3 + b*x^4])/(315*a^5*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} - \frac{(8b)\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx}{9a}$$

$$= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} + \frac{(16b^2)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{21a^2}$$

$$= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4} - \frac{(64b^3)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{105a^3}$$

$$= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4} + \frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3} + \frac{(12}{}$$

$$= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4} + \frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3} - \frac{256}{}$$

Mathematica [**A**]
time = 0.09, size = 64, normalized size = 0.47

$$-\frac{2\sqrt{x^3(a+bx)}\,(35a^4-40a^3bx+48a^2b^2x^2-64ab^3x^3+128b^4x^4)}{315a^5x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x^
3 + 128*b^4*x^4))/(315*a^5*x^6)
```

Maple [**A**]
time = 0.49, size = 83, normalized size = 0.61

method	result	size
trager	$-\frac{2(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4)\sqrt{b\,x^4+a\,x^3}}{315a^5x^6}$	63
risch	$-\frac{2(bx+a)(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4)}{315x^3\sqrt{x^3\,(bx+a)}\,a^5}$	66
gosper	$-\frac{2(bx+a)(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4)}{315x^3a^5\sqrt{b\,x^4+a\,x^3}}$	68
default	$-\frac{2\sqrt{x\,(bx+a)}\,\sqrt{b\,x^2+ax}\,(128b^4x^4-64a\,b^3x^3+48a^2b^2x^2-40a^3bx+35a^4)}{315x^4\sqrt{b\,x^4+a\,x^3}\,a^5}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/x^4*(x*(b*x+a))^(1/2)*(b*x^2+a*x)^(1/2)*(128*b^4*x^4-64*a*b^3*x^3+48
*a^2*b^2*x^2-40*a^3*b*x+35*a^4)/(b*x^4+a*x^3)^(1/2)/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^4), x)
```

Fricas [A]
time = 3.32, size = 62, normalized size = 0.46

$$-\frac{2\,(128\,b^4x^4-64\,ab^3x^3+48\,a^2b^2x^2-40\,a^3bx+35\,a^4)\sqrt{bx^4+ax^3}}{315\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] -2/315*(128*b^4*x^4 - 64*a*b^3*x^3 + 48*a^2*b^2*x^2 - 40*a^3*b*x + 35*a^4)*
sqrt(b*x^4 + a*x^3)/(a^5*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^4\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(x**3*(a + b*x))), x)
```

Giac [A]
time = 0.61, size = 140, normalized size = 1.03

$$\frac{2\left(1008\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)^4b^2+1680\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)^3ab^{\frac{3}{2}}+1080\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)^2a^2b+315\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)a^3\sqrt{b}+35\,a^4\right)}{315\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)^9\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 2/315*(1008*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2 + 1680*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^3*a*b^(3/2) + 1080*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b +
```

```
315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + 35*a^4)/((sqrt(b)*x - sqr
t(b*x^2 + a*x))^9*sgn(x))
```

Mupad [B]
time = 5.14, size = 116, normalized size = 0.85

$$\frac{16\,b\,\sqrt{b\,x^4+a\,x^3}}{63\,a^2\,x^5}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{9\,a\,x^6}-\frac{32\,b^2\,\sqrt{b\,x^4+a\,x^3}}{105\,a^3\,x^4}+\frac{128\,b^3\,\sqrt{b\,x^4+a\,x^3}}{315\,a^4\,x^3}-\frac{256\,b^4\,\sqrt{b\,x^4+a\,x^3}}{315\,a^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (16*b*(a*x^3 + b*x^4)^(1/2))/(63*a^2*x^5) - (2*(a*x^3 + b*x^4)^(1/2))/(9*a*
x^6) - (32*b^2*(a*x^3 + b*x^4)^(1/2))/(105*a^3*x^4) + (128*b^3*(a*x^3 + b*x
^4)^(1/2))/(315*a^4*x^3) - (256*b^4*(a*x^3 + b*x^4)^(1/2))/(315*a^5*x^2)
```

3.319 $\int \frac{1}{x^3+bx^5}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)$$

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1607, 272, 46}

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3+bx^5}\,dx &= \int \frac{1}{x^3(1+bx^2)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx, x, x^2\right)\\
&= -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$-\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Maple [A]
time = 0.38, size = 23, normalized size = 0.88

method	result	size
default	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(b\,x^2+1)}{2}$	23
norman	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(b\,x^2+1)}{2}$	23
risch	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(-b\,x^2-1)}{2}$	24
meijerg	$\frac{b\left(\ln(b\,x^2+1)-2\ln(x)-\ln(b)-\frac{1}{x^2 b}\right)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.85

$$\frac{1}{2}\,b\log\left(bx^2+1\right)-b\log\left(x\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + 1) - b*log(x) - 1/2/x^2
```

Fricas [A]
time = 1.71, size = 28, normalized size = 1.08

$$\frac{bx^2\log\left(bx^2+1\right)-2\,bx^2\log\left(x\right)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

Sympy [A]
time = 0.09, size = 22, normalized size = 0.85

$$-b\log\left(x\right)+\frac{b\log\left(x^2+\frac{1}{b}\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+x**3),x)
```

```
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

Giac [A]
time = 0.60, size = 32, normalized size = 1.23

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

Mupad [B]
time = 0.05, size = 22, normalized size = 0.85

$$\frac{b\,\ln\left(b\,x^2+1\right)}{2}-b\,\ln\left(x\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5 + x^3),x)
```

```
[Out] (b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

3.320 $\int \frac{1}{-x^3+bx^5} dx$

Optimal. Leaf size=27

$$\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1-bx^2\right)$$

[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 46}

$$\frac{1}{2}b\log\left(1-bx^2\right) - b\log(x) + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(-x^3 + b*x^5)^(-1),x]

[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-x^3+bx^5}\,dx=\int \frac{1}{x^3(-1+bx^2)}\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)\,dx,x,x^2\right)$$
$$=\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + b*x^5)^(-1),x]
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Maple [A]
time = 0.38, size = 23, normalized size = 0.85

method	result	size
default	$\frac{b\ln(b\,x^2-1)}{2}+\frac{1}{2x^2}-b\ln(x)$	23
norman	$\frac{b\ln(b\,x^2-1)}{2}+\frac{1}{2x^2}-b\ln(x)$	23
risch	$\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(-b\,x^2+1)}{2}$	24
meijerg	$\frac{b\left(\ln(-b\,x^2+1)-2\ln(x)-\ln(-b)+\frac{1}{x^2 b}\right)}{2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5-x^3),x,method=_RETURNVERBOSE)
[Out] 1/2*b*ln(b*x^2-1)+1/2/x^2-b*ln(x)
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.81

$$\frac{1}{2}\,b\log\left(bx^2-1\right)-b\log\left(x\right)+\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2
```

Fricas [**A**]
time = 1.91, size = 28, normalized size = 1.04

$$\frac{bx^2 \log(bx^2 - 1) - 2\,bx^2 \log(x) + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

Sympy [**A**]
time = 0.09, size = 22, normalized size = 0.81

$$-b\log(x) + \frac{b\log\left(x^2 - \frac{1}{b}\right)}{2} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5-x**3),x)
```

```
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

Giac [**A**]
time = 0.58, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,b\log\left(x^2\right) + \frac{1}{2}\,b\log\left(\left|bx^2 - 1\right|\right) + \frac{bx^2 + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

Mupad [**B**]
time = 5.21, size = 22, normalized size = 0.81

$$\frac{b\,\ln\left(b\,x^2 - 1\right)}{2} - b\,\ln\left(x\right) + \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5 - x^3),x)
```

```
[Out] (b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

3.321 $\int \frac{1}{ax+bx}\,dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{a+b}$$

```
[Out] ln(x)/(a+b)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{6, 12, 29\}$

$$\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-1),x]
```

```
[Out] Log[x]/(a + b)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx}\,dx &= \int \frac{1}{(a+b)x}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{a+b}\\
&= \frac{\log(x)}{a+b}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.75

$$\frac{\log(ax+bx)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-1),x]
[Out] Log[a*x + b*x]/(a + b)
```

Maple [A]
time = 0.02, size = 9, normalized size = 1.12

method	result	size
default	$\frac{\ln(x)}{a+b}$	9
norman	$\frac{\ln(x)}{a+b}$	9
risch	$\frac{\ln(x)}{a+b}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x)/(a+b)
```

Maxima [A]
time = 0.34, size = 14, normalized size = 1.75

$$\frac{\log(ax+bx)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="maxima")
[Out] log(a*x + b*x)/(a + b)
```

Fricas [A]
time = 2.79, size = 8, normalized size = 1.00

$$\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="fricas")
[Out] log(x)/(a + b)
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x)
[Out] log(x)/(a + b)
```

Giac [A]
time = 0.53, size = 15, normalized size = 1.88

$$\frac{\log(|ax+bx|)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="giac")
[Out] log(abs(a*x + b*x))/(a + b)
```

Mupad [B]
time = 5.27, size = 8, normalized size = 1.00

$$\frac{\ln(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x),x)
[Out] log(x)/(a + b)
```

3.322 $\int \frac{1}{(ax+bx)^2} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{(a+b)^2 x}$$

```
[Out] -1/(a+b)^2/x
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-2),x]
```

```
[Out] -(1/((a + b)^2*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax+bx)^2} \, dx &= \int \frac{1}{(a+b)^2 x^2} \, dx \\
&= \frac{\int \frac{1}{x^2} \, dx}{(a+b)^2} \\
&= -\frac{1}{(a+b)^2 x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-2),x]
```

```
[Out] -(1/((a + b)^2*x))
```

Maple [A]
time = 0.01, size = 11, normalized size = 1.10

method	result	size
gosper	$-\frac{1}{(a+b)^2 x}$	11
default	$-\frac{1}{(a+b)^2 x}$	11
norman	$-\frac{1}{(a+b)^2 x}$	11
risch	$-\frac{1}{(a+b)^2 x}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)^2/x
```

Maxima [A]
time = 0.29, size = 16, normalized size = 1.60

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/((a*x + b*x)*(a + b))
```

Fricas [A]
time = 2.23, size = 18, normalized size = 1.80

$$-\frac{1}{(a^2+2\,ab+b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/((a^2 + 2*a*b + b^2)*x)
```

Sympy [A]
time = 0.01, size = 15, normalized size = 1.50

$$-\frac{1}{x\left(a^{2}+2ab+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**2,x)
[Out] -1/(x*(a**2 + 2*a*b + b**2))
```

Giac [A]
time = 0.53, size = 16, normalized size = 1.60

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="giac")
[Out] -1/((a*x + b*x)*(a + b))
```

Mupad [B]
time = 0.03, size = 10, normalized size = 1.00

$$-\frac{1}{x\left(a+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x)^2,x)
[Out] -1/(x*(a + b)^2)
```

3.323 $\int \frac{1}{(ax+bx)^3} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{2(a+b)^3x^2}$$

[Out] -1/2/(a+b)^3/x^2

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x)^(-3),x]

[Out] -1/2*1/((a + b)^3*x^2)

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx)^3} \, dx &= \int \frac{1}{(a+b)^3x^3} \, dx\\ &= \frac{\int \frac{1}{x^3} \, dx}{(a+b)^3}\\ &= -\frac{1}{2(a+b)^3x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2(a+b)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-3),x]
```

```
[Out] -1/2*1/((a + b)^3*x^2)
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.92

method	result	size
gosper	$-\frac{1}{2(a+b)^3x^2}$	11
default	$-\frac{1}{2(a+b)^3x^2}$	11
norman	$-\frac{1}{2(a+b)^3x^2}$	11
risch	$-\frac{1}{2(a+b)^3x^2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a+b)^3/x^2
```

Maxima [A]
time = 0.27, size = 16, normalized size = 1.33

$$-\frac{1}{2\,(ax+bx)^2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.
time = 2.33, size = 26, normalized size = 2.17

$$-\frac{1}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 0.02, size = 27, normalized size = 2.25

$$-\frac{1}{2x^2\left(a^3+3a^2b+3ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**3,x)
```

```
[Out] -1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

Giac [A]
time = 0.54, size = 16, normalized size = 1.33

$$-\frac{1}{2\left(ax+bx\right)^2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="giac")
```

```
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

Mupad [B]
time = 0.04, size = 26, normalized size = 2.17

$$-\frac{1}{2\,x^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x)^3,x)
```

```
[Out] -1/(2*x^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
```

3.324 $\int \frac{1}{ax^2+bx^2} dx$

Optimal. Leaf size=10

$$-\frac{1}{(a+b)x}$$

```
[Out] -1/(a+b)/x
```

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^2)^(-1),x]
```

```
[Out] -(1/((a + b)*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2+bx^2}\,dx &= \int \frac{1}{(a+b)x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{a+b}\\ &= -\frac{1}{(a+b)x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^2)^(-1),x]
[Out] -(1/((a + b)*x))
```

Maple [A]
time = 0.38, size = 11, normalized size = 1.10

method	result	size
gosper	$-\frac{1}{(a+b)x}$	11
default	$-\frac{1}{(a+b)x}$	11
norman	$-\frac{1}{(a+b)x}$	11
risch	$-\frac{1}{(a+b)x}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^2),x,method=_RETURNVERBOSE)
[Out] -1/(a+b)/x
```

Maxima [A]
time = 0.30, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")
[Out] -1/((a + b)*x)
```

Fricas [A]
time = 1.55, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/((a + b)*x)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$-\frac{1}{x\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**2),x)
[Out] -1/(x*(a + b))
```

Giac [A]
time = 0.47, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="giac")
[Out] -1/((a + b)*x)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 1.00

$$-\frac{1}{x\;(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^2),x)
[Out] -1/(x*(a + b))
```

3.325 $\int \frac{1}{ax^n+bx^n} \, dx$

Optimal. Leaf size=20

$$\frac{x^{1-n}}{(a+b)(1-n)}$$

[Out] x^(1-n)/(a+b)/(1-n)

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^n + b*x^n)^(-1),x]

[Out] x^(1 - n)/((a + b)*(1 - n))

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax^n+bx^n} \, dx &= \int \frac{x^{-n}}{a+b} \, dx \\
&= \frac{\int x^{-n} \, dx}{a+b} \\
&= \frac{x^{1-n}}{(a+b)(1-n)}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-n}}{(a+b)(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-1),x]
[Out] x^(1 - n)/((a + b)*(1 - n))
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.95

method	result	size
gosper	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
risch	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
norman	$-\frac{x\,\mathrm{e}^{-n\ln(x)}}{an+bn-a-b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n),x,method=_RETURNVERBOSE)
[Out] -x/(-1+n)/(x^n)/(a+b)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 1.05

$$-\frac{x}{(a(n-1)+b(n-1))x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")
[Out] -x/((a*(n - 1) + b*(n - 1))*x^n)
```

Fricas [A]
time = 1.33, size = 22, normalized size = 1.10

$$-\frac{x}{((a+b)n-a-b)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")
[Out] -x/(((a + b)*n - a - b)*x^n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.
time = 0.26, size = 32, normalized size = 1.60

$$\begin{cases} -\frac{x}{anx^n - ax^n + bnx^n - bx^n} & \text{for } n \neq 1 \\ \frac{\log(x)}{a+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n),x)
```

```
[Out] Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/(
a + b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x^n + b*x^n), x)
```

Mupad [B]
time = 5.18, size = 19, normalized size = 0.95

$$-\frac{x^{1-n}}{(a+b)\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n),x)
```

```
[Out] -x^(1 - n)/((a + b)*(n - 1))
```

3.326 $\int \frac{1}{(ax^n+bx^n)^2} dx$

Optimal. Leaf size=20

$$\frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

```
[Out] x^(1-2*n)/(a+b)^2/(1-2*n)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-2),x]
```

```
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^n+bx^n)^2}\,dx &= \int \frac{x^{-2n}}{(a+b)^2}\,dx\\ &= \frac{\int x^{-2n}\,dx}{(a+b)^2}\\ &= \frac{x^{1-2n}}{(a+b)^2(1-2n)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-2),x]
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Maple [A]
time = 0.02, size = 21, normalized size = 1.05

method	result	size
gosper	$-\frac{x\,x^{-2n}}{(-1+2n)(a+b)^2}$	21
risch	$-\frac{x\,x^{-2n}}{(a^2+2ab+b^2)(-1+2n)}$	29
norman	$-\frac{x\,\mathrm{e}^{-2n\ln(x)}}{(2an+2bn-a-b)(a+b)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^2,x,method=_RETURNVERBOSE)
[Out] -x/(-1+2*n)/(x^n)^2/(a+b)^2
```

Maxima [A]
time = 0.28, size = 40, normalized size = 2.00

$$-\frac{x}{(a^2(2\,n-1)+2\,ab(2\,n-1)+b^2(2\,n-1))x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")
[Out] -x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))
```

Fricas [A]
time = 2.18, size = 36, normalized size = 1.80

$$\frac{x}{(a^2+2\,ab+b^2-2\,(a^2+2\,ab+b^2)n)x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")
[Out] x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(15) = 30$.
time = 0.38, size = 82, normalized size = 4.10

$$\begin{cases} -\frac{x}{2a^2nx^{2n}-a^2x^{2n}+4abnx^{2n}-2abx^{2n}+2b^2nx^{2n}-b^2x^{2n}} & \text{for } n \neq \frac{1}{2} \\ \frac{\log(x)}{a^2+2ab+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**2,x)
```

```
[Out] Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*b
*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**2
+ 2*a*b + b**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-2), x)
```

Mupad [B]
time = 5.14, size = 21, normalized size = 1.05

$$-\frac{x^{1-2n}}{(a+b)^2 (2n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n)^2,x)
```

```
[Out] -x^(1 - 2*n)/((a + b)^2*(2*n - 1))
```

3.327 $\int \frac{1}{(ax^n+bx^n)^3}\,dx$

Optimal. Leaf size=20

$$\frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

```
[Out] x^(1-3*n)/(a+b)^3/(1-3*n)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6, 12, 30}

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax^n+bx^n)^3}\,dx &= \int \frac{x^{-3n}}{(a+b)^3}\,dx\\
&= \frac{\int x^{-3n}\,dx}{(a+b)^3}\\
&= \frac{x^{1-3n}}{(a+b)^3(1-3n)}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Maple [A]
time = 0.02, size = 21, normalized size = 1.05

method	result	size
gosper	$-\frac{x\,x^{-3n}}{(-1+3n)(a+b)^3}$	21
norman	$-\frac{x\,\mathrm{e}^{-3n\ln(x)}}{(3an+3bn-a-b)(a+b)^2}$	33
risch	$-\frac{x\,x^{-3n}}{(a^3+3a^2b+3a\,b^2+b^3)(-1+3n)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x/(-1+3*n)/(x^n)^3/(a+b)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.
time = 0.30, size = 53, normalized size = 2.65

$$-\frac{x}{(a^3(3\,n-1)+3\,a^2b(3\,n-1)+3\,ab^2(3\,n-1)+b^3(3\,n-1))x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1))
*x^(3*n))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.
time = 3.24, size = 52, normalized size = 2.60

$$\frac{x}{(a^3+3\,a^2b+3\,ab^2+b^3-3\,(a^3+3\,a^2b+3\,ab^2+b^3)n)x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x^
(3*n))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(15) = 30$.
time = 0.46, size = 119, normalized size = 5.95

$$\begin{cases} -\frac{x}{3a^3nx^{3n}-a^3x^{3n}+9a^2bnx^{3n}-3a^2bx^{3n}+9ab^2nx^{3n}-3ab^2x^{3n}+3b^3nx^{3n}-b^3x^{3n}} & \text{for } n \neq \frac{1}{3} \\ \frac{\log(x)}{a^3+3a^2b+3ab^2+b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**3,x)
```

```
[Out] Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3*
a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(3*
n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 + b**
3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-3), x)
```

Mupad [B]
time = 5.12, size = 21, normalized size = 1.05

$$-\frac{x^{1-3\,n}}{{\left(a+b\right)}^3\,\left(3\,n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n)^3,x)
```

```
[Out] -x^(1 - 3*n)/((a + b)^3*(3*n - 1))
```

3.328 $\int (ax+bx^{14})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{13})^{13}}{169b}$$

[Out] 1/169*(b*x^13+a)^13/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 267}

$$\frac{(a+bx^{13})^{13}}{169b}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^14)^12,x]

[Out] (a + b*x^13)^13/(169*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (ax+bx^{14})^{12}\,dx &= \int x^{12}(a+bx^{13})^{12}\,dx\\ &= \frac{(a+bx^{13})^{13}}{169b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.36, size = 135, normalized size = 8.44

method	result
default	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}+\frac{132}{13}$
gosper	$\frac{x^{13}\left(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^{2}b^{10}x^{130}+286a^{3}b^{9}x^{117}+715a^{4}b^{8}x^{104}+1287a^{5}b^{7}x^{91}+1716a^{6}b^{6}x^{78}+1716b^{5}a^{7}x^{65}+1287a^{8}b^{4}x^{52}+715\right.}{169}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^{2}b^{10}x^{143}}{13}+\frac{22a^{3}b^{9}x^{130}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{132b^{5}a^{7}x^{78}}{13}+\frac{99a^{8}b^{4}x^{6}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*b^5*a^7*
x^78+99/13*a^8*b^4*x^65+55/13*b^3*a^9*x^52+22/13*a^10*b^2*x^39+6/13*b*a^11*
x^26+1/13*a^12*x^13
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.28, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 2.57, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

time = 0.03, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

time = 0.48, size = 134, normalized size = 8.38

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}ab^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^{11}bx^{26}+\frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Mupad [B]

time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a^4\,b^8\,x^{117}}{13}+\frac{22\,a^3\,b^9\,x^{130}}{13}+\frac{6\,a^2\,b^{10}\,x^{143}}{13}+\frac{a\,b^{11}\,x^{156}}{13}+\frac{b^{12}\,x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^14)^12,x)
```

```
[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
+ (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (13
2*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^
4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

3.329 $\int x^{12}(ax+bx^{26})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{25})^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 267}

$$\frac{(a+bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx &= \int x^{24}\left(a+bx^{25}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{25}\right)^{13}}{325b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.56, size = 135, normalized size = 8.44

method	result
default	$\frac{132}{25}a^6b^6x^{175} + \frac{11}{5}b^3a^9x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}b\,a^{11}x^{50} + \frac{1}{25}a^{12}x^{25} + \frac{132}{25}b^5a^7x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{1}{325}b^{12}$
gosper	$\frac{x^{25}\left(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716b^5a^7x^{125}+1287a^8b^4x^{100}+7\right.}{325}$
risch	$\frac{b^{12}x^{325}}{325} + \frac{a\,b^{11}x^{300}}{25} + \frac{6a^2b^{10}x^{275}}{25} + \frac{22a^3b^9x^{250}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{99a^5b^7x^{200}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{132b^5a^7x^{150}}{25} + \frac{99a^8b^4x}{25}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 132/25*a^6*b^6*x^175+11/5*b^3*a^9*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^5
0+1/25*a^12*x^25+132/25*b^5*a^7*x^150+99/25*a^8*b^4*x^125+1/325*b^12*x^325+
1/25*a*b^11*x^300+11/5*a^4*b^8*x^225+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^25
0+99/25*a^5*b^7*x^200
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.27, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325} + \frac{1}{25}\,ab^{11}x^{300} + \frac{6}{25}\,a^2b^{10}x^{275} + \frac{22}{25}\,a^3b^9x^{250} + \frac{11}{5}\,a^4b^8x^{225} + \frac{99}{25}\,a^5b^7x^{200} + \frac{132}{25}\,a^6b^6x^{175} + \frac{132}{25}\,a^7b^5x^{150} + \frac{99}{25}\,a^8b^4x^{125} + \frac{11}{5}\,a^9b^3x^{100} + \frac{22}{25}\,a^{10}b^2x^{75} + \frac{6}{25}\,a^{11}bx^{50} + \frac{1}{25}\,a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.68, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325} + \frac{1}{25}\,ab^{11}x^{300} + \frac{6}{25}\,a^2b^{10}x^{275} + \frac{22}{25}\,a^3b^9x^{250} + \frac{11}{5}\,a^4b^8x^{225} + \frac{99}{25}\,a^5b^7x^{200} + \frac{132}{25}\,a^6b^6x^{175} + \frac{132}{25}\,a^7b^5x^{150} + \frac{99}{25}\,a^8b^4x^{125} + \frac{11}{5}\,a^9b^3x^{100} + \frac{22}{25}\,a^{10}b^2x^{75} + \frac{6}{25}\,a^{11}bx^{50} + \frac{1}{25}\,a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

time = 0.04, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

time = 0.47, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{11}{5}a^{9}b^{3}x^{100}+\frac{22}{25}a^{10}b^{2}x^{75}+\frac{6}{25}a^{11}bx^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B]

time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.330 $\int x^{24}(ax+bx^{38})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{37})^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 267}

$$\frac{(a+bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{(a+bx^{37})^{13}}{481b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.05, size = 135, normalized size = 8.44

method	result
default	$\frac{1}{37}a^{12}x^{37}+\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{6}{37}b\,a^{11}x^{74}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b$
gosper	$\frac{x^{37}\left(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^{2}b^{10}x^{370}+286a^{3}b^{9}x^{333}+715a^{4}b^{8}x^{296}+1287a^{5}b^{7}x^{259}+1716a^{6}b^{6}x^{222}+1716b^{5}a^{7}x^{185}+1287a^{8}b^{4}x^{148}\right.}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{132b^{5}a^{7}x^{222}}{37}+\frac{99a^{8}b^{4}}{37}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/37*a^12*x^37+1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+6/37*
b*a^11*x^74+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132
/37*a^6*b^6*x^259+132/37*b^5*a^7*x^222+22/37*a^10*b^2*x^111+99/37*a^8*b^4*x
^185+55/37*b^3*a^9*x^148
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.28, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 2.25, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.
time = 0.04, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.47, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B]
time = 5.15, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37} + \frac{6\,a^{11}\,b\,x^{74}}{37} + \frac{22\,a^{10}\,b^2\,x^{111}}{37} + \frac{55\,a^9\,b^3\,x^{148}}{37} + \frac{99\,a^8\,b^4\,x^{185}}{37} + \frac{132\,a^7\,b^5\,x^{222}}{37} + \frac{132\,a^6\,b^6\,x^{259}}{37} + \frac{99\,a^5\,b^7\,x^{296}}{37} + \frac{55\,a^4\,b^8\,x^{333}}{37} + \frac{22\,a^3\,b^9\,x^{370}}{37} + \frac{6\,a^2\,b^{10}\,x^{407}}{37} + \frac{a\,b^{11}\,x^{444}}{37} + \frac{b^{12}\,x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.331 $\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1598, 267}

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx &= \int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(27) = 54.
time = 0.07, size = 193, normalized size = 7.15

$$\frac{x^{1+12m}\left(13a^{12}+78a^{11}bx^{1+12m}+286a^{10}b^2x^{2+24m}+715a^9b^3x^{3+36m}+1287a^8b^4x^{4+48m}+1716a^7b^5x^{5+60m}+1716a^6b^6x^{6+72m}+1287a^5b^7x^{7+84m}+715a^4b^8x^{8+96m}+286a^3b^9x^{9+108m}+78a^2b^{10}x^{10+120m}+13ab^{11}x^{11+132m}+b^{12}x^{12+144m}\right)}{13+156m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (x^(1 + 12*m)*(13*a^12 + 78*a^11*b*x^(1 + 12*m) + 286*a^10*b^2*x^(2 + 24*m)
 + 715*a^9*b^3*x^(3 + 36*m) + 1287*a^8*b^4*x^(4 + 48*m) + 1716*a^7*b^5*x^(5
 + 60*m) + 1716*a^6*b^6*x^(6 + 72*m) + 1287*a^5*b^7*x^(7 + 84*m) + 715*a^4*
b^8*x^(8 + 96*m) + 286*a^3*b^9*x^(9 + 108*m) + 78*a^2*b^10*x^(10 + 120*m) +
 13*a*b^11*x^(11 + 132*m) + b^12*x^(12 + 144*m)))/(13 + 156*m)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(25) = 50$.
time = 0.40, size = 339, normalized size = 12.56

method	result
risch	$\frac{b^{12}x^{26+156m}}{13(1+12m)x^{13}} + \frac{a\,b^{11}x^{24+144m}}{(1+12m)x^{12}} + \frac{6a^2b^{10}x^{22+132m}}{(1+12m)x^{11}} + \frac{22a^3b^9x^{20+120m}}{(1+12m)x^{10}} + \frac{55a^4b^8x^{18+108m}}{(1+12m)x^9} + \frac{99a^5b^7x^{16+96m}}{(1+12m)x^8} + \frac{132a^6b^6x}{(1+12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13/(1+12*m)*b^12/x^13*(x^(2+12*m))^13+1/(1+12*m)*a*b^11/x^12*(x^(2+12*m))
^12+6/(1+12*m)*a^2*b^10/x^11*(x^(2+12*m))^11+22/(1+12*m)*a^3*b^9/x^10*(x^(2
+12*m))^10+55/(1+12*m)*a^4*b^8/x^9*(x^(2+12*m))^9+99/(1+12*m)*a^5*b^7/x^8*(
x^(2+12*m))^8+132/(1+12*m)*a^6*b^6/x^7*(x^(2+12*m))^7+132/(1+12*m)*b^5*a^7/
x^6*(x^(2+12*m))^6+99/(1+12*m)*a^8*b^4/x^5*(x^(2+12*m))^5+55/(1+12*m)*b^3*a
^9/x^4*(x^(2+12*m))^4+22/(1+12*m)*a^10*b^2/x^3*(x^(2+12*m))^3+6/(1+12*m)*b*
a^11/x^2*(x^(2+12*m))^2+1/(1+12*m)*a^12/x*x^(2+12*m)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(25) = 50$.
time = 0.29, size = 275, normalized size = 10.19

$$\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)} + \frac{ab^{11}x^{144\,m+12}}{12\,m+1} + \frac{6\,a^2b^{10}x^{132\,m+11}}{12\,m+1} + \frac{22\,a^3b^9x^{120\,m+10}}{12\,m+1} + \frac{55\,a^4b^8x^{108\,m+9}}{12\,m+1} + \frac{99\,a^5b^7x^{96\,m+8}}{12\,m+1} + \frac{132\,a^6b^6x^{84\,m+7}}{12\,m+1} + \frac{132\,a^7b^5x^{72\,m+6}}{12\,m+1} + \frac{99\,a^8b^4x^{60\,m+5}}{12\,m+1} + \frac{55\,a^9b^3x^{48\,m+4}}{12\,m+1} + \frac{22\,a^{10}b^2x^{36\,m+3}}{12\,m+1} + \frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1} + \frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(25) = 50$.
time = 1.69, size = 231, normalized size = 8.56

$$\frac{13\,a^{12}x^{12}x^{12\,m+2}+78\,a^{11}bx^{11}x^{24\,m+4}+286\,a^{10}b^{2}x^{10}x^{36\,m+6}+715\,a^{9}b^{3}x^{9}x^{48\,m+8}+1287\,a^{8}b^{4}x^{8}x^{60\,m+10}+1716\,a^{7}b^{5}x^{7}x^{72\,m+12}+1716\,a^{6}b^{6}x^{6}x^{84\,m+14}+1287\,a^{5}b^{7}x^{5}x^{96\,m+16}+715\,a^{4}b^{8}x^{4}x^{108\,m+18}+286\,a^{3}b^{9}x^{3}x^{120\,m+20}+78\,a^{2}b^{10}x^{2}x^{132\,m+22}+13\,ab^{11}xx^{144\,m+24}+b^{12}x^{156\,m+26}}{13\,(12\,m+1)x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(13*a^12*x^12*x^(12*m + 2) + 78*a^11*b*x^11*x^(24*m + 4) + 286*a^10*b^
2*x^10*x^(36*m + 6) + 715*a^9*b^3*x^9*x^(48*m + 8) + 1287*a^8*b^4*x^8*x^(60
*m + 10) + 1716*a^7*b^5*x^7*x^(72*m + 12) + 1716*a^6*b^6*x^6*x^(84*m + 14)
+ 1287*a^5*b^7*x^5*x^(96*m + 16) + 715*a^4*b^8*x^4*x^(108*m + 18) + 286*a^3
*b^9*x^3*x^(120*m + 20) + 78*a^2*b^10*x^2*x^(132*m + 22) + 13*a*b^11*x*x^(1
44*m + 24) + b^12*x^(156*m + 26))/((12*m + 1)*x^13)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-12+12*m)*(a*x+b*x**(2+12*m))**12,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(25) = 50$.
time = 0.54, size = 285, normalized size = 10.56

$$\frac{13\,a^{12}x^{12}e^{\left(12\,m\log\left(x\right)+2\,\log\left(x\right)\right)}+78\,a^{11}bx^{11}e^{\left(24\,m\log\left(x\right)+4\,\log\left(x\right)\right)}+286\,a^{10}b^{2}x^{10}e^{\left(36\,m\log\left(x\right)+6\,\log\left(x\right)\right)}+715\,a^{9}b^{3}x^{9}e^{\left(48\,m\log\left(x\right)+8\,\log\left(x\right)\right)}+1287\,a^{8}b^{4}x^{8}e^{\left(60\,m\log\left(x\right)+10\,\log\left(x\right)\right)}+1716\,a^{7}b^{5}x^{7}e^{\left(72\,m\log\left(x\right)+12\,\log\left(x\right)\right)}+1716\,a^{6}b^{6}x^{6}e^{\left(84\,m\log\left(x\right)+14\,\log\left(x\right)\right)}+1287\,a^{5}b^{7}x^{5}e^{\left(96\,m\log\left(x\right)+16\,\log\left(x\right)\right)}+715\,a^{4}b^{8}x^{4}e^{\left(108\,m\log\left(x\right)+18\,\log\left(x\right)\right)}+286\,a^{3}b^{9}x^{3}e^{\left(120\,m\log\left(x\right)+20\,\log\left(x\right)\right)}+78\,a^{2}b^{10}x^{2}e^{\left(132\,m\log\left(x\right)+22\,\log\left(x\right)\right)}+13\,ab^{11}xe^{\left(144\,m\log\left(x\right)+24\,\log\left(x\right)\right)}+b^{12}e^{\left(156\,m\log\left(x\right)+26\,\log\left(x\right)\right)}}{13\,\left(12\,mx^{13}+x^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(13*a^12*x^12*e^(12*m*log(x) + 2*log(x)) + 78*a^11*b*x^11*e^(24*m*log(
x) + 4*log(x)) + 286*a^10*b^2*x^10*e^(36*m*log(x) + 6*log(x)) + 715*a^9*b^3
*x^9*e^(48*m*log(x) + 8*log(x)) + 1287*a^8*b^4*x^8*e^(60*m*log(x) + 10*log(
x)) + 1716*a^7*b^5*x^7*e^(72*m*log(x) + 12*log(x)) + 1716*a^6*b^6*x^6*e^(84
*m*log(x) + 14*log(x)) + 1287*a^5*b^7*x^5*e^(96*m*log(x) + 16*log(x)) + 715
*a^4*b^8*x^4*e^(108*m*log(x) + 18*log(x)) + 286*a^3*b^9*x^3*e^(120*m*log(x)
 + 20*log(x)) + 78*a^2*b^10*x^2*e^(132*m*log(x) + 22*log(x)) + 13*a*b^11*x*
e^(144*m*log(x) + 24*log(x)) + b^12*e^(156*m*log(x) + 26*log(x)))/(12*m*x^1
3 + x^13)
```

Mupad [B]
time = 5.87, size = 287, normalized size = 10.63

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{13\,a^{12}\,x\,x^{12\,m}}{156\,m+13}+\frac{78\,a^{11}\,b\,x^{24\,m}\,x^2}{156\,m+13}+\frac{13\,a\,b^{11}\,x^{144\,m}\,x^{12}}{156\,m+13}+\frac{286\,a^{10}\,b^2\,x^{36\,m}\,x^3}{156\,m+13}+\frac{715\,a^9\,b^3\,x^{48\,m}\,x^4}{156\,m+13}+\frac{1287\,a^8\,b^4\,x^{60\,m}\,x^5}{156\,m+13}+\frac{1716\,a^7\,b^5\,x^{72\,m}\,x^6}{156\,m+13}+\frac{1716\,a^6\,b^6\,x^{84\,m}\,x^7}{156\,m+13}+\frac{1287\,a^5\,b^7\,x^{96\,m}\,x^8}{156\,m+13}+\frac{715\,a^4\,b^8\,x^{108\,m}\,x^9}{156\,m+13}+\frac{286\,a^3\,b^9\,x^{120\,m}\,x^{10}}{156\,m+13}+\frac{78\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{156\,m+13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m - 12)*(a*x + b*x^(12*m + 2))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (13*a^12*x*x^(12*m))/(156*m + 13) + (7
8*a^11*b*x^(24*m)*x^2)/(156*m + 13) + (13*a*b^11*x^(144*m)*x^12)/(156*m + 1
3) + (286*a^10*b^2*x^(36*m)*x^3)/(156*m + 13) + (715*a^9*b^3*x^(48*m)*x^4)/
(156*m + 13) + (1287*a^8*b^4*x^(60*m)*x^5)/(156*m + 13) + (1716*a^7*b^5*x^(
72*m)*x^6)/(156*m + 13) + (1716*a^6*b^6*x^(84*m)*x^7)/(156*m + 13) + (1287*
a^5*b^7*x^(96*m)*x^8)/(156*m + 13) + (715*a^4*b^8*x^(108*m)*x^9)/(156*m + 1
3) + (286*a^3*b^9*x^(120*m)*x^10)/(156*m + 13) + (78*a^2*b^10*x^(132*m)*x^1
1)/(156*m + 13)
```

3.332 $\int (ax+bx^{14})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{13})^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 267}

$$\frac{(a+bx^{13})^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (ax+bx^{14})^{12}\,dx &= \int x^{12}(a+bx^{13})^{12}\,dx\\ &= \frac{(a+bx^{13})^{13}}{169b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.34, size = 135, normalized size = 8.44

method	result
default	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}b$
gosper	$\frac{x^{13}\left(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716b^5a^7x^{65}+1287a^8b^4x^{52}+715b\right.}{169}$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132b^5a^7x^{78}}{13}+\frac{99a^8b^4x^{65}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*b^5*a^7*
x^78+99/13*a^8*b^4*x^65+55/13*b^3*a^9*x^52+22/13*a^10*b^2*x^39+6/13*b*a^11*
x^26+1/13*a^12*x^13
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.28, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130}+\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78}+\frac{99}{13}\,a^8b^4x^{65}+\frac{55}{13}\,a^9b^3x^{52}+\frac{22}{13}\,a^{10}b^2x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 2.03, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130}+\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78}+\frac{99}{13}\,a^8b^4x^{65}+\frac{55}{13}\,a^9b^3x^{52}+\frac{22}{13}\,a^{10}b^2x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

time = 0.03, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

time = 0.48, size = 134, normalized size = 8.38

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}ab^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^{11}bx^{26}+\frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Mupad [B]

time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a^4\,b^8\,x^{117}}{13}+\frac{22\,a^3\,b^9\,x^{130}}{13}+\frac{6\,a^2\,b^{10}\,x^{143}}{13}+\frac{a\,b^{11}\,x^{156}}{13}+\frac{b^{12}\,x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^14)^12,x)
```

```
[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
+ (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (13
2*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^
4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

3.333 $\int (ax^2 + bx^{27})^{12} \, dx$

Optimal. Leaf size=16

$$\frac{(a + bx^{25})^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 267}

$$\frac{(a + bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int (ax^2 + bx^{27})^{12} \, dx &= \int x^{24} (a + bx^{25})^{12} \, dx \\ &= \frac{(a + bx^{25})^{13}}{325b} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25} + \frac{6}{25}a^{11}bx^{50} + \frac{22}{25}a^{10}b^2x^{75} + \frac{11}{5}a^9b^3x^{100} + \frac{99}{25}a^8b^4x^{125} + \frac{132}{25}a^7b^5x^{150} + \frac{132}{25}a^6b^6x^{175} + \frac{99}{25}a^5b^7x^{200} + \frac{11}{5}a^4b^8x^{225} + \frac{22}{25}a^3b^9x^{250} + \frac{6}{25}a^2b^{10}x^{275} + \frac{1}{25}ab^{11}x^{300} + \frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.54, size = 135, normalized size = 8.44

method	result
default	$\frac{132}{25}a^6b^6x^{175}+\frac{11}{5}b^3a^9x^{100}+\frac{22}{25}a^{10}b^2x^{75}+\frac{6}{25}b\,a^{11}x^{50}+\frac{1}{25}a^{12}x^{25}+\frac{132}{25}b^5a^7x^{150}+\frac{99}{25}a^8b^4x^{125}+\frac{1}{325}b$
gosper	$\frac{x^{25}\left(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716b^5a^7x^{125}+1287a^8b^4x^{100}\right.}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132b^5a^7x^{150}}{25}+\frac{99a^8b^4}{25}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^27+a*x^2)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 132/25*a^6*b^6*x^175+11/5*b^3*a^9*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^5
0+1/25*a^12*x^25+132/25*b^5*a^7*x^150+99/25*a^8*b^4*x^125+1/325*b^12*x^325+
1/25*a*b^11*x^300+11/5*a^4*b^8*x^225+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^25
0+99/25*a^5*b^7*x^200
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.28, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.45, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.
time = 0.03, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{22a^3b^9x^{250}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**27+a*x**2)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.50, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}a^8b^4x^{125}+\frac{11}{5}a^9b^3x^{100}+\frac{22}{25}a^{10}b^2x^{75}+\frac{6}{25}a^{11}bx^{50}+\frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B]
time = 5.19, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^27)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.334 $\int (ax^3+bx^{40})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{37})^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 267}

$$\frac{(a+bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^3+bx^{40}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{(a+bx^{37})^{13}}{481b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.08, size = 135, normalized size = 8.44

method	result
default	$\frac{1}{37}a^{12}x^{37}+\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{6}{37}b\,a^{11}x^{74}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}$
gosper	$\frac{x^{37}\left(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^{2}b^{10}x^{370}+286a^{3}b^{9}x^{333}+715a^{4}b^{8}x^{296}+1287a^{5}b^{7}x^{259}+1716a^{6}b^{6}x^{222}+1716b^{5}a^{7}x^{185}+1287a^{8}b^{4}x^{148}+\right.}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{132b^{5}a^{7}x^{222}}{37}+\frac{99a^{8}b^{4}x}{37}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^40+a*x^3)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/37*a^12*x^37+1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+6/37*
b*a^11*x^74+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132
/37*a^6*b^6*x^259+132/37*b^5*a^7*x^222+22/37*a^10*b^2*x^111+99/37*a^8*b^4*x
^185+55/37*b^3*a^9*x^148
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.29, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.59, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

time = 0.04, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**40+a*x**3)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

time = 0.46, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B]

time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37} + \frac{6\,a^{11}\,b\,x^{74}}{37} + \frac{22\,a^{10}\,b^2\,x^{111}}{37} + \frac{55\,a^9\,b^3\,x^{148}}{37} + \frac{99\,a^8\,b^4\,x^{185}}{37} + \frac{132\,a^7\,b^5\,x^{222}}{37} + \frac{132\,a^6\,b^6\,x^{259}}{37} + \frac{99\,a^5\,b^7\,x^{296}}{37} + \frac{55\,a^4\,b^8\,x^{333}}{37} + \frac{22\,a^3\,b^9\,x^{370}}{37} + \frac{6\,a^2\,b^{10}\,x^{407}}{37} + \frac{a\,b^{11}\,x^{444}}{37} + \frac{b^{12}\,x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^40)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.335 $\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {1607, 267}

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx &= \int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 193 vs. $2(27)=54$.
time = 0.04, size = 193, normalized size = 7.15

$$\frac{x^{1+12m}\left(13a^{12}+78a^{11}bx^{1+12m}+286a^{10}b^2x^{2+24m}+715a^9b^3x^{3+36m}+1287a^8b^4x^{4+48m}+1716a^7b^5x^{5+60m}+1716a^6b^6x^{6+72m}+1287a^5b^7x^{7+84m}+715a^4b^8x^{8+96m}+286a^3b^9x^{9+108m}+78a^2b^{10}x^{10+120m}+13ab^{11}x^{11+132m}+b^{12}x^{12+144m}\right)}{13+156m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (x^(1 + 12*m)*(13*a^12 + 78*a^11*b*x^(1 + 12*m) + 286*a^10*b^2*x^(2 + 24*m)
 + 715*a^9*b^3*x^(3 + 36*m) + 1287*a^8*b^4*x^(4 + 48*m) + 1716*a^7*b^5*x^(5
 + 60*m) + 1716*a^6*b^6*x^(6 + 72*m) + 1287*a^5*b^7*x^(7 + 84*m) + 715*a^4*
b^8*x^(8 + 96*m) + 286*a^3*b^9*x^(9 + 108*m) + 78*a^2*b^10*x^(10 + 120*m) +
 13*a*b^11*x^(11 + 132*m) + b^12*x^(12 + 144*m)))/(13 + 156*m)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(25) = 50$.
time = 0.36, size = 287, normalized size = 10.63

method	result
risch	$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{1+12m}+\frac{6a^{2}b^{10}x^{11}x^{132m}}{1+12m}+\frac{22a^{3}b^{9}x^{10}x^{120m}}{1+12m}+\frac{55a^{4}b^{8}x^{9}x^{108m}}{1+12m}+\frac{99a^{5}b^{7}x^{8}x^{96m}}{1+12m}+\frac{132a^{6}b^{6}x^{7}x}{1+12m}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+13*m))^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^12*x^13/(1+12*m)*(x^m)^156+a*b^11*x^12/(1+12*m)*(x^m)^144+6*a^2*b^10
*x^11/(1+12*m)*(x^m)^132+22*a^3*b^9*x^10/(1+12*m)*(x^m)^120+55*a^4*b^8*x^9/
(1+12*m)*(x^m)^108+99*a^5*b^7*x^8/(1+12*m)*(x^m)^96+132*a^6*b^6*x^7/(1+12*m
)*(x^m)^84+132*b^5*a^7*x^6/(1+12*m)*(x^m)^72+99*a^8*b^4*x^5/(1+12*m)*(x^m)^
60+55*b^3*a^9*x^4/(1+12*m)*(x^m)^48+22*a^10*b^2*x^3/(1+12*m)*(x^m)^36+6*b*a
^11*x^2/(1+12*m)*(x^m)^24+a^12/(1+12*m)*x*(x^m)^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(25) = 50$.
time = 0.29, size = 275, normalized size = 10.19

$$\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^{2}b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^{3}b^{9}x^{120\,m+10}}{12\,m+1}+\frac{55\,a^{4}b^{8}x^{108\,m+9}}{12\,m+1}+\frac{99\,a^{5}b^{7}x^{96\,m+8}}{12\,m+1}+\frac{132\,a^{6}b^{6}x^{84\,m+7}}{12\,m+1}+\frac{132\,a^{7}b^{5}x^{72\,m+6}}{12\,m+1}+\frac{99\,a^{8}b^{4}x^{60\,m+5}}{12\,m+1}+\frac{55\,a^{9}b^{3}x^{48\,m+4}}{12\,m+1}+\frac{22\,a^{10}b^{2}x^{36\,m+3}}{12\,m+1}+\frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1}+\frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(25) = 50$.

time = 1.92, size = 205, normalized size = 7.59

$$\frac{b^{12}x^{13}x^{156\,m}+13\,ab^{11}x^{12}x^{144\,m}+78\,a^{2}b^{10}x^{11}x^{132\,m}+286\,a^{3}b^{9}x^{10}x^{120\,m}+715\,a^{4}b^{8}x^{9}x^{108\,m}+1287\,a^{5}b^{7}x^{8}x^{96\,m}+1716\,a^{6}b^{6}x^{7}x^{84\,m}+1716\,a^{7}b^{5}x^{6}x^{72\,m}+1287\,a^{8}b^{4}x^{5}x^{60\,m}+715\,a^{9}b^{3}x^{4}x^{48\,m}+286\,a^{10}b^{2}x^{3}x^{36\,m}+78\,a^{11}bx^{2}x^{24\,m}+13\,a^{12}xx^{12\,m}}{13\,(12\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+13*m))**12,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(25) = 50.
time = 0.48, size = 205, normalized size = 7.59

$$\frac{b^{12}x^{13}x^{156\,m}+13\,ab^{11}x^{12}x^{144\,m}+78\,a^{2}b^{10}x^{11}x^{132\,m}+286\,a^{3}b^{9}x^{10}x^{120\,m}+715\,a^{4}b^{8}x^{9}x^{108\,m}+1287\,a^{5}b^{7}x^{8}x^{96\,m}+1716\,a^{6}b^{6}x^{7}x^{84\,m}+1716\,a^{7}b^{5}x^{6}x^{72\,m}+1287\,a^{8}b^{4}x^{5}x^{60\,m}+715\,a^{9}b^{3}x^{4}x^{48\,m}+286\,a^{10}b^{2}x^{3}x^{36\,m}+78\,a^{11}bx^{2}x^{24\,m}+13\,a^{12}xx^{12\,m}}{13\,(12\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

Mupad [B]
time = 5.95, size = 285, normalized size = 10.56

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1}+\frac{132\,a^7\,b^5\,x^{72\,m}\,x^6}{12\,m+1}+\frac{132\,a^6\,b^6\,x^{84\,m}\,x^7}{12\,m+1}+\frac{99\,a^5\,b^7\,x^{96\,m}\,x^8}{12\,m+1}+\frac{55\,a^4\,b^8\,x^{108\,m}\,x^9}{12\,m+1}+\frac{22\,a^3\,b^9\,x^{120\,m}\,x^{10}}{12\,m+1}+\frac{6\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(13*m + 1))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^11
*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^10
*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (99*
a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1) +
(132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*m + 1
) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^10)/(12
*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

3.336 $\int (ax^m+bx^{1+6m})^5 \, dx$

Optimal. Leaf size=27

$$\frac{(a+bx^{1+5m})^6}{6b(1+5m)}$$

```
[Out] 1/6*(a+b*x^(1+5*m))^6/b/(1+5*m)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1607, 267}

$$\frac{(a+bx^{5m+1})^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (ax^m+bx^{1+6m})^5 \, dx &= \int x^{5m}(a+bx^{1+5m})^5 \, dx\\ &= \frac{(a+bx^{1+5m})^6}{6b(1+5m)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(27) = 54.
time = 0.05, size = 88, normalized size = 3.26

$$\frac{x^{1+5m}(6a^5+15a^4bx^{1+5m}+20a^3b^2x^{2+10m}+15a^2b^3x^{3+15m}+6ab^4x^{4+20m}+b^5x^{5+25m})}{6+30m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (x^(1 + 5*m)*(6*a^5 + 15*a^4*b*x^(1 + 5*m) + 20*a^3*b^2*x^(2 + 10*m) + 15*a
^2*b^3*x^(3 + 15*m) + 6*a*b^4*x^(4 + 20*m) + b^5*x^(5 + 25*m)))/(6 + 30*m)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(25) = 50$.
time = 0.40, size = 126, normalized size = 4.67

method	result	size
risch	$\frac{b^5 x^6 x^{30m}}{6+30m}+\frac{a\,b^4 x^5 x^{25m}}{1+5m}+\frac{5a^2 b^3 x^4 x^{20m}}{2(1+5m)}+\frac{10a^3 b^2 x^3 x^{15m}}{3(1+5m)}+\frac{5a^4 b\,x^2 x^{10m}}{2(1+5m)}+\frac{a^5 x\,x^{5m}}{1+5m}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+6*m))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^5*x^6/(1+5*m)*(x^m)^30+a*b^4*x^5/(1+5*m)*(x^m)^25+5/2*a^2*b^3*x^4/(1+
5*m)*(x^m)^20+10/3*a^3*b^2*x^3/(1+5*m)*(x^m)^15+5/2*a^4*b*x^2/(1+5*m)*(x^m)
^10+a^5/(1+5*m)*x*(x^m)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(25) = 50$.
time = 0.27, size = 121, normalized size = 4.48

$$\frac{b^5 x^{30\,m+6}}{6\,(5\,m+1)}+\frac{ab^4 x^{25\,m+5}}{5\,m+1}+\frac{5\,a^2 b^3 x^{20\,m+4}}{2\,(5\,m+1)}+\frac{10\,a^3 b^2 x^{15\,m+3}}{3\,(5\,m+1)}+\frac{5\,a^4 b x^{10\,m+2}}{2\,(5\,m+1)}+\frac{a^5 x^{5\,m+1}}{5\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^(30*m + 6)/(5*m + 1) + a*b^4*x^(25*m + 5)/(5*m + 1) + 5/2*a^2*b^3
*x^(20*m + 4)/(5*m + 1) + 10/3*a^3*b^2*x^(15*m + 3)/(5*m + 1) + 5/2*a^4*b*x
^(10*m + 2)/(5*m + 1) + a^5*x^(5*m + 1)/(5*m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.
time = 1.68, size = 93, normalized size = 3.44

$$\frac{b^5 x^6 x^{30\,m}+6\,ab^4 x^5 x^{25\,m}+15\,a^2 b^3 x^4 x^{20\,m}+20\,a^3 b^2 x^3 x^{15\,m}+15\,a^4 b x^2 x^{10\,m}+6\,a^5 x x^{5\,m}}{6\,(5\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(19) = 38$.
time = 35.48, size = 178, normalized size = 6.59

$$\begin{cases} \frac{6a^5xx^{5m}}{30m+6} + \frac{15a^4bx^2x^{10m}}{30m+6} + \frac{20a^3b^2x^3x^{15m}}{30m+6} + \frac{15a^2b^3x^4x^{20m}}{30m+6} + \frac{6ab^4x^5x^{25m}}{30m+6} + \frac{b^5x^6x^{30m}}{30m+6} & \text{for } m \neq -\frac{1}{5} \\ a^5\log(x) + 5a^4b\log(x) + 10a^3b^2\log(x) + 10a^2b^3\log(x) + 5ab^4\log(x) + b^5\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+6*m))**5,x)
```

```
[Out] Piecewise((6*a**5*x*x**(5*m)/(30*m + 6) + 15*a**4*b*x**2*x**(10*m)/(30*m + 6) + 20*a**3*b**2*x**3*x**(15*m)/(30*m + 6) + 15*a**2*b**3*x**4*x**(20*m)/(30*m + 6) + 6*a*b**4*x**5*x**(25*m)/(30*m + 6) + b**5*x**6*x**(30*m)/(30*m + 6), Ne(m, -1/5)), (a**5*log(x) + 5*a**4*b*log(x) + 10*a**3*b**2*log(x) + 10*a**2*b**3*log(x) + 5*a*b**4*log(x) + b**5*log(x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.
time = 0.50, size = 93, normalized size = 3.44

$$\frac{b^5x^6x^{30\,m} + 6\,ab^4x^5x^{25\,m} + 15\,a^2b^3x^4x^{20\,m} + 20\,a^3b^2x^3x^{15\,m} + 15\,a^4bx^2x^{10\,m} + 6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

Mupad [B]
time = 5.44, size = 124, normalized size = 4.59

$$\frac{b^5\,x^{30\,m}\,x^6}{30\,m+6} + \frac{a^5\,x\,x^{5\,m}}{5\,m+1} + \frac{5\,a^4\,b\,x^{10\,m}\,x^2}{10\,m+2} + \frac{a\,b^4\,x^{25\,m}\,x^5}{5\,m+1} + \frac{5\,a^2\,b^3\,x^{20\,m}\,x^4}{10\,m+2} + \frac{10\,a^3\,b^2\,x^{15\,m}\,x^3}{15\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(6*m + 1))^5,x)
```

```
[Out] (b^5*x^(30*m)*x^6)/(30*m + 6) + (a^5*x*x^(5*m))/(5*m + 1) + (5*a^4*b*x^(10*m)*x^2)/(10*m + 2) + (a*b^4*x^(25*m)*x^5)/(5*m + 1) + (5*a^2*b^3*x^(20*m)*x^4)/(10*m + 2) + (10*a^3*b^2*x^(15*m)*x^3)/(15*m + 3)
```

3.337 $\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

```
[Out] -1/2/b/(1-3*m)/(a+b*x^(1-3*m))^2
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1607, 267}

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] -1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx &= \int \frac{x^{-3m}}{\left(a+bx^{1-3m}\right)^3}\,dx\\ &= -\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 27, normalized size = 1.00

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] -1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Maple [A]
time = 0.44, size = 39, normalized size = 1.44

method	result	size
risch	$-\frac{x\left(2 a\,x^{3 m}+b x\right)}{2\left(3 m-1\right) a^{2} \left(a\,x^{3 m}+b x\right)^{2}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1-2*m)+a*x^m)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(2*a*(x^m)^3+b*x)/(3*m-1)/a^2/(a*(x^m)^3+b*x)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.
time = 0.30, size = 66, normalized size = 2.44

$$-\frac{2\,axx^{3\,m}+bx^{2}}{2\,\left(2\,a^{3}b(3\,m-1)xx^{3\,m}+a^{2}b^{2}(3\,m-1)x^{2}+a^{4}(3\,m-1)x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m -
1)*x^2 + a^4*(3*m - 1)*x^(6*m))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(25) = 50$.
time = 1.88, size = 82, normalized size = 3.04

$$-\frac{2\,axx^{3\,m}+bx^{2}}{2\,\left(2\,(3\,a^{3}bm-a^{3}b)xx^{3\,m}+(3\,a^{2}b^{2}m-a^{2}b^{2})x^{2}+(3\,a^{4}m-a^{4})x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2*
m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^(-2*m + 1))^(-3), x)
```

Mupad [**B**]
time = 5.21, size = 38, normalized size = 1.41

$$-\frac{x\,\left(b\,x+2\,a\,x^{3\,m}\right)}{2\,a^2\,\left(3\,m-1\right)\,{\left(b\,x+a\,x^{3\,m}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^m + b*x^(1 - 2*m))^3,x)
```

```
[Out] -(x*(b*x + 2*a*x^(3*m)))/(2*a^2*(3*m - 1)*(b*x + a*x^(3*m))^2)
```

3.338 $\int \frac{1}{\frac{b}{x}+ax}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+ax^2)}{2a}$$

```
[Out] 1/2*ln(a*x^2+b)/a
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 266}

$$\frac{\log(ax^2+b)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x}+ax}\,dx &= \int \frac{x}{b+ax^2}\,dx\\ &= \frac{\log(b+ax^2)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b+ax^2)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\ln(a\,x^2+b)}{2a}$	14
norman	$\frac{\ln(a\,x^2+b)}{2a}$	14
risch	$\frac{\ln(a\,x^2+b)}{2a}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x*b+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(a*x^2+b)/a
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(a*x^2 + b)/a
```

Fricas [A]
time = 1.82, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(a*x^2 + b)/a
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x)
```

```
[Out] log(a*x**2 + b)/(2*a)
```

Giac [A]
time = 0.50, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|ax^{2}+b\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*x^2 + b))/a
```

Mupad [B]
time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^{2}+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x),x)
```

```
[Out] log(b + a*x^2)/(2*a)
```

3.339 $\int \frac{1}{\frac{b}{x^2}+ax}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+ax^3)}{3a}$$

```
[Out] 1/3*ln(a*x^3+b)/a
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 266}

$$\frac{\log(ax^3+b)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^3]/(3*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x^2}+ax}\,dx &= \int \frac{x^2}{b+ax^3}\,dx\\ &= \frac{\log(b+ax^3)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b+ax^3)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^3]/(3*a)
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\ln\left(a\,x^{3}+b\right)}{3a}$	14
norman	$\frac{\ln\left(a\,x^{3}+b\right)}{3a}$	14
risch	$\frac{\ln\left(a\,x^{3}+b\right)}{3a}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(a*x^3+b)/a
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^3+b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(a*x^3 + b)/a
```

Fricas [A]
time = 1.72, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^3+b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(a*x^3 + b)/a
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^3+b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2+a*x),x)
```

```
[Out] log(a*x**3 + b)/(3*a)
```

Giac [A]
time = 0.49, size = 14, normalized size = 0.93

$$\frac{\log\left(|ax^3+b|\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(a*x^3 + b))/a
```

Mupad [B]
time = 5.12, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^3+b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x^2),x)
```

```
[Out] log(b + a*x^3)/(3*a)
```

3.340 $\int \frac{1}{\frac{b}{x^3}+ax} \, dx$

Optimal. Leaf size=15

$$\frac{\log(b+ax^4)}{4a}$$

[Out] 1/4*ln(a*x^4+b)/a

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 266}

$$\frac{\log(ax^4+b)}{4a}$$

Antiderivative was successfully verified.

[In] Int[(b/x^3 + a*x)^(-1),x]

[Out] Log[b + a*x^4]/(4*a)

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x^3}+ax} \, dx &= \int \frac{x^3}{b+ax^4} \, dx\\ &= \frac{\log(b+ax^4)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b+ax^4)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\ln(a\,x^4+b)}{4a}$	14
norman	$\frac{\ln(a\,x^4+b)}{4a}$	14
risch	$\frac{\ln(a\,x^4+b)}{4a}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(a*x^4+b)/a
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^4+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(a*x^4 + b)/a
```

Fricas [A]
time = 1.53, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^4+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(a*x^4 + b)/a
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x),x)
```

```
[Out] log(a*x**4 + b)/(4*a)
```

Giac [A]
time = 0.83, size = 14, normalized size = 0.93

$$\frac{\log\left(|ax^4+b|\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x^4 + b))/a
```

Mupad [B]
time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^4+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x^3),x)
```

```
[Out] log(b + a*x^4)/(4*a)
```

3.341 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b\left(b+ax^2\right)^2}$$

```
[Out] 1/4*x^4/b/(a*x^2+b)^2
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 270}

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-3),x]
```

```
[Out] x^4/(4*b*(b + a*x^2)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx &= \int \frac{x^3}{\left(b+ax^2\right)^3}\,dx\\ &= \frac{x^4}{4b\left(b+ax^2\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.26

$$-\frac{b+2ax^2}{4a^2\left(b+ax^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-3),x]
```

```
[Out] -1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

Maple [A]
time = 0.04, size = 31, normalized size = 1.63

method	result	size
gosper	$-\frac{2 a\,x^{2}+b}{4 \left(a\,x^{2}+b \right)^{2} a^{2}}$	23
norman	$\frac{-\frac{x^{2}}{2 a}-\frac{b}{4 a^{2}}}{\left(a\,x^{2}+b \right)^{2}}$	26
risch	$\frac{-\frac{x^{2}}{2 a}-\frac{b}{4 a^{2}}}{\left(a\,x^{2}+b \right)^{2}}$	26
default	$-\frac{1}{2 a^{2} \left(a\,x^{2}+b \right)}+\frac{b}{4 a^{2} \left(a\,x^{2}+b \right)^{2}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x*b+a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/(a*x^2+b)+1/4*b/a^2/(a*x^2+b)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 0.28, size = 36, normalized size = 1.89

$$-\frac{2\,ax^2+b}{4\,(a^4x^4+2\,a^3bx^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 1.95, size = 36, normalized size = 1.89

$$-\frac{2\,ax^2+b}{4\,(a^4x^4+2\,a^3bx^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.13, size = 36, normalized size = 1.89

$$\frac{-2ax^2 - b}{4a^4x^4 + 8a^3bx^2 + 4a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)**3,x)
```

```
[Out] (-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

Giac [A]
time = 0.61, size = 22, normalized size = 1.16

$$-\frac{2\,ax^2 + b}{4\,(ax^2 + b)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

Mupad [B]
time = 0.04, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{4\,a^2} + \frac{x^2}{2\,a}}{a^2\,x^4 + 2\,a\,b\,x^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x)^3,x)
```

```
[Out] -(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

3.342 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} \, dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10b\left(b+ax^5\right)^2}$$

```
[Out] 1/10*x^10/b/(a*x^5+b)^2
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 270}

$$\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] x^10/(10*b*(b + a*x^5)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3} \, dx &= \int \frac{x^9}{\left(b+ax^5\right)^3} \, dx\\ &= \frac{x^{10}}{10b\left(b+ax^5\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.26

$$-\frac{b+2ax^5}{10a^2\left(b+ax^5\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] -1/10*(b + 2*a*x^5)/(a^2*(b + a*x^5)^2)
```

Maple [A]
time = 0.05, size = 31, normalized size = 1.63

method	result	size
gosper	$-\frac{2 a \,x^{5}+b}{10\left(a \,x^{5}+b \right)^{2} a^{2}}$	23
norman	$\frac{-\frac{x^{5}}{5 a}-\frac{b}{10 a^{2}}}{\left(a \,x^{5}+b \right)^{2}}$	26
risch	$\frac{-\frac{x^{5}}{5 a}-\frac{b}{10 a^{2}}}{\left(a \,x^{5}+b \right)^{2}}$	26
default	$-\frac{1}{5 a^{2} \left(a \,x^{5}+b \right)}+\frac{b}{10 a^{2} \left(a \,x^{5}+b \right)^{2}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a^2/(a*x^5+b)+1/10*b/a^2/(a*x^5+b)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 0.29, size = 36, normalized size = 1.89

$$-\frac{2 \, a x^{5} + b}{10 \, {\left(a^{4} x^{10} + 2 \, a^{3} b x^{5} + a^{2} b^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 2.50, size = 36, normalized size = 1.89

$$-\frac{2 \, a x^{5} + b}{10 \, {\left(a^{4} x^{10} + 2 \, a^{3} b x^{5} + a^{2} b^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.22, size = 36, normalized size = 1.89

$$\frac{-2ax^5 - b}{10a^4x^{10} + 20a^3bx^5 + 10a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x**2)**3,x)
```

```
[Out] (-2*a*x**5 - b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)
```

Giac [A]
time = 0.59, size = 22, normalized size = 1.16

$$-\frac{2\,ax^5 + b}{10\,(ax^5 + b)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)
```

Mupad [B]
time = 0.06, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{10\,a^2} + \frac{x^5}{5\,a}}{a^2\,x^{10} + 2\,a\,b\,x^5 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b/x^3)^3,x)
```

```
[Out] -(b/(10*a^2) + x^5/(5*a))/(b^2 + a^2*x^10 + 2*a*b*x^5)
```

3.343 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3} dx$

Optimal. Leaf size=19

$$\frac{x^{16}}{16b\left(b+ax^8\right)^2}$$

```
[Out] 1/16*x^16/b/(a*x^8+b)^2
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 270}

$$\frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^5 + a*x^3)^(-3),x]
```

```
[Out] x^16/(16*b*(b + a*x^8)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3} dx &= \int \frac{x^{15}}{\left(b+ax^8\right)^3} dx\\ &= \frac{x^{16}}{16b\left(b+ax^8\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.26

$$-\frac{b+2ax^8}{16a^2\left(b+ax^8\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^5 + a*x^3)^(-3),x]
[Out] -1/16*(b + 2*a*x^8)/(a^2*(b + a*x^8)^2)
```

Maple [A]
time = 0.06, size = 31, normalized size = 1.63

method	result	size
gosper	$-\frac{2 a \,x^{8}+b}{16 \left(a \,x^{8}+b \right)^{2} a^{2}}$	23
norman	$\frac{-\frac{b}{16 a^{2}}-\frac{x^{8}}{8 a}}{\left(a \,x^{8}+b \right)^{2}}$	26
risch	$\frac{-\frac{b}{16 a^{2}}-\frac{x^{8}}{8 a}}{\left(a \,x^{8}+b \right)^{2}}$	26
default	$-\frac{1}{8 a^{2} \left(a \,x^{8}+b \right)}+\frac{b}{16 a^{2} \left(a \,x^{8}+b \right)^{2}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^5+a*x^3)^3,x,method=_RETURNVERBOSE)
[Out] -1/8/a^2/(a*x^8+b)+1/16*b/a^2/(a*x^8+b)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 0.29, size = 36, normalized size = 1.89

$$-\frac{2\,ax^8+b}{16\,(a^4x^{16}+2\,a^3bx^8+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 1.39, size = 36, normalized size = 1.89

$$-\frac{2\,ax^8+b}{16\,(a^4x^{16}+2\,a^3bx^8+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.32, size = 36, normalized size = 1.89

$$\frac{-2ax^8 - b}{16a^4x^{16} + 32a^3bx^8 + 16a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**5+a*x**3)**3,x)
[Out] (-2*a*x**8 - b)/(16*a**4*x**16 + 32*a**3*b*x**8 + 16*a**2*b**2)
```

Giac [A]
time = 0.66, size = 22, normalized size = 1.16

$$-\frac{2\,ax^8 + b}{16\,(ax^8 + b)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")
[Out] -1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)
```

Mupad [B]
time = 5.18, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{16\,a^2} + \frac{x^8}{8\,a}}{a^2\,x^{16} + 2\,a\,b\,x^8 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b/x^5)^3,x)
[Out] -(b/(16*a^2) + x^8/(8*a))/(b^2 + a^2*x^16 + 2*a*b*x^8)
```

3.344 $\int \left(\frac{a}{x}+bx\right)^2 \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 276}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a/x + b*x)^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^2 \, dx &= \int \frac{\left(a+bx^2\right)^2}{x^2} \, dx\\ &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right) \, dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Maple [A]
time = 0.03, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Fricas [A]
time = 1.45, size = 25, normalized size = 1.04

$$\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.79

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Giac [A]
time = 0.66, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

3.345 $\int \left(\frac{a}{x}+bx\right)^3 \, dx$

Optimal. Leaf size=40

$$-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1607, 272, 45}

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^3,x]
```

```
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \left(\frac{a}{x}+bx\right)^3 dx = \int \frac{(a+bx^2)^3}{x^3} dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^2} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right) dx, x, x^2\right)$$
$$= -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^3,x]
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Maple [A]
time = 0.08, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{a^3}{2x^2}+\frac{3a\,b^2x^2}{2}+\frac{b^3x^4}{4}+3a^2b\ln(x)$	35
norman	$\frac{-\frac{1}{2}a^3+\frac{1}{4}b^3x^6+\frac{3}{2}a\,b^2x^4}{x^2}+3a^2b\ln(x)$	37
risch	$\frac{b^3x^4}{4}+\frac{3a\,b^2x^2}{2}+\frac{9a^2b}{4}-\frac{a^3}{2x^2}+3a^2b\ln(x)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.85

$$\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+3\,a^2b\log(x)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

Fricas [A]
time = 1.56, size = 38, normalized size = 0.95

$$\frac{b^{3}x^{6} + 6\,ab^{2}x^{4} + 12\,a^{2}bx^{2}\log\left(x\right) - 2\,a^{3}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.92

$$-\frac{a^{3}}{2x^{2}} + 3a^{2}b\log\left(x\right) + \frac{3ab^{2}x^{2}}{2} + \frac{b^{3}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

Giac [A]
time = 0.69, size = 46, normalized size = 1.15

$$\frac{1}{4}\,b^{3}x^{4} + \frac{3}{2}\,ab^{2}x^{2} + \frac{3}{2}\,a^{2}b\log\left(x^{2}\right) - \frac{3\,a^{2}bx^{2} + a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

Mupad [B]
time = 0.04, size = 34, normalized size = 0.85

$$\frac{b^{3}\,x^{4}}{4} - \frac{a^{3}}{2\,x^{2}} + \frac{3\,a\,b^{2}\,x^{2}}{2} + 3\,a^{2}\,b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^3,x)
```

```
[Out] (b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

3.346 $\int \left(\frac{a}{x}+bx\right)^4\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {1607, 276}

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^4,x]
```

```
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^4\,dx &= \int \frac{\left(a+bx^2\right)^4}{x^4}\,dx\\ &= \int \left(6a^2b^2+\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+4ab^3x^2+b^4x^4\right)\,dx\\ &= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^4,x]
```

```
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.90

method	result	size
default	$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4a\,b^3x^3}{3}+\frac{b^4x^5}{5}$	45
risch	$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x+\frac{-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	47
norman	$\frac{\frac{1}{5}b^4x^8+\frac{4}{3}a\,b^3x^6+6a^2b^2x^4-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	48
gosper	$-\frac{-3b^4x^8-20a\,b^3x^6-90a^2b^2x^4+60a^3b\,x^2+5a^4}{15x^3}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Maxima [A]
time = 0.29, size = 44, normalized size = 0.88

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{4\,a^3b}{x}-\frac{a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3
```

Fricas [A]
time = 1.20, size = 48, normalized size = 0.96

$$\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

Sympy [A]
time = 0.06, size = 49, normalized size = 0.98

$$6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5} + \frac{-a^4 - 12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(3
*x**3)
```

Giac [A]
time = 0.56, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5 + \frac{4}{3}\,ab^3x^3 + 6\,a^2b^2x - \frac{12\,a^3bx^2 + a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Mupad [B]
time = 0.05, size = 47, normalized size = 0.94

$$\frac{b^4\,x^5}{5} - \frac{\frac{a^4}{3} + 4\,b\,a^3\,x^2}{x^3} + 6\,a^2\,b^2\,x + \frac{4\,a\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^4,x)
```

```
[Out] (b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

3.347 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$

Optimal. Leaf size=185

$$-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+2\sqrt{\frac{2}{5+\sqrt{5}}}\,x\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{\right.}\right.\right.$$

```
[Out] 1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(1+x^2-1/2
*x*(-5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*x*2^(1
/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/10*arctan(1/5*x*(50+10*5^(1/2
))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {1607, 299, 648, 632, 210, 642, 31}

$$-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\text{ArcTan}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\,x+\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\,x\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-2) + x^3)^(-1),x]
```

```
[Out] -1/5*(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 +
Sqrt[5])]*x]) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqr
t[(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 -
Sqrt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2
])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
```

```
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx &= \int \frac{x^2}{1+x^5}\,dx \\
&= \frac{2}{5}\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx+\frac{2}{5}\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx+\frac{1}{5}\int \\
&= \frac{1}{5}\log(1+x)+\frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}+\frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{}{1+} \\
&= \frac{1}{5}\log(1+x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}x+2x^2\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}\right. \\
&= \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right.}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 144, normalized size = 0.78

$$
\frac{1}{20}\left(-2\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1+\sqrt{5}-4x}{\sqrt{10-2\sqrt{5}}}\right)-2\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{-1+\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+4\log(1+x)-\left(1+\sqrt{5}\right)\log\left(1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2\right)+\left(-1+\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-2) + x^3)^(-1),x]
```

```
[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
- 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

Maple [A]
time = 0.07, size = 165, normalized size = 0.89

method	result
risch	$\frac{\ln(x+1)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^3+\_Z^2+\_Z+1\right)}\_R\ln\left(\_R^2+x\right)\right)}{5}$

default	$\frac{\ln(x+1)}{5}+\frac{\left(-\sqrt{5}-1\right)\ln\left(x\sqrt{5}+2x^2-x+2\right)}{20}+\frac{2\left(-\sqrt{5}-1-\frac{\left(-\sqrt{5}-1\right)\left(\sqrt{5}-1\right)}{4}\right)\arctan\left(\frac{\sqrt{5}+4x-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*ln(x+1)+1/20*(-5^(1/2)-1)*ln(x*5^(1/2)+2*x^2-x+2)+2/5*(-5^(1/2)-1-1/4*(
-5^(1/2)-1)*(5^(1/2)-1))/(10+2*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*
5^(1/2))^(1/2))-1/20*(-5^(1/2)+1)*ln(-x*5^(1/2)+2*x^2-x+2)-2/5*(-5^(1/2)+1-
1/4*(-5^(1/2)+1)*(-5^(1/2)-1))/(10-2*5^(1/2))^(1/2)*arctan((-5^(1/2)+4*x-1)
/(10-2*5^(1/2))^(1/2))
```

Maxima [A]
time = 0.51, size = 124, normalized size = 0.67

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+\frac{\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}-\frac{\log\left(2x^2+x\left(\sqrt{5}-1\right)+2\right)}{5\left(\sqrt{5}-1\right)}+\frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5
) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqr
t(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1
/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(122) = 244$.
time = 4.33, size = 637, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/16*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + x) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5)
- 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2
*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5)
 + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(
5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/2*sqrt
```

```
(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(
sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x - 1) + 1/20*(sqrt(5)
- 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqr
t(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sq
rt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/
2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(
5) - 1)^2 - 1/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)
^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x -
 1) + 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(1/16*(2*sqrt(1
/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + x) + 1/5*log(x + 1)
```

Sympy [**A**]
time = 0.83, size = 36, normalized size = 0.19

$$\frac{\log(x+1)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1, \left(t \mapsto t\log\left(25t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**2+x**3),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(25*_t**2 + x)))
```

Giac [**A**]
time = 0.56, size = 112, normalized size = 0.61

$$\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{5}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x
 + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4
*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

Mupad [**B**]
time = 5.91, size = 197, normalized size = 1.06

$$\frac{\ln(x+1)}{5}-\ln\left(1-\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)+\ln\left(\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}+\sqrt{5}-1\right)^3}{64}+1\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}+\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-\ln\left(1-\frac{x\left(\sqrt{5}+\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2}\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right)-\ln\left(1-\frac{x\left(\sqrt{5}-\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}-\frac{\sqrt{2}\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2 + x^3),x)
```

```
[Out] log(x + 1)/5 - log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)/
64)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log((x*(2^(1
/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2) -
5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(1 - (x*(5^(1/2) + 2^(1/2)*(5^(1/2)
- 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/2
0) - log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)
/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

3.348 $\int x^p(ax^n + bx^{1+13n+p})^{12}\,dx$

Optimal. Leaf size=29

$$\frac{(a+bx^{1+12n+p})^{13}}{13b(1+12n+p)}$$

```
[Out] 1/13*(a+b*x^(1+12*n+p))^13/b/(1+12*n+p)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1598, 267}

$$\frac{(a+bx^{12n+p+1})^{13}}{13b(12n+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx &= \int x^{12n+p}\left(a+bx^{1+12n+p}\right)^{12}\,dx\\ &= \frac{(a+bx^{1+12n+p})^{13}}{13b(1+12n+p)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(29) = 58.
time = 0.11, size = 232, normalized size = 8.00

$$\frac{x^{1+12n+p}(13a^{12}+78a^{11}bx^{1+12n+p}+286a^{10}b^2x^{2+24n+2p}+715a^9b^3x^{3+36n+3p}+1287a^8b^4x^{4+48n+4p}+1716a^7b^5x^{5+60n+5p}+1716a^6b^6x^{6+72n+6p}+1287a^5b^7x^{7+84n+7p}+715a^4b^8x^{8+96n+8p}+286a^3b^9x^{9+108n+9p}+78a^2b^{10}x^{10+120n+10p}+13ab^{11}x^{11+132n+11p}+b^{12}x^{12+144n+12p})}{13(1+12n+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (x^(1 + 12*n + p)*(13*a^12 + 78*a^11*b*x^(1 + 12*n + p) + 286*a^10*b^2*x^(2
 + 24*n + 2*p) + 715*a^9*b^3*x^(3 + 36*n + 3*p) + 1287*a^8*b^4*x^(4 + 48*n
+ 4*p) + 1716*a^7*b^5*x^(5 + 60*n + 5*p) + 1716*a^6*b^6*x^(6 + 72*n + 6*p)
+ 1287*a^5*b^7*x^(7 + 84*n + 7*p) + 715*a^4*b^8*x^(8 + 96*n + 8*p) + 286*a^
3*b^9*x^(9 + 108*n + 9*p) + 78*a^2*b^10*x^(10 + 120*n + 10*p) + 13*a*b^11*x
^(11 + 132*n + 11*p) + b^12*x^(12 + 144*n + 12*p)))/(13*(1 + 12*n + p))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(27) = 54$.
time = 0.64, size = 363, normalized size = 12.52

method	result
risch	$\frac{b^{12} x^{13} x^{156 n} x^{13 p}}{13+156 n+13 p}+\frac{a\,b^{11} x^{12} x^{144 n} x^{12 p}}{1+12 n+p}+\frac{6 a^{2} b^{10} x^{11} x^{132 n} x^{11 p}}{1+12 n+p}+\frac{22 a^{3} b^{9} x^{10} x^{120 n} x^{10 p}}{1+12 n+p}+\frac{55 a^{4} b^{8} x^{9} x^{108 n} x^{9 p}}{1+12 n+p}+\frac{99 a^{5} b^{7} x^{8} x^{96 n}}{1+12 n+p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n+
p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10*(
x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9+99
*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12*n+p
)*(x^p)^7+132*b^5*a^7*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(x^n)^
60/(1+12*n+p)*(x^p)^5+55*b^3*a^9*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a^10*b^
2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*b*a^11*x^2*(x^n)^24/(1+12*n+p)*(x^p)^2+
a^12/(1+12*n+p)*x*(x^n)^12*x^p
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(27) = 54$.
time = 0.29, size = 325, normalized size = 11.21

$$\frac{b^{12}x^{156\,n+13\,p+13}}{13\,(12\,n+p+1)}+\frac{ab^{11}x^{144\,n+12\,p+12}}{12\,n+p+1}+\frac{6\,a^{2}b^{10}x^{132\,n+11\,p+11}}{12\,n+p+1}+\frac{22\,a^{3}b^{9}x^{120\,n+10\,p+10}}{12\,n+p+1}+\frac{55\,a^{4}b^{8}x^{108\,n+9\,p+9}}{12\,n+p+1}+\frac{99\,a^{5}b^{7}x^{96\,n+8\,p+8}}{12\,n+p+1}+\frac{132\,a^{6}b^{6}x^{84\,n+7\,p+7}}{12\,n+p+1}+\frac{132\,a^{7}b^{5}x^{72\,n+6\,p+6}}{12\,n+p+1}+\frac{99\,a^{8}b^{4}x^{60\,n+5\,p+5}}{12\,n+p+1}+\frac{55\,a^{9}b^{3}x^{48\,n+4\,p+4}}{12\,n+p+1}+\frac{22\,a^{10}b^{2}x^{36\,n+3\,p+3}}{12\,n+p+1}+\frac{6\,a^{11}bx^{24\,n+2\,p+2}}{12\,n+p+1}+\frac{a^{12}x^{12\,n+p+1}}{12\,n+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*n + 13*p + 13)/(12*n + p + 1) + a*b^11*x^(144*n + 12*p + 1
2)/(12*n + p + 1) + 6*a^2*b^10*x^(132*n + 11*p + 11)/(12*n + p + 1) + 22*a^
3*b^9*x^(120*n + 10*p + 10)/(12*n + p + 1) + 55*a^4*b^8*x^(108*n + 9*p + 9)
/(12*n + p + 1) + 99*a^5*b^7*x^(96*n + 8*p + 8)/(12*n + p + 1) + 132*a^6*b^
6*x^(84*n + 7*p + 7)/(12*n + p + 1) + 132*a^7*b^5*x^(72*n + 6*p + 6)/(12*n
+ p + 1) + 99*a^8*b^4*x^(60*n + 5*p + 5)/(12*n + p + 1) + 55*a^9*b^3*x^(48*
```

```
n + 4*p + 4)/(12*n + p + 1) + 22*a^10*b^2*x^(36*n + 3*p + 3)/(12*n + p + 1)
 + 6*a^11*b*x^(24*n + 2*p + 2)/(12*n + p + 1) + a^12*x^(12*n + p + 1)/(12*n
 + p + 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(27) = 54$.
time = 2.24, size = 297, normalized size = 10.24

$$\frac{78 a^{2} b^{10} x^{2 n} x^{143 n+11 p+11}+286 a^{3} b^{9} x^{3 n} x^{130 n+10 p+10}+715 a^{4} b^{8} x^{4 n} x^{117 n+9 p+9}+1287 a^{5} b^{7} x^{5 n} x^{104 n+8 p+8}+1716 a^{6} b^{6} x^{6 n} x^{91 n+7 p+7}+1716 a^{7} b^{5} x^{7 n} x^{78 n+6 p+6}+1287 a^{8} b^{4} x^{8 n} x^{65 n+5 p+5}+715 a^{9} b^{3} x^{9 n} x^{52 n+4 p+4}+286 a^{10} b^{2} x^{10 n} x^{39 n+3 p+3}+78 a^{11} b x^{11 n} x^{26 n+2 p+2}+13 a^{12} x^{12 n} x^{13 n+p+1}+13 a b^{11} x^{156 n+12 p+12} x^{n}+b^{12} x^{169 n+13 p+13}}{13 (12 n+p+1) x^{13 n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(78*a^2*b^10*x^(2*n)*x^(143*n + 11*p + 11) + 286*a^3*b^9*x^(3*n)*x^(13
0*n + 10*p + 10) + 715*a^4*b^8*x^(4*n)*x^(117*n + 9*p + 9) + 1287*a^5*b^7*x
^(5*n)*x^(104*n + 8*p + 8) + 1716*a^6*b^6*x^(6*n)*x^(91*n + 7*p + 7) + 1716
*a^7*b^5*x^(7*n)*x^(78*n + 6*p + 6) + 1287*a^8*b^4*x^(8*n)*x^(65*n + 5*p +
5) + 715*a^9*b^3*x^(9*n)*x^(52*n + 4*p + 4) + 286*a^10*b^2*x^(10*n)*x^(39*n
 + 3*p + 3) + 78*a^11*b*x^(11*n)*x^(26*n + 2*p + 2) + 13*a^12*x^(12*n)*x^(1
3*n + p + 1) + 13*a*b^11*x^(156*n + 12*p + 12)*x^n + b^12*x^(169*n + 13*p +
 13))/((12*n + p + 1)*x^(13*n))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(27) = 54$.
time = 1.06, size = 269, normalized size = 9.28

$$\frac{b^{12} x^{13} x^{156 n} x^{13 p}+13 a b^{11} x^{12} x^{144 n} x^{12 p}+78 a^{2} b^{10} x^{11} x^{132 n} x^{11 p}+286 a^{3} b^{9} x^{10} x^{120 n} x^{10 p}+715 a^{4} b^{8} x^{9} x^{108 n} x^{9 p}+1287 a^{5} b^{7} x^{8} x^{96 n} x^{8 p}+1716 a^{6} b^{6} x^{7} x^{84 n} x^{7 p}+1716 a^{7} b^{5} x^{6} x^{72 n} x^{6 p}+1287 a^{8} b^{4} x^{5} x^{60 n} x^{5 p}+715 a^{9} b^{3} x^{4} x^{48 n} x^{4 p}+286 a^{10} b^{2} x^{3} x^{36 n} x^{3 p}+78 a^{11} b x^{2} x^{24 n} x^{2 p}+13 a^{12} x x^{12 n} x^{p}}{13 (12 n+p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*n)*x^(13*p) + 13*a*b^11*x^12*x^(144*n)*x^(12*p) + 78
*a^2*b^10*x^11*x^(132*n)*x^(11*p) + 286*a^3*b^9*x^10*x^(120*n)*x^(10*p) + 7
15*a^4*b^8*x^9*x^(108*n)*x^(9*p) + 1287*a^5*b^7*x^8*x^(96*n)*x^(8*p) + 1716
*a^6*b^6*x^7*x^(84*n)*x^(7*p) + 1716*a^7*b^5*x^6*x^(72*n)*x^(6*p) + 1287*a^
8*b^4*x^5*x^(60*n)*x^(5*p) + 715*a^9*b^3*x^4*x^(48*n)*x^(4*p) + 286*a^10*b^
```

```
2*x^3*x^(36*n)*x^(3*p) + 78*a^11*b*x^2*x^(24*n)*x^(2*p) + 13*a^12*x*x^(12*n
)*x^p)/(12*n + p + 1)
```

Mupad [B]
time = 6.78, size = 363, normalized size = 12.52

$$\frac{a^{12}\,x\,x^p\,x^{12\,n}}{12\,n+p+1}+\frac{b^{12}\,x^{156\,n}\,x^{13\,p}\,x^{13}}{156\,n+13\,p+13}+\frac{22\,a^{10}\,b^2\,x^{36\,n}\,x^{3\,p}\,x^3}{12\,n+p+1}+\frac{55\,a^9\,b^3\,x^{48\,n}\,x^{4\,p}\,x^4}{12\,n+p+1}+\frac{99\,a^8\,b^4\,x^{60\,n}\,x^{5\,p}\,x^5}{12\,n+p+1}+\frac{132\,a^7\,b^5\,x^{72\,n}\,x^{6\,p}\,x^6}{12\,n+p+1}+\frac{132\,a^6\,b^6\,x^{84\,n}\,x^{7\,p}\,x^7}{12\,n+p+1}+\frac{99\,a^5\,b^7\,x^{96\,n}\,x^{8\,p}\,x^8}{12\,n+p+1}+\frac{55\,a^4\,b^8\,x^{108\,n}\,x^{9\,p}\,x^9}{12\,n+p+1}+\frac{22\,a^3\,b^9\,x^{120\,n}\,x^{10\,p}\,x^{10}}{12\,n+p+1}+\frac{6\,a^2\,b^{10}\,x^{132\,n}\,x^{11\,p}\,x^{11}}{12\,n+p+1}+\frac{6\,a^{11}\,b\,x^{24\,n}\,x^{2\,p}\,x^2}{12\,n+p+1}+\frac{a\,b^{11}\,x^{144\,n}\,x^{12\,p}\,x^{12}}{12\,n+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(a*x^n + b*x^(13*n + p + 1))^12,x)
```

```
[Out] (a^12*x*x^p*x^(12*n))/(12*n + p + 1) + (b^12*x^(156*n)*x^(13*p)*x^13)/(156*
n + 13*p + 13) + (22*a^10*b^2*x^(36*n)*x^(3*p)*x^3)/(12*n + p + 1) + (55*a^
9*b^3*x^(48*n)*x^(4*p)*x^4)/(12*n + p + 1) + (99*a^8*b^4*x^(60*n)*x^(5*p)*x
^5)/(12*n + p + 1) + (132*a^7*b^5*x^(72*n)*x^(6*p)*x^6)/(12*n + p + 1) + (1
32*a^6*b^6*x^(84*n)*x^(7*p)*x^7)/(12*n + p + 1) + (99*a^5*b^7*x^(96*n)*x^(8
*p)*x^8)/(12*n + p + 1) + (55*a^4*b^8*x^(108*n)*x^(9*p)*x^9)/(12*n + p + 1)
 + (22*a^3*b^9*x^(120*n)*x^(10*p)*x^10)/(12*n + p + 1) + (6*a^2*b^10*x^(132
*n)*x^(11*p)*x^11)/(12*n + p + 1) + (6*a^11*b*x^(24*n)*x^(2*p)*x^2)/(12*n +
 p + 1) + (a*b^11*x^(144*n)*x^(12*p)*x^12)/(12*n + p + 1)
```

3.349 $\int x^{12}(a+bx^{13})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{13})^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{(a+bx^{13})^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{12}(a+bx^{13})^{12}\,dx = \frac{(a+bx^{13})^{13}}{169b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
```

```
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Maple [A]
time = 0.40, size = 15, normalized size = 0.94

method	result
default	$\frac{(b\,x^{13}+a)^{13}}{169b}$
gosper	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}b$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132b^5a^7x^{78}}{13}+\frac{99a^8b^4x^{65}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^13+a)^12,x,method=_RETURNVERBOSE)
[Out] 1/169*(b*x^13+a)^13/b
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$\frac{(bx^{13}+a)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
[Out] 1/169*(b*x^13 + a)^13/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.20, size = 134, normalized size = 8.38

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}ab^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a^{10}b^2x^{39}+\frac{6}{13}a^{11}bx^{26}+\frac{1}{13}a^{12}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.
time = 0.03, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**13+a)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

Giac [A]
time = 0.58, size = 14, normalized size = 0.88

$$\frac{(bx^{13}+a)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

Mupad [B]
time = 5.33, size = 14, normalized size = 0.88

$$\frac{(b\,x^{13}+a)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b*x^13)^12,x)
```

```
[Out] (a + b*x^13)^13/(169*b)
```

3.350 $\int x^{12}(ax+bx^{26})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{25})^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 267}

$$\frac{(a+bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx &= \int x^{24}\left(a+bx^{25}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{25}\right)^{13}}{325b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.54, size = 135, normalized size = 8.44

method	result
default	$\frac{132}{25}a^6b^6x^{175}+\frac{11}{5}b^3a^9x^{100}+\frac{22}{25}a^{10}b^2x^{75}+\frac{6}{25}b\,a^{11}x^{50}+\frac{1}{25}a^{12}x^{25}+\frac{132}{25}b^5a^7x^{150}+\frac{99}{25}a^8b^4x^{125}+\frac{1}{325}b$
gosper	$\frac{x^{25}\left(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716b^5a^7x^{125}+1287a^8b^4x^{100}\right.}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132b^5a^7x^{150}}{25}+\frac{99a^8b^4}{25}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 132/25*a^6*b^6*x^175+11/5*b^3*a^9*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^5
0+1/25*a^12*x^25+132/25*b^5*a^7*x^150+99/25*a^8*b^4*x^125+1/325*b^12*x^325+
1/25*a*b^11*x^300+11/5*a^4*b^8*x^225+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^25
0+99/25*a^5*b^7*x^200
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.28, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.47, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

time = 0.04, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{22a^3b^9x^{250}}{25} + \frac{6a^2b^{10}x^{275}}{25} + \frac{ab^{11}x^{300}}{25} + \frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

time = 0.59, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325} + \frac{1}{25}ab^{11}x^{300} + \frac{6}{25}a^2b^{10}x^{275} + \frac{22}{25}a^3b^9x^{250} + \frac{11}{5}a^4b^8x^{225} + \frac{99}{25}a^5b^7x^{200} + \frac{132}{25}a^6b^6x^{175} + \frac{132}{25}a^7b^5x^{150} + \frac{99}{25}a^8b^4x^{125} + \frac{11}{5}a^9b^3x^{100} + \frac{22}{25}a^{10}b^2x^{75} + \frac{6}{25}a^{11}bx^{50} + \frac{1}{25}a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B]

time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25} + \frac{6\,a^{11}\,b\,x^{50}}{25} + \frac{22\,a^{10}\,b^2\,x^{75}}{25} + \frac{11\,a^9\,b^3\,x^{100}}{5} + \frac{99\,a^8\,b^4\,x^{125}}{25} + \frac{132\,a^7\,b^5\,x^{150}}{25} + \frac{132\,a^6\,b^6\,x^{175}}{25} + \frac{99\,a^5\,b^7\,x^{200}}{25} + \frac{11\,a^4\,b^8\,x^{225}}{5} + \frac{22\,a^3\,b^9\,x^{250}}{25} + \frac{6\,a^2\,b^{10}\,x^{275}}{25} + \frac{a\,b^{11}\,x^{300}}{25} + \frac{b^{12}\,x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.351 $\int x^{12}(ax^2+bx^{39})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{37})^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {1598, 267}

$$\frac{(a+bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx&=\int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\&=\frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.13, size = 135, normalized size = 8.44

method	result
default	$\frac{1}{37}a^{12}x^{37}+\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{6}{37}b\,a^{11}x^{74}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}$
gosper	$\frac{x^{37}\left(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^{2}b^{10}x^{370}+286a^{3}b^{9}x^{333}+715a^{4}b^{8}x^{296}+1287a^{5}b^{7}x^{259}+1716a^{6}b^{6}x^{222}+1716b^{5}a^{7}x^{185}+1287a^{8}b^{4}x^{148}+\right.}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{132b^{5}a^{7}x^{222}}{37}+\frac{99a^{8}b^{4}x}{37}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^39+a*x^2)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/37*a^12*x^37+1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+6/37*
b*a^11*x^74+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132
/37*a^6*b^6*x^259+132/37*b^5*a^7*x^222+22/37*a^10*b^2*x^111+99/37*a^8*b^4*x
^185+55/37*b^3*a^9*x^148
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.28, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.87, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.
time = 0.04, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**39+a*x**2)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.56, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+\frac{22}{37}a^{10}b^2x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B]
time = 5.22, size = 134, normalized size = 8.38

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x^2 + b*x^39)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.352 $\int x^{24}(a+bx^{25})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{25})^{13}}{325b}$$

[Out] 1/325*(b*x^25+a)^13/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\frac{(a+bx^{25})^{13}}{325b}$$

Antiderivative was successfully verified.

[In] Int[x^24*(a + b*x^25)^12,x]

[Out] (a + b*x^25)^13/(325*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

Antiderivative was successfully verified.

[In] Integrate[x^24*(a + b*x^25)^12,x]

[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1

```
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [A]
time = 0.53, size = 15, normalized size = 0.94

method	result
default	$\frac{\left(b\,x^{25}+a\right)^{13}}{325 b}$
gosper	$\frac{132}{25} a^{6} b^{6} x^{175}+\frac{11}{5} b^{3} a^{9} x^{100}+\frac{22}{25} a^{10} b^{2} x^{75}+\frac{6}{25} b\,a^{11} x^{50}+\frac{1}{25} a^{12} x^{25}+\frac{132}{25} b^{5} a^{7} x^{150}+\frac{99}{25} a^{8} b^{4} x^{125}+\frac{1}{325} b$
risch	$\frac{b^{12} x^{325}}{325}+\frac{a\,b^{11} x^{300}}{25}+\frac{6 a^{2} b^{10} x^{275}}{25}+\frac{22 a^{3} b^{9} x^{250}}{25}+\frac{11 a^{4} b^{8} x^{225}}{5}+\frac{99 a^{5} b^{7} x^{200}}{25}+\frac{132 a^{6} b^{6} x^{175}}{25}+\frac{132 b^{5} a^{7} x^{150}}{25}+\frac{99 a^{8} b^{4}}{25}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^25+a)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/325*(b*x^25+a)^13/b
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.88

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.62, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.
time = 0.03, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**25+a)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Giac [A]
time = 0.58, size = 14, normalized size = 0.88

$$\frac{(bx^{25} + a)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

Mupad [B]
time = 5.20, size = 14, normalized size = 0.88

$$\frac{(b\,x^{25} + a)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a + b*x^25)^12,x)
```

```
[Out] (a + b*x^25)^13/(325*b)
```

3.353 $\int x^{24}(ax+bx^{38})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{37})^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1598, 267}

$$\frac{(a+bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{24}(ax+bx^{38})^{12}\,dx &= \int x^{36}(a+bx^{37})^{12}\,dx\\ &= \frac{(a+bx^{37})^{13}}{481b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.10, size = 135, normalized size = 8.44

method	result
default	$\frac{1}{37}a^{12}x^{37}+\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{6}{37}b\,a^{11}x^{74}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}$
gosper	$\frac{x^{37}\left(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^{2}b^{10}x^{370}+286a^{3}b^{9}x^{333}+715a^{4}b^{8}x^{296}+1287a^{5}b^{7}x^{259}+1716a^{6}b^{6}x^{222}+1716b^{5}a^{7}x^{185}+1287a^{8}b^{4}x^{148}+\right.}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{132b^{5}a^{7}x^{222}}{37}+\frac{99a^{8}b^{4}x}{37}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/37*a^12*x^37+1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+6/37*
b*a^11*x^74+22/37*a^3*b^9*x^370+55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132
/37*a^6*b^6*x^259+132/37*b^5*a^7*x^222+22/37*a^10*b^2*x^111+99/37*a^8*b^4*x
^185+55/37*b^3*a^9*x^148
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.28, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.65, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.
time = 0.04, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8x^{333}}{37} + \frac{22a^3b^9x^{370}}{37} + \frac{6a^2b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 0.50, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481} + \frac{1}{37}ab^{11}x^{444} + \frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}a^{10}b^2x^{111} + \frac{6}{37}a^{11}bx^{74} + \frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B]
time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37} + \frac{6\,a^{11}\,b\,x^{74}}{37} + \frac{22\,a^{10}\,b^2\,x^{111}}{37} + \frac{55\,a^9\,b^3\,x^{148}}{37} + \frac{99\,a^8\,b^4\,x^{185}}{37} + \frac{132\,a^7\,b^5\,x^{222}}{37} + \frac{132\,a^6\,b^6\,x^{259}}{37} + \frac{99\,a^5\,b^7\,x^{296}}{37} + \frac{55\,a^4\,b^8\,x^{333}}{37} + \frac{22\,a^3\,b^9\,x^{370}}{37} + \frac{6\,a^2\,b^{10}\,x^{407}}{37} + \frac{a\,b^{11}\,x^{444}}{37} + \frac{b^{12}\,x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.354 $\int x^{36}(a+bx^{37})^{12}\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^{37})^{13}}{481b}$$

[Out] 1/481*(b*x^37+a)^13/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{(a+bx^{37})^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[x^36*(a + b*x^37)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.
time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

Antiderivative was successfully verified.

[In] Integrate[x^36*(a + b*x^37)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x

```
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [A]
time = 1.07, size = 15, normalized size = 0.94

method	result
default	$\frac{(b x^{37}+a)^{13}}{481 b}$
gosper	$\frac{1}{37}a^{12}x^{37}+\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{6}{37}b\,a^{11}x^{74}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{132b^{5}a^{7}x^{222}}{37}+\frac{99a^{8}b^{4}}{37}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(b*x^37+a)^12,x,method=_RETURNVERBOSE)
[Out] 1/481*(b*x^37+a)^13/b
```

Maple [A]

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$\frac{(bx^{37}+a)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
[Out] 1/481*(b*x^37 + a)^13/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.
time = 1.64, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}b^{4}x^{185}+\frac{55}{37}a^{9}b^{3}x^{148}+\frac{22}{37}a^{10}b^{2}x^{111}+\frac{6}{37}a^{11}bx^{74}+\frac{1}{37}a^{12}x^{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.
time = 0.03, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}+\frac{22a^{3}b^{9}x^{370}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**36*(b*x**37+a)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Giac [A]
time = 0.54, size = 14, normalized size = 0.88

$$\frac{(bx^{37}+a)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

Mupad [B]
time = 5.18, size = 14, normalized size = 0.88

$$\frac{(b\,x^{37}+a)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(a + b*x^37)^12,x)
```

```
[Out] (a + b*x^37)^13/(481*b)
```

3.355 $\int \frac{1}{ax+bx^n}\,dx$

Optimal. Leaf size=23

$$\frac{\log(b+ax^{1-n})}{a(1-n)}$$

[Out] ln(b+a*x^(1-n))/a/(1-n)

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 266}

$$\frac{\log(ax^{1-n}+b)}{a(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^n)^(-1),x]

[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx^n}\,dx &= \int \frac{x^{-n}}{b+ax^{1-n}}\,dx\\ &= \frac{\log(b+ax^{1-n})}{a(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(b+ax^{1-n})}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Maple [A]
time = 0.42, size = 35, normalized size = 1.52

method	result	size
risch	$\frac{n\ln(x)}{a(-1+n)}-\frac{\ln\left(x^{n}+\frac{ax}{b}\right)}{a(-1+n)}$	35
norman	$\frac{n\ln(x)}{a(-1+n)}-\frac{\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)}{a(-1+n)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] n/a/(-1+n)*ln(x)-1/a/(-1+n)*ln(x^n+a/b*x)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 1.61

$$\frac{n\log\left(x\right)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^{n}}{b}\right)}{a(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="maxima")
```

```
[Out] n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))
```

Fricas [A]
time = 1.89, size = 27, normalized size = 1.17

$$\frac{n\log\left(x\right)-\log\left(ax+bx^{n}\right)}{an-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(14) = 28$.

time = 0.30, size = 48, normalized size = 2.09

$$\begin{cases} \frac{\log(x)}{b} & \text{for } a = 0 \wedge n = 1 \\ -\frac{x}{b(nx^n - x^n)} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 1 \\ \frac{n\log(x)}{an-a} - \frac{\log\left(x + \frac{bx^n}{a}\right)}{an-a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 1)), (-x/(b*(n*x**n - x**n)), Eq(a, 0
)), (log(x)/(a + b), Eq(n, 1)), (n*log(x)/(a*n - a) - log(x + b*x**n/a)/(a*
n - a), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^n), x)
```

Mupad [B]
time = 5.26, size = 26, normalized size = 1.13

$$-\frac{\ln\left(b\,x^n+a\,x\right)-n\,\ln\left(x\right)}{a\,\left(n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n + a*x),x)
```

```
[Out] -(log(b*x^n + a*x) - n*log(x))/(a*(n - 1))
```

3.356 $\int \frac{1}{ax+bx^{1+n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

```
[Out] ln(x)/a-ln(a+b*x^n)/a/n
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1607, 272, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{ax+bx^{1+n}}\,dx &= \int \frac{1}{x\left(a+bx^n\right)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{an}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^n\right)}{an}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.09

$$\frac{\log\left(x^n\right)-\log\left(an(a+bx^n)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 + n))^(-1),x]
```

```
[Out] (Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

Maple [A]
time = 0.41, size = 36, normalized size = 1.57

method	result	size
norman	$\frac{(1+n)\ln(x)}{an}-\frac{\ln\left(ax+b\,\mathrm{e}^{(1+n)\ln(x)}\right)}{an}$	36
risch	$\frac{\ln(x)}{an}+\frac{\ln(x)}{a}-\frac{\ln\left(x^{1+n}+\frac{ax}{b}\right)}{an}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1+n)),x,method=_RETURNVERBOSE)
```

```
[Out] (1+n)/a/n*ln(x)-1/a/n*ln(a*x+b*exp((1+n)*ln(x)))
```

Maxima [A]
time = 0.28, size = 27, normalized size = 1.17

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Fricas [A]
time = 2.11, size = 28, normalized size = 1.22

$$\frac{(n+1)\log(x) - \log(ax + bx^{n+1})}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")
```

```
[Out] ((n + 1)*log(x) - log(a*x + b*x^(n + 1)))/(a*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.
time = 0.63, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty}\log(x) & \text{for}\ a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for}\ b = 0 \\ -\frac{x^{-n}}{bn} & \text{for}\ a = 0 \\ \frac{\log(x)}{a+b} & \text{for}\ n = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^n\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1+n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (-1/(b*n*x**n), Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log(a
/b + x**n)/(a*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(n + 1)), x)
```

Mupad [B]
time = 5.23, size = 31, normalized size = 1.35

$$\frac{\ln(x)\,(n+1)}{a\,n} - \frac{\ln\left(x\,(a + b\,x^n)\right)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(n + 1)),x)
[Out] (log(x)*(n + 1))/(a*n) - log(x*(a + b*x^n))/(a*n)
```

3.357 $\int \frac{1}{ax+bx^{1-n}} \, dx$

Optimal. Leaf size=15

$$\frac{\log(b+ax^n)}{an}$$

```
[Out] ln(b+a*x^n)/a/n
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 266}

$$\frac{\log(ax^n+b)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx^{1-n}} \, dx &= \int \frac{x^{-1+n}}{b+ax^n} \, dx \\ &= \frac{\log(b+ax^n)}{an}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.20

$$\frac{\log(bn+anx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b*n + a*n*x^n]/(a*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.
time = 0.40, size = 37, normalized size = 2.47

method	result	size
norman	$\frac{(-1+n)\ln(x)}{an}+\frac{\ln\left(ax+b\,\mathrm{e}^{(1-n)\ln(x)}\right)}{an}$	37
risch	$-\frac{\ln(x)}{an}+\frac{\ln(x)}{a}+\frac{\ln\left(x^{1-n}+\frac{ax}{b}\right)}{an}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1-n)),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+n)/a/n*ln(x)+1/a/n*ln(a*x+b*exp((1-n)*ln(x)))
```

Maxima [A]
time = 0.30, size = 19, normalized size = 1.27

$$\frac{\log\left(\frac{ax^n+b}{a}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="maxima")
```

```
[Out] log((a*x^n + b)/a)/(a*n)
```

Fricas [A]
time = 2.08, size = 28, normalized size = 1.87

$$\frac{(n-1)\log(x)+\log\left(ax+bx^{-n+1}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")
```

```
[Out] ((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

time = 0.67, size = 39, normalized size = 2.60

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{x^n}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} + \frac{\log\left(\frac{a}{b}+x^{-n}\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1-n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (x**n/(b*n), Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a + log(a/b
+ x**(-n))/(a*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(-n + 1)), x)
```

Mupad [B]
time = 5.22, size = 34, normalized size = 2.27

$$\frac{\ln\left(a\,x+b\,x^{1-n}\right)}{a\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1 - n)),x)
```

```
[Out] log(a*x + b*x^(1 - n))/(a*n) + (log(x)*(n - 1))/(a*n)
```

3.358 $\int \frac{1}{2x+3x^{1+n}}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{2}-\frac{\log(2+3x^n)}{2n}$$

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x^n)/n
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1607, 272, 36, 29, 31}

$$\frac{\log(x)}{2}-\frac{\log(3x^n+2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{2x+3x^{1+n}}\,dx &= \int \frac{1}{x\left(2+3x^n\right)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(2+3x)}\,dx,x,x^n\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{2n}-\frac{3\operatorname{Subst}\left(\int \frac{1}{2+3x}\,dx,x,x^n\right)}{2n}\\
&= \frac{\log(x)}{2}-\frac{\log\left(2+3x^n\right)}{2n}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.09

$$\frac{\log\left(x^n\right)-\log\left(n(2+3x^n)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 + n))^(-1),x]
```

```
[Out] (Log[x^n] - Log[n*(2 + 3*x^n)])/(2*n)
```

Maple [A]
time = 0.54, size = 27, normalized size = 1.23

method	result	size
meijerg	$\frac{-\ln\left(1+\frac{3x^n}{2}\right)+n\ln(x)-\ln(2)+\ln(3)}{2n}$	27
risch	$\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}-\frac{\ln\left(\frac{2x}{3}+x^{1+n}\right)}{2n}$	28
norman	$\frac{(1+n)\ln(x)}{2n}-\frac{\ln\left(2x+3\,e^{(1+n)\ln(x)}\right)}{2n}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(1+n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/n*(-ln(1+3/2*x^n)+n*ln(x)-ln(2)+ln(3))
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.73

$$-\frac{\log\left(x^n+\frac{2}{3}\right)}{2\,n}+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^n + 2/3)/n + 1/2*log(x)
```

Fricas [A]
time = 2.28, size = 26, normalized size = 1.18

$$\frac{(n+1)\log(x) - \log(3\,x^{n+1} + 2\,x)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")
```

```
[Out] 1/2*((n + 1)*log(x) - log(3*x^(n + 1) + 2*x))/n
```

Sympy [A]
time = 0.51, size = 20, normalized size = 0.91

$$\begin{cases} \frac{\log(x)}{2} - \frac{\log(3x^n+2)}{2n} & \text{for}\ n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1+n)),x)
```

```
[Out] Piecewise((log(x)/2 - log(3*x**n + 2)/(2*n), Ne(n, 0)), (log(x)/5, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate(1/(3*x^(n + 1) + 2*x), x)
```

Mupad [B]
time = 5.23, size = 26, normalized size = 1.18

$$\frac{\ln(x)\,(n+1)}{2\,n} - \frac{\ln\left(\frac{2\,x}{3} + x^{n+1}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 3*x^(n + 1)),x)
```

```
[Out] (log(x)*(n + 1))/(2*n) - log((2*x)/3 + x^(n + 1))/(2*n)
```

3.359 $\int \frac{1}{2x+3x^{1-n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(3+2x^n)}{2n}$$

```
[Out] 1/2*ln(3+2*x^n)/n
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 266}

$$\frac{\log(2x^n+3)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2x+3x^{1-n}}\,dx &= \int \frac{x^{-1+n}}{3+2x^n}\,dx\\ &= \frac{\log(3+2x^n)}{2n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.20

$$\frac{\log(3n+2nx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3*n + 2*n*x^n]/(2*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.
time = 0.40, size = 30, normalized size = 2.00

method	result	size
meijerg	$-\frac{-\ln\left(1+\frac{3x^{-n}}{2}\right)-n\ln(x)-\ln(2)+\ln(3)}{2n}$	30
risch	$-\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}+\frac{\ln\left(\frac{2x}{3}+x^{1-n}\right)}{2n}$	30
norman	$\frac{(-1+n)\ln(x)}{2n}+\frac{\ln\left(2x+3\,\mathrm{e}^{(1-n)\ln(x)}\right)}{2n}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(1-n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/n*(-ln(1+3/2*x^(-n))-n*ln(x)-ln(2)+ln(3))
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$\frac{\log\left(x^n+\frac{3}{2}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^n + 3/2)/n
```

Fricas [A]
time = 2.41, size = 26, normalized size = 1.73

$$\frac{(n-1)\log(x)+\log\left(3\,x^{-n+1}+2\,x\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")
```

```
[Out] 1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n
```

Sympy [A]
time = 0.55, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\log(x)}{2}+\frac{\log\left(2+3x^{-n}\right)}{2n} & \text{for}\: n\neq 0\\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1-n)),x)
```

```
[Out] Piecewise((log(x)/2 + log(2 + 3/x**n)/(2*n), Ne(n, 0)), (log(x)/5, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate(1/(3*x^(-n + 1) + 2*x), x)
```

Mupad [B]
time = 5.20, size = 28, normalized size = 1.87

$$\frac{\ln\left(\frac{2\,x}{3}+x^{1-n}\right)}{2\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 3*x^(1 - n)),x)
```

```
[Out] log((2*x)/3 + x^(1 - n))/(2*n) + (log(x)*(n - 1))/(2*n)
```

3.360 $\int \frac{1}{-\sqrt{x}+x} \, dx$

Optimal. Leaf size=12

$$2 \log \left(1-\sqrt{x}\right)$$

```
[Out] 2*ln(1-x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 266}

$$2 \log \left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{-\sqrt{x}+x} \, dx &= \int \frac{1}{\left(-1+\sqrt{x}\right)\sqrt{x}} \, dx \\ &= 2 \log \left(1-\sqrt{x}\right) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 0.83

$$2 \log \left(-1+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[x] + x)^(-1),x]
[Out] 2*Log[-1 + Sqrt[x]]
```

Maple [A]
time = 0.36, size = 12, normalized size = 1.00

method	result	size
derivativedivides	$2\ln\left(-1+\sqrt{x}\right)$	9
meijerg	$2\ln\left(1-\sqrt{x}\right)$	11
default	$\ln\left(x-1\right)-2\,\mathrm{arctanh}\left(\sqrt{x}\right)$	12
trager	$\ln\left(2\sqrt{x}-1-x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-x^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(x-1)-2*arctanh(x^(1/2))
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.67

$$2\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="maxima")
[Out] 2*log(sqrt(x) - 1)
```

Fricas [A]
time = 1.40, size = 8, normalized size = 0.67

$$2\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="fricas")
[Out] 2*log(sqrt(x) - 1)
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.67

$$2\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x**(1/2)),x)
```

```
[Out] 2*log(sqrt(x) - 1)
```

Giac [A]
time = 0.56, size = 9, normalized size = 0.75

$$2 \log \left(\left| \sqrt{x} - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="giac")
[Out] 2*log(abs(sqrt(x) - 1))
```

Mupad [B]
time = 0.10, size = 8, normalized size = 0.67

$$2 \ln \left(\sqrt{x} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^(1/2)),x)
[Out] 2*log(x^(1/2) - 1)
```

3.361 $\int \frac{1}{-x^{3/5}+x} dx$

Optimal. Leaf size=14

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

[Out] 5/2*ln(1-x^(2/5))

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 266}

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Antiderivative was successfully verified.

[In] Int[(-x^(3/5) + x)^(-1),x]

[Out] (5*Log[1 - x^(2/5)])/2

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^{3/5}+x}\,dx &= \int \frac{1}{\left(-1+x^{2/5}\right)x^{3/5}}\,dx\\ &= \frac{5}{2}\log\left(1-x^{2/5}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.79

$$\frac{5}{2}\log\left(-1+\sqrt[5]{x}\right)+\frac{5}{2}\log\left(1+\sqrt[5]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(3/5) + x)^(-1),x]
```

```
[Out] (5*Log[-1 + x^(1/5)])/2 + (5*Log[1 + x^(1/5)])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(10) = 20$.

time = 0.69, size = 116, normalized size = 8.29

method	result
meijerg	$\frac{5\ln\left(1-x^{\frac{2}{5}}\right)}{2}$
derivativedivides	$\frac{5\ln\left(x^{\frac{1}{5}}-1\right)}{2}+\frac{5\ln\left(1+x^{\frac{1}{5}}\right)}{2}$
trager	$\frac{\ln\left(-10x^{\frac{4}{5}}-5x^{\frac{8}{5}}+5x^{\frac{2}{5}}+10x^{\frac{6}{5}}+x^{2}-1\right)}{2}$
default	$-\frac{\ln\left(-\sqrt{5}\, x^{\frac{1}{5}}+2x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{2}+2\ln\left(x^{\frac{1}{5}}-1\right)-\frac{\ln\left(\sqrt{5}\, x^{\frac{1}{5}}+2x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{2}-\frac{\ln\left(-\sqrt{5}\, x^{\frac{1}{5}}+2x^{\frac{2}{5}}-x\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(3/5)+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)+2*ln(x^(1/5)-1)-1/2*ln(5^(1/2
)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)-1/2*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)-x^(1/5)+2)
+1/2*ln(x-1)+2*ln(1+x^(1/5))+1/2*ln(x+1)-1/2*ln(5^(1/2)*x^(1/5)+2*x^(2/5)-x
^(1/5)+2)
```

Maxima [A]

time = 0.27, size = 17, normalized size = 1.21

$$\frac{5}{2}\,\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\,\log\left(x^{\frac{1}{5}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="maxima")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)
```

Fricas [A]

time = 1.19, size = 8, normalized size = 0.57

$$\frac{5}{2}\,\log\left(x^{\frac{2}{5}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="fricas")
```

```
[Out] 5/2*log(x^(2/5) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.10, size = 22, normalized size = 1.57

$$\frac{5\log\left(\sqrt[5]{x} - 1\right)}{2} + \frac{5\log\left(\sqrt[5]{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(3/5)+x),x)
```

```
[Out] 5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2
```

Giac [A]
time = 0.56, size = 18, normalized size = 1.29

$$\frac{5}{2}\log\left(x^{\frac{1}{5}} + 1\right) + \frac{5}{2}\log\left(\left|x^{\frac{1}{5}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="giac")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))
```

Mupad [B]
time = 5.30, size = 8, normalized size = 0.57

$$\frac{5\ln\left(x^{2/5} - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^(3/5)),x)
```

```
[Out] (5*log(x^(2/5) - 1))/2
```

3.362 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$

Optimal. Leaf size=12

$$\frac{3}{4}\log\left(1+x^{4/3}\right)$$

```
[Out] 3/4*ln(1+x^(4/3))
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1607, 266}

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}}\,dx\\ &= \frac{3}{4}\log\left(1+x^{4/3}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{3}{4}\log\left(1+x^{4/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

Maple [A]
time = 0.35, size = 9, normalized size = 0.75

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
default	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
trager	$-\frac{\ln\left(\frac{3x^{\frac{20}{3}}-x^8-6x^{\frac{16}{3}}-6x^{\frac{8}{3}}+7x^4+3x^{\frac{4}{3}}-1}{\left(x^4+1\right)^3}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+x),x,method=_RETURNVERBOSE)
[Out] 3/4*ln(1+x^(4/3))
```

Maxima [A]
time = 0.50, size = 8, normalized size = 0.67

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
[Out] 3/4*log(x^(4/3) + 1)
```

Fricas [A]
time = 2.00, size = 8, normalized size = 0.67

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
[Out] 3/4*log(x^(4/3) + 1)
```

Sympy [A]
time = 0.07, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(8) = 16.
time = 0.61, size = 32, normalized size = 2.67

$$\frac{3}{4}\log\left(\sqrt{2}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{3}{4}\log\left(-\sqrt{2}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

Mupad [B]
time = 0.09, size = 8, normalized size = 0.67

$$\frac{3\,\ln\left(x^{4/3}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1/x^(1/3)),x)
```

```
[Out] (3*log(x^(4/3) + 1))/4
```

3.363 $\int \frac{1}{x+x^{\sqrt{2}}} \, dx$

Optimal. Leaf size=24

$$\log(x) - \left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)$$

```
[Out] ln(x)-ln(1+x^(2^(1/2)-1))*(1+2^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1607, 272, 36, 29, 31}

$$\log(x) - \left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^Sqrt[2])^(-1),x]
```

```
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x+x^{\sqrt{2}}}\,dx &= \int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)}\,dx \\
&= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^{-1+\sqrt{2}}\right) \\
&= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^{-1+\sqrt{2}}\right)+\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^{-1+\sqrt{2}}\right) \\
&= \log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^Sqrt[2])^(-1),x]
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Maple [A]
time = 0.44, size = 28, normalized size = 1.17

method	result	size
norman	$\left(2+\sqrt{2}\right)\ln\left(x\right)+\left(-1-\sqrt{2}\right)\ln\left(x+{\rm e}^{\sqrt{2}\,\ln(x)}\right)$	28
meijerg	$\frac{-\ln\left(1+x^{\sqrt{2}-1}\right)+\left(\sqrt{2}-1\right)\ln(x)}{\sqrt{2}-1}$	30
risch	$2\ln\left(x\right)+\sqrt{2}\,\ln\left(x\right)-\ln\left(x+x^{\sqrt{2}}\right)\sqrt{2}-\ln\left(x+x^{\sqrt{2}}\right)$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(2^(1/2))),x,method=_RETURNVERBOSE)
[Out] (2+2^(1/2))*ln(x)+(-1-2^(1/2))*ln(x+exp(2^(1/2)*ln(x)))
```

Maxima [A]
time = 0.50, size = 31, normalized size = 1.29

$$\frac{\sqrt{2}\,\log\left(x\right)}{\sqrt{2}-1}-\frac{\log\left(x+x^{\left(\sqrt{2}\right)}\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")
[Out] sqrt(2)*log(x)/(sqrt(2) - 1) - log(x + x^sqrt(2))/(sqrt(2) - 1)
```

Fricas [A]
time = 0.99, size = 24, normalized size = 1.00

$$-\left(\sqrt{2}+1\right)\log\left(x+x^{\left(\sqrt{2}\right)}\right)+\left(\sqrt{2}+2\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")
[Out] -(sqrt(2) + 1)*log(x + x^sqrt(2)) + (sqrt(2) + 2)*log(x)
```

Sympy [A]
time = 0.18, size = 31, normalized size = 1.29

$$\frac{\sqrt{2}\log\left(x\right)}{-1+\sqrt{2}}-\frac{\log\left(x+x^{\sqrt{2}}\right)}{-1+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(2**(1/2))),x)
[Out] sqrt(2)*log(x)/(-1 + sqrt(2)) - log(x + x**(sqrt(2)))/(-1 + sqrt(2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")
[Out] integrate(1/(x + x^sqrt(2)), x)
```

Mupad [B]
time = 5.25, size = 26, normalized size = 1.08

$$\ln\left(x\right)\,\left(\sqrt{2}+2\right)-\frac{\ln\left(x+x^{\sqrt{2}}\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^(2^(1/2))),x)
[Out] log(x)*(2^(1/2) + 2) - log(x + x^(2^(1/2)))/(2^(1/2) - 1)
```

3.364 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal. Leaf size=75

$$-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}$$

```
[Out] 2*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/(j-n)-2*(a*x^j+b*x
^n)^(1/2)/(j-n)/(x^(1/2*j))
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2053, 2054, 212}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^
(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+a\int\frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$$
$$=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{(2a)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}$$
$$=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}$$

Mathematica [A]
time = 0.18, size = 104, normalized size = 1.39

$$-\frac{2x^{-j/2}\left(ax^j+bx^n-\sqrt{a}\,\sqrt{b}\,x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/((j - n)*x^(j/2)*Sqrt[a*x^j +
b*x^n])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int x^{-1-\frac{j}{2}}\sqrt{a\,x^j+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
[Out] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(x**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^j+b\,x^n}}{x^{\frac{j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1),x)
```

```
[Out] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1), x)
```

3.365 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal. Leaf size=99

$$-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

```
[Out] 2*x^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/c/(j-n)/
((c*x)^(1/2*j))-2*(a*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(1/2*j))
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {2056, 2053, 2054, 212}

$$\frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^(j/2)) + (2*Sqrt[a]*x^(j/2)*ArcTa
nh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^(j/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx &= \frac{\left(x^{j/2}(cx)^{-j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c}\\ &= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{\left(ax^{j/2}(cx)^{-j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}\\ &= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{\left(2ax^{j/2}(cx)^{-j/2}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}\\ &= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 109, normalized size = 1.10

$$-\frac{2(cx)^{-j/2}\left(ax^j+bx^n-\sqrt{a}\,\sqrt{b}\,x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(c*(j - n)*(c*x)^(j/2)*Sqrt[a*
x^j + b*x^n])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{a\,x^j+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
[Out] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^j+b\,x^n}}{(c\,x)^{\frac{j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1), x)
```

3.366 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$

```
[Out] 2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*a^(1/2)*(c*x)^(1/2)/c^3/(3-n
)/x^(1/2)-2*(a*x^3+b*x^n)^(1/2)/c/(3-n)/(c*x)^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2056, 2054, 212}

$$\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{a\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^3}\\
&= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^3\sqrt{x}}\\
&= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(2a\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}\\
&= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 103, normalized size = 1.13

$$\frac{2x\left(ax^3+bx^n-\sqrt{a}\,\sqrt{b}\,x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-3+n)(cx)^{5/2}\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
[Out] (2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/((-3 + n)*(c*x)^(5/2)*Sqrt[a*
x^3 + b*x^n])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^3+b\,x^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2),x)
```

```
[Out] Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^n+a\,x^3}}{(c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2),x)
[Out] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2), x)
```

3.367 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$

```
[Out] 2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))*a^(1/2)/c^2/(2-n)-2*(a*x^2+b*x^n)^
(1/2)/c^2/(2-n)/x
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2053, 2033, 212}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/S
qrt[a*x^2 + b*x^n]])/(c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
```

```
Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx &= \frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}\\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{a\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^2}\\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}\\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 99, normalized size = 1.39

$$\frac{2\left(ax^2+bx^n-\sqrt{a}\,\sqrt{b}\,x^{1+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{c^2(-2+n)x\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^n]
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2+b\,x^n}}{c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

```
[Out] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^2 + bx^n}}{x^2}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)
[Out] Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")
[Out] integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^n + a\,x^2}}{c^2\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2),x)
[Out] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2), x)
```

3.368 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}$$

```
[Out] 2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/c/(1-n)/(c*x)^
(1/2)-2*(a*x+b*x^n)^(1/2)/c/(1-n)/(c*x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$,
Rules used = {2053, 2056, 2054, 212}

$$\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(
Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{a\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx}{c} \\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{\left(a\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^n}}\,dx}{c\sqrt{cx}} \\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{\left(2a\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} \\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A]
time = 0.36, size = 100, normalized size = 1.15

$$
\frac{x\left(2ax+2bx^n-2\sqrt{a}\,\sqrt{b}\,x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-1+n)(cx)^{3/2}\sqrt{ax+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n)
)/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/((-1 + n)*(c*x)^(3/2)*Sqrt[
a*x + b*x^n])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{ax+b\,x^n}}{(cx)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
[Out] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2),x)
[Out] Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^n+a\,x}}{{\left(c\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2),x)
[Out] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2), x)
```

3.369 $\int \frac{\sqrt{a+bx^n}}{cx}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))*a^(1/2)/c/n+2*(a+b*x^n)^(1/2)/c/n
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 272, 52, 65, 214}

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^n}}{cx}\,dx &= \frac{\int \frac{\sqrt{a+bx^n}}{x}\,dx}{c}\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^n\right)}{cn}\\ &= \frac{2\sqrt{a+bx^n}}{cn}+\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^n\right)}{cn}\\ &= \frac{2\sqrt{a+bx^n}}{cn}+\frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^n}\right)}{bcn}\\ &= \frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.88

$$\frac{2\left(\sqrt{a+bx^n}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/(c*x),x]
[Out] (2*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]]))/(c*n)
```

Maple [A]
time = 0.65, size = 39, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
default	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
risch	$\frac{2\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{nc}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)}{nc}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^(1/2)/c/x,x,method=_RETURNVERBOSE)
[Out] 1/c/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 58, normalized size = 1.14

$$\frac{\frac{\sqrt{a}\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{n}+\frac{2\sqrt{bx^{n}+a}}{n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="maxima")
[Out] (sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
*sqrt(b*x^n + a)/n)/c
```

Fricas [A]
time = 1.74, size = 97, normalized size = 1.90

$$\left[\frac{\sqrt{a}\log\left(\frac{bx^{n}-2\sqrt{bx^{n}+a}\sqrt{a}+2a}{x^{n}}\right)+2\sqrt{bx^{n}+a}}{cn},\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx^{n}+a}\sqrt{-a}}{a}\right)+\sqrt{bx^{n}+a}\right)}{cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="fricas")
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n
 + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n +
 a))/(c*n)]
```

Sympy [A]
time = 0.77, size = 78, normalized size = 1.53

$$\frac{-\frac{2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}\,n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/c/x,x)
```

```
[Out] (-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/n + 2*a/(sqrt(b)*n*x**(n/2)*s
qrt(a/(b*x**n) + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a/(b*x**n) + 1)))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x^n}}{c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/(c*x),x)
```

```
[Out] int((a + b*x^n)^(1/2)/(c*x), x)
```

3.370 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=84

$$\frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(1+n)/(c*x)^(
1/2)+2*(c*x)^(1/2)*(a/x+b*x^n)^(1/2)/c/(1+n)
```

Rubi [A]
time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2056, 2054, 212}

$$\frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sq
rt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx &= \frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} + (ac)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx \\
&= \frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} + \frac{\left(a\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{\sqrt{cx}} \\
&= \frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} - \frac{\left(2a\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}} \\
&= \frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} - \frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 84, normalized size = 1.00

$$\frac{2x\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{(1+n)\sqrt{cx}\,\sqrt{a+bx^{1+n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (2*x*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)] - Sqrt[a]*ArcTanh[Sqrt[a + b*
x^(1 + n)]/Sqrt[a]]))/((1 + n)*Sqrt[c*x]*Sqrt[a + b*x^(1 + n)])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}+b\,x^{n}}}{\sqrt{c x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}+bx^{n}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^n+\frac{a}{x}}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2),x)
[Out] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2), x)
```

3.371 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))*a^(1/2)/(2+n)+2*x*(a/x^2+b*x^n)^(
1/2)/(2+n)
```

Rubi [A]

time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2032, 2054, 212}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^
2 + b*x^n])])/(2 + n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx = \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$$

$$=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

$$=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

Mathematica [**A**]
time = 0.07, size = 77, normalized size = 1.26

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{(2+n)\sqrt{a+bx^{2+n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2 + b*x^n],x]
[Out] (2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)] - Sqrt[a]*ArcTanh[Sqrt[a +
b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a}{x^2}+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2+b*x^n)^(1/2),x)
[Out] int((a/x^2+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a}{x^2} + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2 + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

Mupad [B]
time = 5.17, size = 97, normalized size = 1.59

$$\frac{x\,\sqrt{b\,x^n+\frac{a}{x^2}}}{\frac{n}{2}+1}+\frac{\sqrt{a}\,\,x\,\mathrm{asin}\left(\frac{\sqrt{a}\,\,1{}\mathrm{i}}{\sqrt{b}\,\,x^{\frac{n}{2}+1}}\right)\,\sqrt{b\,x^n+\frac{a}{x^2}}\,\,1{}\mathrm{i}}{\sqrt{b}\,\,x^{\frac{n}{2}+1}\,\left(\frac{n}{2}+1\right)\,\sqrt{\frac{a}{b\,x^{n+2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a/x^2)^(1/2),x)
```

```
[Out] (x*(b*x^n + a/x^2)^(1/2))/(n/2 + 1) + (a^(1/2)*x*asin((a^(1/2)*1i)/(b^(1/2)
*x^(n/2 + 1)))*(b*x^n + a/x^2)^(1/2)*1i)/(b^(1/2)*x^(n/2 + 1)*(n/2 + 1)*(a/
(b*x^(n + 2)) + 1)^(1/2))
```

3.372 $\int \sqrt{cx}\,\sqrt{\frac{a}{x^3}+bx^n}\,dx$

Optimal. Leaf size=85

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)}-\frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

```
[Out] -2*c*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(3+n)/(c*
x)^(1/2)+2*(c*x)^(3/2)*(a/x^3+b*x^n)^(1/2)/c/(3+n)
```

Rubi [A]
time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2056, 2054, 212}

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcT
anh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \left(ac^3\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \frac{\left(ac\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{\sqrt{cx}} \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{\left(2ac\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}} \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 84, normalized size = 0.99

$$
\frac{2x\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{(3+n)\sqrt{a+bx^{3+n}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)] - Sqrt[a]*ArcTanh
[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/((3 + n)*Sqrt[a + b*x^(3 + n)])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{c x}\, \sqrt{\frac{a}{x^{3}}+b\,x^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c x} \sqrt{\frac{a}{x^{3}} + b x^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\sqrt{b\,x^n+\frac{a}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2),x)
[Out] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2), x)
```

3.373 $\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx$

Optimal. Leaf size=141

$$-\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}+\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

```
[Out] -2/3*(a*x^j+b*x^n)^(3/2)/c/(j-n)/((c*x)^(3/2*j))+2*a^(3/2)*x^(3/2*j)*arctan
h(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/((c*x)^(3/2*j))-2*a*x^j*(a
*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(3/2*j))
```

Rubi [A]
time = 0.17, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2056, 2053, 2054, 212}

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}-\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*a*x^j*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^((3*j)/2)) - (2*(a*x^j + b*
x^n)^(3/2))/(3*c*(j - n)*(c*x)^((3*j)/2)) + (2*a^(3/2)*x^((3*j)/2)*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^((3*j)/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}\,dx=\frac{\left(x^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}\,dx}{c}$$

$$=-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}+\frac{\left(ax^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c}$$

$$=-\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}+\frac{\left(a^2x^{3j/2}(cx)^{-}\right.}{}$$

$$=-\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}+\frac{\left(2a^2x^{3j/2}(cx)\right.}{}$$

$$=-\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}+\frac{2a^{3/2}x^{3j/2}(cx)}{}$$

Mathematica [A]
time = 0.19, size = 131, normalized size = 0.93

$$-\frac{2(cx)^{-3j/2}\left(4a^2x^{2j}+b^2x^{2n}+5abx^{j+n}-3a^{3/2}\sqrt{b}\,x^{\frac{1}{2}(3j+n)}\sqrt{1+\frac{ax^{j-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{3c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
[Out] (-2*(4*a^2*x^(2*j) + b^2*x^(2*n) + 5*a*b*x^(j + n) - 3*a^(3/2)*Sqrt[b]*x^((
3*j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[
b]]))/(3*c*(j - n)*(c*x)^((3*j)/2)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (cx)^{-1-\frac{3j}{2}}\left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^j+b\,x^n\right)^{3/2}}{\left(c\,x\right)^{\frac{3\,j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1), x)
```

3.374 $\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{2a^{3/2}\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$

```
[Out] -2/3*(a*x^3+b*x^n)^(3/2)/c/(3-n)/(c*x)^(9/2)+2*a^(3/2)*arctanh(x^(3/2)*a^(1
/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/c^6/(3-n)/x^(1/2)-2*a*(a*x^3+b*x^n)^(1
/2)/c^4/(3-n)/(c*x)^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2056, 2054, 212}

$$\frac{2a^{3/2}\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*Sqrt[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^(3/2)) - (2*(a*x^3 + b*x^n)^(
3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (2*a^(3/2)*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(
3/2))/Sqrt[a*x^3 + b*x^n]])/(c^6*(3 - n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx &= -\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx}{c^3} \\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a^2\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^6} \\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{\left(a^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^6\sqrt{x}} \\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{\left(2a^2\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+b}}\right.}{c^6(3-n)\sqrt{x}} \\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}
\end{aligned}$$

Mathematica [A]
time = 0.55, size = 126, normalized size = 0.98

$$\frac{2\sqrt{cx}\left(4a^2x^6+b^2x^{2n}+5abx^{3+n}-3a^{3/2}\sqrt{b}\,x^{\frac{9+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^6(-3+n)x^5\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
[Out] (2*Sqrt[c*x]*(4*a^2*x^6 + b^2*x^(2*n) + 5*a*b*x^(3 + n) - 3*a^(3/2)*Sqrt[b]
*x^((9 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sq
rt[b]]))/(3*c^6*(-3 + n)*x^5*Sqrt[a*x^3 + b*x^n])
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{3}+b\,x^{n}\right)^{\frac{3}{2}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
[Out] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^n+a\,x^3\right)^{3/2}}{{\left(c\,x\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2), x)
```

3.375 $\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx$

Optimal. Leaf size=104

$$-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}$$

```
[Out] -2/3*(a*x^2+b*x^n)^(3/2)/c^4/(2-n)/x^3+2*a^(3/2)*arctanh(x*a^(1/2)/(a*x^2+b
*x^n)^(1/2))/c^4/(2-n)-2*a*(a*x^2+b*x^n)^(1/2)/c^4/(2-n)/x
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2053, 2033, 212}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (-2*a*Sqrt[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^(3/2))/(3*c
^4*(2 - n)*x^3) + (2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(c^4
*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx &= \frac{\int \frac{(ax^2+bx^n)^{3/2}}{x^4}\,dx}{c^4} \\
&= -\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{a\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^4} \\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{a^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^4} \\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)} \\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 117, normalized size = 1.12

$$\frac{2\left(4a^2x^4+b^2x^{2n}+5abx^{2+n}-3a^{3/2}\sqrt{b}\,x^{3+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^4(-2+n)x^3\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n/
2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(3*c^
4*(-2 + n)*x^3*Sqrt[a*x^2 + b*x^n])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+b\,x^n\right)^{\frac{3}{2}}}{c^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

```
[Out] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{ax^2+bx^n}}{x^2}\,dx + \int \frac{bx^n\sqrt{ax^2+bx^n}}{x^4}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)
```

```
[Out] (Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 +
b*x**n)/x**4, x))/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^n+a\,x^2\right)^{3/2}}{c^4\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4),x)
[Out] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4), x)
```

3.376 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}} dx$

Optimal. Leaf size=122

$$-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}$$

```
[Out] -2/3*(a*x+b*x^n)^(3/2)/c/(1-n)/(c*x)^(3/2)+2*a^(3/2)*arctanh(a^(1/2)*x^(1/2
)/(a*x+b*x^n)^(1/2))*x^(1/2)/c^2/(1-n)/(c*x)^(1/2)-2*a*(a*x+b*x^n)^(1/2)/c^
2/(1-n)/(c*x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2053, 2056, 2054, 212}

$$\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a*x + b*x^n])/(c^2*(1 - n)*Sqrt[c*x]) - (2*(a*x + b*x^n)^(3/2))/
(3*c*(1 - n)*(c*x)^(3/2)) + (2*a^(3/2)*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sq
rt[a*x + b*x^n]])/(c^2*(1 - n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx &= -\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx}{c} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a^2\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx}{c^2} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{\left(a^2\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^n}}\,dx}{c^2\sqrt{cx}} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{\left(2a^2\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{2a^{3/2}\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 120, normalized size = 0.98

$$\frac{x\left(8a^2x^2+2b^2x^{2n}+10abx^{1+n}-6a^{3/2}\sqrt{b}\,x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3(-1+n)(cx)^{5/2}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3
+ n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]])
)/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(ax + b\,x^{n})^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax + bx^{n})^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)
```

```
[Out] Integral((a*x + b*x**n)**(3/2)/(c*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^n + a\,x)^{3/2}}{(c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2),x)
[Out] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2), x)
```

3.377 $\int \frac{(a+bx^n)^{3/2}}{cx} dx$

Optimal. Leaf size=73

$$\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

```
[Out] 2/3*(a+b*x^n)^(3/2)/c/n-2*a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n+2*a*
(a+b*x^n)^(1/2)/c/n
```

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 272, 52, 65, 214}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*Ar
cTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^n)^{3/2}}{cx}\,dx &= \frac{\int \frac{(a+bx^n)^{3/2}}{x}\,dx}{c}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^n\right)}{cn}\\
&= \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{cn}\\
&= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}\\
&= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}\\
&= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.79

$$\frac{2\sqrt{a+bx^n}\,(4a+bx^n) - 6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2)/(c*x),x]
```

[Out] (2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(3*c*n)

Maple [A]
time = 0.65, size = 51, normalized size = 0.70

method	result	size
derivativedivides	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	51
default	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	51
risch	$\frac{2(b\,\mathrm{e}^{n\ln(x)}+4a)\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{3nc}-\frac{2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)}{nc}$	59

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^n)^(3/2)/c/x,x,method=_RETURNVERBOSE)

[Out] 1/c/n*(2/3*(a+b*x^n)^(3/2)+2*a*(a+b*x^n)^(1/2)-2*a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))

Maxima [A]
time = 0.49, size = 73, normalized size = 1.00

$$\frac{\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{n}+\frac{2\left((bx^{n}+a)^{\frac{3}{2}}+3\sqrt{bx^{n}+a}\,a\right)}{n}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="maxima")

[Out] 1/3*(3*a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n)/c

Fricas [A]
time = 2.81, size = 120, normalized size = 1.64

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^{n}-2\sqrt{bx^{n}+a}\,\sqrt{a}+2\,a}{x^{n}}\right)+2\,(bx^{n}+4\,a)\sqrt{bx^{n}+a}}{3\,cn},\ \frac{2\left(3\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx^{n}+a}\,\sqrt{-a}}{a}\right)+(bx^{n}+4\,a)\sqrt{bx^{n}+a}\right)}{3\,cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")

```
[Out] [1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*x
^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a)*
sqrt(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]
```

Sympy [**A**]
time = 1.57, size = 88, normalized size = 1.21

$$\frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n}-\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n}+\frac{2\sqrt{a}\,bx^n\sqrt{1+\frac{bx^n}{a}}}{3n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2)/c/x,x)
```

```
[Out] (8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2
)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n)
)/c
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)/(c*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x^n\right)}^{3/2}}{c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)/(c*x),x)
```

```
[Out] int((a + b*x^n)^(3/2)/(c*x), x)
```

3.378 $\int \sqrt{cx} \left(\frac{a}{x}+bx^n\right)^{3/2} dx$

Optimal. Leaf size=117

$$\frac{2a\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{2a^{3/2}c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

```
[Out] 2/3*(c*x)^(3/2)*(a/x+b*x^n)^(3/2)/c/(1+n)-2*a^(3/2)*c*arctanh(a^(1/2)/x^(1/
2)/(a/x+b*x^n)^(1/2))*x^(1/2)/(1+n)/(c*x)^(1/2)+2*a*(c*x)^(1/2)*(a/x+b*x^n)
^(1/2)/(1+n)
```

Rubi [A]
time = 0.20, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2056, 2054, 212}

$$-\frac{2a^{3/2}c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3
/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x
 + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}dx = \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}+(ac)\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}dx$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}+\left(a^2c^2\right)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}dx$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}+\frac{\left(a^2c\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}dx}{\sqrt{cx}}$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{\left(2a^2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}dx,x\right.}{(1+n)\sqrt{cx}}$$

$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+}}\right.}{(1+n)\sqrt{cx}}$$

Mathematica [A]
time = 0.09, size = 97, normalized size = 0.83

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}\left(4a+bx^{1+n}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{3(1+n)\sqrt{a+bx^{1+n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b*x
^(1 + n)])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\, \left(\frac{a}{x}+b\,x^{n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx} \left(\frac{a}{x}+bx^{n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)
[Out] Integral(sqrt(c*x)*(a/x + b*x**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\left(b\,x^n+\frac{a}{x}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2),x)
[Out] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2), x)
```

3.379 $\int c^2 x^2 \left(\frac{a}{x^2}+bx^n\right)^{3/2} dx$

Optimal. Leaf size=98

$$\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

```
[Out] 2/3*c^2*x^3*(a/x^2+b*x^n)^(3/2)/(2+n)-2*a^(3/2)*c^2*arctanh(a^(1/2)/x/(a/x^
2+b*x^n)^(1/2))/(2+n)+2*a*c^2*x*(a/x^2+b*x^n)^(1/2)/(2+n)
```

Rubi [A]
time = 0.21, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {12, 2053, 2032, 2054, 212}

$$-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))
/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2
+ n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
```

```
x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx &= c^2\int x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx\\
&= \frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(ac^2\right)\int\sqrt{\frac{a}{x^2}+bx^n}\,dx\\
&= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(a^2c^2\right)\int\frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx\\
&= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{\left(2a^2c^2\right)\operatorname{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\\
&= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}
\end{aligned}
$$

Mathematica [A]

time = 0.09, size = 94, normalized size = 0.96

$$\frac{2c^2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}\,(4a+bx^{2+n})-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{3(2+n)\sqrt{a+bx^{2+n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) - 3
*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x^(
2 + n)])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int c^2x^2\left(\frac{a}{x^2}+b\,x^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

```
[Out] int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int a\sqrt{\frac{a}{x^2}+bx^n}\,dx+\int bx^2x^n\sqrt{\frac{a}{x^2}+bx^n}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2),x)
```

```
[Out] c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x*
*2 + b*x**n), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int c^2\,x^2\,{\left(b\,x^n+\frac{a}{x^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2*(b*x^n + a/x^2)^(3/2),x)
```

```
[Out] int(c^2*x^2*(b*x^n + a/x^2)^(3/2), x)
```

3.380 $\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx$

Optimal. Leaf size=122

$$\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{2a^{3/2}c^4\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

```
[Out] 2/3*(c*x)^(9/2)*(a/x^3+b*x^n)^(3/2)/c/(3+n)-2*a^(3/2)*c^4*arctanh(a^(1/2)/x
^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/(3+n)/(c*x)^(1/2)+2*a*c^2*(c*x)^(3/2)*(
a/x^3+b*x^n)^(1/2)/(3+n)
```

Rubi [A]
time = 0.41, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2056, 2054, 212}

$$-\frac{2a^{3/2}c^4\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}+\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^(9/2)*(a/x^3 +
 b*x^n)^(3/2))/(3*c*(3 + n)) - (2*a^(3/2)*c^4*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3
/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}dx=\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(ac^3\right)\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(a^2c^6\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+\cdots}}$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\frac{\left(a^2c^4\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+\cdots}}}{\sqrt{cx}}$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{\left(2a^2c^4\sqrt{x}\right)\text{Subst}\left(\int \frac{\cdots}{1\cdots}\cdots\right)}{(3+\cdots}$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\cdots}{x^{3/\cdots}}\cdots\right)}{(3+n)\sqrt{c\cdots}}$$

Mathematica [A]
time = 0.10, size = 100, normalized size = 0.82

$$\frac{2c^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}\left(4a+bx^{3+n}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{3(3+n)\sqrt{a+bx^{3+n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(3
 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sqrt
[a + b*x^(3 + n)])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{2}} \left(\frac{a}{x^3}+b\,x^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/2}\,{\left(b\,x^n+\frac{a}{x^3}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2), x)
```

3.381 $\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$

```
[Out] 2/3*c^5*x^6*(a/x^4+b*x^n)^(3/2)/(4+n)-2*a^(3/2)*c^5*arctanh(a^(1/2)/x^2/(a/
x^4+b*x^n)^(1/2))/(4+n)+2*a*c^5*x^2*(a/x^4+b*x^n)^(1/2)/(4+n)
```

Rubi [A]
time = 0.34, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2053, 2054, 212}

$$-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2
))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])
/(4 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
```

```
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx &= c^5\int x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx \\
&= \frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(ac^5\right)\int x\sqrt{\frac{a}{x^4}+bx^n}\,dx \\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(a^2c^5\right)\int\frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx \\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{\left(2a^2c^5\right)\mathrm{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{}{x^2\sqrt{}}\right.}{4+n} \\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 96, normalized size = 0.96

$$\frac{2c^5x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\sqrt{a+bx^{4+n}}\left(4a+bx^{4+n}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{3(4+n)\sqrt{a+bx^{4+n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]))/(3*(4 + n)*Sqrt[a + b*x
^(4 + n)])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int c^5 x^5 \left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

```
[Out] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^5\left(\int ax\sqrt{\frac{a}{x^4}+bx^n}\,dx+\int bx^5x^n\sqrt{\frac{a}{x^4}+bx^n}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a/
x**4 + b*x**n), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int c^5\,x^5\,{\left(b\,x^n+\frac{a}{x^4}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^5*x^5*(b*x^n + a/x^4)^(3/2),x)
[Out] int(c^5*x^5*(b*x^n + a/x^4)^(3/2), x)
```

3.382 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$

Optimal. Leaf size=51

$$2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x}\right)$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(a/x^2+b/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2004, 2032, 2038, 634, 212}

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
```

```
x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+bx}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+\frac{b}{x}}\,dx\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x + a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x^2}\,dx\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - a\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \frac{1}{x}\right)\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - (2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x}\right)\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 1.14

$$\frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(a + b*x)/x^2]*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]]))/Sqrt[a + b*x]
```

Maple [A]
time = 0.08, size = 47, normalized size = 0.92

method	result	size
default	$\frac{2\sqrt{\frac{bx+a}{x^2}}\, x\left(-\sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\right)}{\sqrt{bx+a}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((b*x+a)/x^2)^(1/2)*x*(-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1
/2))/(b*x+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)/x^2), x)
```

Fricas [A]
time = 2.59, size = 93, normalized size = 1.82

$$\left[2\,x\sqrt{\frac{bx+a}{x^2}}+\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{a}\,x\sqrt{\frac{bx+a}{x^2}}+2\,a}{x}\right),2\,x\sqrt{\frac{bx+a}{x^2}}+2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx+a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x^
2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((
b*x + a)/x^2)/a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)/x**2), x)
```

Giac [A]
time = 0.47, size = 67, normalized size = 1.31

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,\mathrm{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B]
time = 5.18, size = 67, normalized size = 1.31

$$2\,x\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}+\frac{\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/x^2)^(1/2),x)
```

```
[Out] 2*x*(a/x^2 + b/x)^(1/2) + (a^(1/2)*x^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(1/
2)))*(a/x^2 + b/x)^(1/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

3.383 $\int \sqrt{\frac{a+bx^2}{x^2}}\, dx$

Optimal. Leaf size=42

$$\sqrt{b+\frac{a}{x^2}}\, x-\sqrt{a}\, \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}\, x}\right)$$

```
[Out] -arctanh(a^(1/2)/x/(b+a/x^2)^(1/2))*a^(1/2)+x*(b+a/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1997, 248, 283, 223, 212}

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\, \tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{a+bx^2}{x^2}}\,dx &= \int \sqrt{b+\frac{a}{x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{b+ax^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}\,x - a\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}\,x - a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{b+\frac{a}{x^2}}\,x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}\,x - \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}\,x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 1.48

$$\sqrt{b+\frac{a}{x^2}}\,x - \frac{\sqrt{a}\,\sqrt{b+\frac{a}{x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - (Sqrt[a]*Sqrt[b + a/x^2]*x*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/Sqrt[a + b*x^2]
```

Maple [A]
time = 0.05, size = 63, normalized size = 1.50

method	result	size
default	$-\frac{\sqrt{\frac{b\,x^{2}+a}{x^{2}}}\,x\left(\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)-\sqrt{b\,x^{2}+a}\right)}{\sqrt{b\,x^{2}+a}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((b*x^2+a)/x^2)^(1/2)*x*(a^(1/2)*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)-(b*x^
2+a)^(1/2))/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.50, size = 53, normalized size = 1.26

$$\sqrt{b+\frac{a}{x^{2}}}\,x+\frac{1}{2}\,\sqrt{a}\,\log\left(\frac{\sqrt{b+\frac{a}{x^{2}}}\,x-\sqrt{a}}{\sqrt{b+\frac{a}{x^{2}}}\,x+\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b + a/x^2)*x + 1/2*sqrt(a)*log((sqrt(b + a/x^2)*x - sqrt(a))/(sqrt(b +
 a/x^2)*x + sqrt(a)))
```

Fricas [A]
time = 1.24, size = 108, normalized size = 2.57

$$\left[x\sqrt{\frac{bx^{2}+a}{x^{2}}}+\frac{1}{2}\,\sqrt{a}\,\log\left(-\frac{bx^{2}-2\,\sqrt{a}\,x\sqrt{\frac{bx^{2}+a}{x^{2}}}+2\,a}{x^{2}}\right),x\sqrt{\frac{bx^{2}+a}{x^{2}}}+\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx^{2}+a}{x^{2}}}}{bx^{2}+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 + a)/x^2) + 1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(a)*x*sqrt((b*x^
2 + a)/x^2) + 2*a)/x^2), x*sqrt((b*x^2 + a)/x^2) + sqrt(-a)*arctan(sqrt(-a)
*x*sqrt((b*x^2 + a)/x^2)/(b*x^2 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{a+bx^{2}}{x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)/x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.
time = 0.56, size = 69, normalized size = 1.64

$$\frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\sqrt{bx^2+a}\operatorname{sgn}(x)-\frac{\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + sqrt(b*x^2 + a)*sgn(x)
 - (a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B]
time = 5.57, size = 55, normalized size = 1.31

$$x\sqrt{b+\frac{a}{x^2}}+\frac{\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x}\right)\sqrt{b+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\sqrt{\frac{a}{b\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b + a/x^2)^(1/2) + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x))*(b + a/x^2)^(
1/2)*1i)/(b^(1/2)*(a/(b*x^2) + 1)^(1/2))
```

3.384 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

```
[Out] -2/3*arctanh(a^(1/2)/x/(a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(a/x^2+b*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {2004, 2032, 2054, 212}

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x]
)])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx^3}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+bx}\,dx \\
&= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx}}\,dx \\
&= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx}}\right) \\
&= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 1.29

$$
\frac{2x\sqrt{\frac{a}{x^2}+bx}\left(\sqrt{a+bx^3}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^3)/x^2],x]
[Out] (2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]]))/(3*Sqrt[a + b*x^3])
```

Maple [A]
time = 0.06, size = 55, normalized size = 1.08

method	result	size

default	$\frac{2\sqrt{\frac{b\,x^{3}+a}{x^{2}}}\,x\left(-\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{a}+\sqrt{b\,x^{3}+a}\right)}{3\sqrt{b\,x^{3}+a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((b*x^3+a)/x^2)^(1/2)*x*(-arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+(b*x
^3+a)^(1/2))/(b*x^3+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 + a)/x^2), x)
```

Fricas [A]
time = 1.57, size = 104, normalized size = 2.04

$$\left[\frac{2}{3}\,x\sqrt{\frac{bx^{3}+a}{x^{2}}}+\frac{1}{3}\,\sqrt{a}\,\log\left(\frac{bx^{3}-2\,\sqrt{a}\,x\sqrt{\frac{bx^{3}+a}{x^{2}}}+2\,a}{x^{3}}\right),\frac{2}{3}\,x\sqrt{\frac{bx^{3}+a}{x^{2}}}+\frac{2}{3}\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx^{3}+a}{x^{2}}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((b
*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*arct
an(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/a)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 71, normalized size = 1.39

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}}+\frac{2}{3}\,\sqrt{bx^3+a}\,\operatorname{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*sqrt(b*x^3 + a
)*sgn(x) - 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(
-a)
```

Mupad [B]
time = 5.34, size = 63, normalized size = 1.24

$$\frac{2\,x\,\sqrt{b\,x+\frac{a}{x^2}}}{3}+\frac{\sqrt{a}\,\operatorname{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x+\frac{a}{x^2}}\,2\mathrm{i}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{b\,x^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x + a/x^2)^(1/2))/3 + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(3/2)))
*(b*x + a/x^2)^(1/2)*2i)/(3*b^(1/2)*x^(1/2)*(a/(b*x^3) + 1)^(1/2))
```

3.385 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(a/x^2+b*x^(-2
+n))^(1/2)/n
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2004, 2032, 2054, 212}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^(-2 + n)])])/n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
```

```
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+bx^n}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+bx^{-2+n}}\,dx\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\,dx\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.13

$$\frac{2x\sqrt{\frac{a+bx^n}{x^2}}\left(\sqrt{a+bx^n}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[(a + b*x^n)/x^2]*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^
n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

Maple [A]
time = 0.88, size = 74, normalized size = 1.21

method	result	size
risch	$\frac{2\sqrt{\frac{a+b\,\mathrm{e}^{n\ln(x)}}{x^2}}\,x}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)\sqrt{\frac{a+b\,\mathrm{e}^{n\ln(x)}}{x^2}}\,x}{n\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+b*x^n)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*((a+b*exp(n*ln(x)))/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((a+b*exp(n*ln(x)))
^(1/2)/a^(1/2))*((a+b*exp(n*ln(x)))/x^2)^(1/2)/(a+b*exp(n*ln(x)))^(1/2)*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

Fricas [A]
time = 1.68, size = 112, normalized size = 1.84

$$\left[\frac{2\,x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{a}\,\log\left(\frac{bx^n-2\,\sqrt{a}\,x\sqrt{\frac{bx^n+a}{x^2}}+2\,a}{x^n}\right)}{n},\ \frac{2\left(x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx^n+a}{x^2}}}{a}\right)\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n
+ a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sqrt
(-a)*x*sqrt((b*x^n + a)/x^2)/a))/n]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{a+bx^n}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x**n)/x**2)**(1/2),x)
[Out] Integral(sqrt((a + b*x**n)/x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{a+b\,x^n}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^n)/x^2)^(1/2),x)
[Out] int(((a + b*x^n)/x^2)^(1/2), x)
```

3.386 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$

Optimal. Leaf size=53

$$2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x+2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x}\right)$$

```
[Out] 2*arctan(a^(1/2)/x/(-a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(-a/x^2+b/x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2004, 2032, 2038, 634, 209}

$$2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x
)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-a+bx}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,dx\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x - a\int \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x^2}\,dx\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + a\text{Subst}\left(\int \frac{1}{\sqrt{bx-ax^2}}\,dx, x, \frac{1}{x}\right)\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + (2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x}\right)\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 1.25

$$\frac{2x\sqrt{\frac{-a+bx}{x^2}}\left(\sqrt{-a+bx}-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{-a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x)/x^2],x]
```

[Out] (2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]))/Sqrt[-a + b*x]

Maple [A]
time = 0.10, size = 55, normalized size = 1.04

method	result	size
default	$\frac{2\sqrt{-\frac{-bx+a}{x^{2}}}\, x\left(-\sqrt{a}\, \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\right)}{\sqrt{bx-a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(-(-b*x+a)/x^2)^(1/2)*x*(-a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))+(b*x-a)^(1/2))/(b*x-a)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((b*x - a)/x^2), x)

Fricas [A]
time = 1.33, size = 98, normalized size = 1.85

$$\left[2\,x\sqrt{\frac{bx-a}{x^{2}}}+\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{-a}\,x\sqrt{\frac{bx-a}{x^{2}}}-2\,a}{x}\right),2\,x\sqrt{\frac{bx-a}{x^{2}}}-2\,\sqrt{a}\,\arctan\left(\frac{x\sqrt{\frac{bx-a}{x^{2}}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")

[Out] [2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)/x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) - 2*sqrt(a)*arctan(x*sqrt((b*x - a)/x^2)/sqrt(a))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{-a+bx}{x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x)/x**2), x)
```

Giac [A]
time = 0.48, size = 61, normalized size = 1.15

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\operatorname{sgn}(x)+2\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x)+2\sqrt{bx-a}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a))*sgn(x) + 2*(sqrt(a)*arctan(sqrt(-a
)/sqrt(a)) - sqrt(-a))*sgn(x) + 2*sqrt(b*x - a)*sgn(x)
```

Mupad [B]
time = 5.18, size = 67, normalized size = 1.26

$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}+\frac{2\sqrt{a}\sqrt{x}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)\sqrt{\frac{b}{x}-\frac{a}{x^2}}}{\sqrt{b}\sqrt{1-\frac{a}{b\,x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x)/x^2)^(1/2),x)
```

```
[Out] 2*x*(b/x - a/x^2)^(1/2) + (2*a^(1/2)*x^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(1/2))
)*(b/x - a/x^2)^(1/2))/(b^(1/2)*(1 - a/(b*x))^(1/2))
```

3.387 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$

Optimal. Leaf size=43

$$\sqrt{b-\frac{a}{x^2}}\,x+\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}\,x}\right)$$

```
[Out] arctan(a^(1/2)/x/(b-a/x^2)^(1/2))*a^(1/2)+x*(b-a/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1997, 248, 283, 223, 209}

$$\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)+x\sqrt{b-\frac{a}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx &= \int \sqrt{b-\frac{a}{x^2}}\,dx\\ &= -\text{Subst}\left(\int \frac{\sqrt{b-ax^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{b-\frac{a}{x^2}}\,x + a\text{Subst}\left(\int \frac{1}{\sqrt{b-ax^2}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{b-\frac{a}{x^2}}\,x + a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{b-\frac{a}{x^2}}\,x}\right)\\ &= \sqrt{b-\frac{a}{x^2}}\,x + \sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}\,x}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 1.58

$$\sqrt{b-\frac{a}{x^2}}\,x - \frac{\sqrt{a}\,\sqrt{b-\frac{a}{x^2}}\,x\tan^{-1}\left(\frac{\sqrt{-a+bx^2}}{\sqrt{a}}\right)}{\sqrt{-a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x - (Sqrt[a]*Sqrt[b - a/x^2]*x*ArcTan[Sqrt[-a + b*x^2]/Sqrt
[a]])/Sqrt[-a + b*x^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.
time = 0.06, size = 81, normalized size = 1.88

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,x\left(\sqrt{-a}\,\sqrt{b\,x^{2}-a}+a\ln\left(\frac{-2a+2\sqrt{-a}\,\sqrt{b\,x^{2}-a}}{x}\right)\right)}{\sqrt{-a}\,\sqrt{b\,x^{2}-a}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*x^2+a)/x^2)^(1/2)*x*((-a)^(1/2)*(b*x^2-a)^(1/2)+a*ln(2*((-a)^(1/2)*(b
*x^2-a)^(1/2)-a)/x))/(-a)^(1/2)/(b*x^2-a)^(1/2)
```

Maxima [A]
time = 0.50, size = 34, normalized size = 0.79

$$\sqrt{b-\frac{a}{x^{2}}}\,x-\sqrt{a}\,\arctan\left(\frac{\sqrt{b-\frac{a}{x^{2}}}\,x}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b - a/x^2)*x - sqrt(a)*arctan(sqrt(b - a/x^2)*x/sqrt(a))
```

Fricas [A]
time = 2.21, size = 118, normalized size = 2.74

$$\left[x\sqrt{\frac{bx^{2}-a}{x^{2}}}+\frac{1}{2}\sqrt{-a}\,\log\left(-\frac{bx^{2}-2\sqrt{-a}\,x\sqrt{\frac{bx^{2}-a}{x^{2}}}-2\,a}{x^{2}}\right),x\sqrt{\frac{bx^{2}-a}{x^{2}}}+\sqrt{a}\,\arctan\left(\frac{\sqrt{a}\,x\sqrt{\frac{bx^{2}-a}{x^{2}}}}{bx^{2}-a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 - a)/x^2) + 1/2*sqrt(-a)*log(-(b*x^2 - 2*sqrt(-a)*x*sqrt((b*
x^2 - a)/x^2) - 2*a)/x^2), x*sqrt((b*x^2 - a)/x^2) + sqrt(a)*arctan(sqrt(a)
*x*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{-a+bx^{2}}{x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**2)/x**2), x)
```

Giac [A]
time = 0.44, size = 63, normalized size = 1.47

$$-\sqrt{a}\arctan\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)\operatorname{sgn}(x)+\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x)+\sqrt{bx^2-a}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a))*sgn(x) + (sqrt(a)*arctan(sqrt(-a)/
sqrt(a)) - sqrt(-a))*sgn(x) + sqrt(b*x^2 - a)*sgn(x)
```

Mupad [B]
time = 5.38, size = 54, normalized size = 1.26

$$x\sqrt{b-\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)\sqrt{b-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b - a/x^2)^(1/2) + (a^(1/2)*asin(a^(1/2)/(b^(1/2)*x))*(b - a/x^2)^(1/2))
/(b^(1/2)*(1 - a/(b*x^2))^(1/2))
```

3.388 $\int \sqrt{\frac{-a+bx^3}{x^2}}\, dx$

Optimal. Leaf size=53

$$\frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx}+\frac{2}{3}\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)$$

```
[Out] 2/3*arctan(a^(1/2)/x/(-a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(-a/x^2+b*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2004, 2032, 2054, 209}

$$\frac{2}{3}\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)+\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) +
 b*x])])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+bx}\,dx\\ &= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx}}\,dx\\ &= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx}}\right)\\ &= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{2}{3}\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 1.38

$$\frac{2x\sqrt{-\frac{a}{x^2}+bx}\left(\sqrt{-a+bx^3}-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^3)/x^2],x]
[Out] (2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x^
3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

Maple [A]
time = 0.10, size = 73, normalized size = 1.38

method	result	size

default	$\frac{2\sqrt{-\frac{-b\,x^3+a}{x^2}}\,x\left(\sqrt{b\,x^3-a}\,\sqrt{-a}+a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3-a}}{\sqrt{-a}}\right)\right)}{3\sqrt{b\,x^3-a}\,\sqrt{-a}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-(-b*x^3+a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3-
a)^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 - a)/x^2), x)
```

Fricas [A]
time = 1.83, size = 109, normalized size = 2.06

$$\left[\frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}}+\frac{1}{3}\,\sqrt{-a}\,\log\left(\frac{bx^3-2\,\sqrt{-a}\,x\sqrt{\frac{bx^3-a}{x^2}}-2\,a}{x^3}\right),\frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}}-\frac{2}{3}\,\sqrt{a}\,\arctan\left(\frac{x\sqrt{\frac{bx^3-a}{x^2}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt(
(b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) - 2/3*sqrt(a)*arc
tan(x*sqrt((b*x^3 - a)/x^2)/sqrt(a))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3-a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 65, normalized size = 1.23

$$-\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\operatorname{sgn}(x)+\frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x)+\frac{2}{3}\sqrt{bx^3-a}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a))*sgn(x) + 2/3*(sqrt(a)*arctan(s
qrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2/3*sqrt(b*x^3 - a)*sgn(x)
```

Mupad [B]
time = 5.35, size = 63, normalized size = 1.19

$$\frac{2x\sqrt{bx-\frac{a}{x^2}}}{3}+\frac{2\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{3/2}}\right)\sqrt{bx-\frac{a}{x^2}}}{3\sqrt{b}\,\sqrt{x}\,\sqrt{1-\frac{a}{b\,x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x - a/x^2)^(1/2))/3 + (2*a^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(3/2)))*(b
*x - a/x^2)^(1/2))/(3*b^(1/2)*x^(1/2)*(1 - a/(b*x^3))^(1/2))
```

3.389 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n}+\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

```
[Out] 2*arctan(a^(1/2)/x/(-a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(-a/x^2+b*x^(-2
+n))^(1/2)/n
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2004, 2032, 2054, 209}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}+\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-
(a/x^2) + b*x^(-2 + n)])])/n
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
```

```
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+bx^{-2+n}}\,dx\\ &= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\,dx\\ &= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{(2a)\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\\ &= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 1.22

$$\frac{2x\sqrt{\frac{-a+bx^n}{x^2}}\left(\sqrt{-a+bx^n}-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{-a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[(-a + b*x^n)/x^2]*(Sqrt[-a + b*x^n] - Sqrt[a]*ArcTan[Sqrt[-a + b*
x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

Maple [A]
time = 0.74, size = 105, normalized size = 1.67

method	result	size
risch	$-\frac{2\left(a-b\,\mathrm{e}^{n\ln(x)}\right)\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}-a}{x^2}}\,x}{n\left(b\,\mathrm{e}^{n\ln(x)}-a\right)}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{b\,\mathrm{e}^{n\ln(x)}-a}}{\sqrt{a}}\right)\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}-a}{x^2}}\,x}{n\sqrt{b\,\mathrm{e}^{n\ln(x)}-a}}$	105

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^n-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(a-b*exp(n*ln(x)))/n/(b*exp(n*ln(x))-a)*((b*exp(n*ln(x))-a)/x^2)^(1/2)*x -2*a^(1/2)/n*arctan((b*exp(n*ln(x))-a)^(1/2)/a^(1/2))*((b*exp(n*ln(x))-a)/x ^2)^(1/2)/(b*exp(n*ln(x))-a)^(1/2)*x

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((b*x^n - a)/x^2), x)

Fricas [A]
time = 2.32, size = 118, normalized size = 1.87

$$\left[\frac{2\,x\sqrt{\frac{bx^n-a}{x^2}}+\sqrt{-a}\,\log\left(\frac{bx^n-2\sqrt{-a}\,x\sqrt{\frac{bx^n-a}{x^2}}-2\,a}{x^n}\right)}{n},\ \frac{2\left(x\sqrt{\frac{bx^n-a}{x^2}}-\sqrt{a}\,\arctan\left(\frac{x\sqrt{\frac{bx^n-a}{x^2}}}{\sqrt{a}}\right)\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")

[Out] [(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x^ n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) - sqrt(a)*arctan(x*s qrt((b*x^n - a)/x^2)/sqrt(a)))/n]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{-a+bx^n}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**n)/x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-\frac{a-b\,x^n}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^n)/x^2)^(1/2),x)
```

```
[Out] int((-(a - b*x^n)/x^2)^(1/2), x)
```

3.390 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$

Optimal. Leaf size=62

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}$$

```
[Out] 2*(c*x)^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/(x^(
1/2*j))/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2056, 2054, 212}

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(
j - n)*x^(j/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \frac{\left(x^{-j/2}(cx)^{j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}$$
$$= \frac{\left(2x^{-j/2}(cx)^{j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$
$$= \frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}$$

Mathematica [A]
time = 0.13, size = 98, normalized size = 1.58

$$\frac{2\sqrt{b}\,x^{\frac{1}{2}(-j+n)}(cx)^{j/2}\sqrt{1+\frac{ax^{j-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
[Out] (2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sq
rt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
[Out] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{\frac{j}{2}-1}}{\sqrt{a\,x^j + b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2), x)
```

3.391 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}$$

```
[Out] 2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/(3-n)/a^(1/2)/x^
(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2056, 2054, 212}

$$\frac{2\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 -
n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{\sqrt{x}}$$

$$= \frac{\left(2\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{(3-n)\sqrt{x}}$$

$$= \frac{2\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}$$

Mathematica [A]
time = 0.30, size = 89, normalized size = 1.68

$$-\frac{2\sqrt{b}\,x^{\frac{1}{2}(-1+n)}\sqrt{cx}\sqrt{1+\frac{ax^{3-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(-3+n)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
[Out] (-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqr
t[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{a\,x^3+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
[Out] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x}}{\sqrt{b\,x^n + a\,x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2), x)
```

3.392 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}$$

```
[Out] 2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/(2-n)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2033, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.
time = 0.06, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{b}\,x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(-2+n)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^n],x]
[Out] (-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))
/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,x^2+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^n)^(1/2),x)
[Out] int(1/(a*x^2+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

Mupad [B]
time = 5.39, size = 67, normalized size = 1.81

$$\frac{\sqrt{b}\ x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\ x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{b}}\right)\ \sqrt{\frac{a\,x^{2-n}}{b}+1}\ \ 1\mathrm{i}}{\sqrt{a}\ \left(\frac{n}{2}-1\right)\ \sqrt{b\,x^n+a\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n + a*x^2)^(1/2),x)
```

```
[Out] (b^(1/2)*x^(n/2)*asin((a^(1/2)*x^(1 - n/2)*1i)/b^(1/2))*((a*x^(2 - n))/b +
1)^(1/2)*1i)/(a^(1/2)*(n/2 - 1)*(b*x^n + a*x^2)^(1/2))
```

3.393 $\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}$$

[Out] 2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/(1-n)/a^(1/2)/(c*x)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2056, 2054, 212}

$$\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]

[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*Sqrt[c*x])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx = \frac{\sqrt{x}\,\int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^n}}\,dx}{\sqrt{cx}}$$
$$= \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{(1-n)\sqrt{cx}}$$
$$= \frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}$$

Mathematica [A]
time = 0.19, size = 87, normalized size = 1.71

$$-\frac{2\sqrt{b}\,x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(-1+n)\sqrt{cx}\,\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2
 - n/2))/Sqrt[b]])/(Sqrt[a]*(-1 + n)*Sqrt[c*x]*Sqrt[a*x + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^n+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)), x)
```

3.394 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \frac{\int \frac{1}{x\sqrt{a+bx^n}}\,dx}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Mathematica [**A**]
time = 0.03, size = 31, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Maple [**A**]
time = 0.69, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{cn\sqrt{a}}$	26
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{cn\sqrt{a}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

Maxima [A]
time = 0.52, size = 42, normalized size = 1.35

$$\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*c*n)
```

Fricas [A]
time = 1.44, size = 76, normalized size = 2.45

$$\left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}\,cn}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{acn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(-
a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*c*n)]
```

Sympy [A]
time = 0.56, size = 27, normalized size = 0.87

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x**n)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(sqrt(a)*c*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*c*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{c\,x\,\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x^n)^(1/2)),x)
[Out] int(1/(c*x*(a + b*x^n)^(1/2)), x)
```

3.395 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(1+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/c/(1+n)/a^(1/2)/(c*x)
^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2056, 2054, 212}

$$-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 + n
)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
```

```
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{c\sqrt{cx}}$$

$$= -\frac{\left(2\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{c(1+n)\sqrt{cx}}$$

$$= -\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(1+n)\sqrt{cx}}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{1+n}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)}{\sqrt{a}\,(1+n)(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[a
]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{\frac{a}{x}+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{\frac{a}{x} + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{3/2}\,\sqrt{b\,x^n+\frac{a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)),x)
[Out] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)), x)
```

3.396 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(2+n)}$$

[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/c^2/(2+n)/a^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {12, 2054, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(n+2)}$$

Antiderivative was successfully verified.

[In] Int[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]

[Out] (-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{c^2(2+n)}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(2+n)}$$

Mathematica [**A**]
time = 0.06, size = 66, normalized size = 1.65

$$-\frac{2\sqrt{a+bx^{2+n}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
[Out] (-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]*
c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
[Out] int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{c^2\,x^2\,\sqrt{b\,x^n+\frac{a}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)),x)
[Out] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)), x)
```

3.397 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(3+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/c^2/(3+n)/a^(1/2)/(
c*x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2056, 2054, 212}

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(3
 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
```

```
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{c^2\sqrt{cx}}$$

$$= -\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{c^2(3+n)\sqrt{cx}}$$

$$= -\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(3+n)\sqrt{cx}}$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{3+n}}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)}{\sqrt{a}\,(3+n)(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
[Out] (-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[a
]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{\frac{a}{x^3}+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/((c*x)**(5/2)*sqrt(a/x**3 + b*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,\sqrt{b\,x^n+\frac{a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)),x)
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)), x)
```

3.398 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}$$

```
[Out] 2*(c*x)^(3/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/a^(3/2)/c/(j
-n)/(x^(3/2*j))-2*(c*x)^(3/2*j)/a/c/(j-n)/(x^j)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2056, 2055, 2054, 212}

$$\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2))/(a*c*(j - n)*x^j*Sqrt[a*x^j + b*x^n]) + (2*(c*x)^((3*j
)/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*c*(j - n)*x^(
(3*j)/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
```

```
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx &= \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx}{c}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{ac}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{\left(2x^{-3j/2}(cx)^{3j/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{ac(j-n)}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 117, normalized size = 1.09

$$-\frac{2x^{-3j/2}(cx)^{3j/2}\left(\sqrt{a}\,x^{j/2}-\sqrt{b}\,x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j - n
))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(a^(3/2)*c*(j - n)*x^((3*j
)/2)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(a\,x^{j}+b\,x^{n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3j}{2}-1}}{(ax^{j}+bx^{n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**(3*j/2 - 1)/(a*x**j + b*x**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{\frac{3\,j}{2}-1}}{\left(a\,x^j+b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2),x)
[Out] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2), x)
```

3.399 $\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{2c^3\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}$$

```
[Out] 2*c^3*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/a^(3/2)/(3-n
)/x^(1/2)-2*c^2*(c*x)^(3/2)/a/(3-n)/(a*x^3+b*x^n)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2055, 2056, 2054, 212}

$$\frac{2c^3\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
```

```
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{c^3\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{a}\\ &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(c^3\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{a\sqrt{x}}\\ &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(2c^3\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a(3-n)\sqrt{x}}\\ &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{2c^3\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 1.64, size = 109, normalized size = 1.16

$$\frac{2c^3\sqrt{cx}\left(\sqrt{a}\,x^{3/2}-\sqrt{b}\,x^{n/2}\sqrt{1+\frac{ax^{3-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-3+n)\sqrt{x}\,\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-3 + n)*Sqrt[x]*Sqr
t[a*x^3 + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{(a\,x^3+b\,x^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{(b\,x^n+a\,x^3)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2), x)
```

3.400 $\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}$$

```
[Out] 2*c^2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/a^(3/2)/(2-n)-2*c^2*x/a/(2-n)/
(a*x^2+b*x^n)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2055, 2033, 212}

$$\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*c^2*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(a^(3/2)*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx &= c^2\int \frac{x^2}{(ax^2+bx^n)^{3/2}}\,dx\\ &= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{c^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{a}\\ &= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{(2c^2)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^n}}\right)}{a(2-n)}\\ &= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 91, normalized size = 1.26

$$\frac{2c^2\left(\sqrt{a}\,x-\sqrt{b}\,x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-2+n)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt
[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2}{\left(a\,x^2+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

```
[Out] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^2}{ax^2\sqrt{ax^2+bx^n}+bx^n\sqrt{ax^2+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)
```

```
[Out] c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*x
**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c^2\,x^2}{\left(b\,x^n+a\,x^2\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2),x)
[Out] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2), x)
```

3.401 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{2c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}$$

```
[Out] 2*c*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/(1-n)/(c*x)^
(1/2)-2*(c*x)^(1/2)/a/(1-n)/(a*x+b*x^n)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2055, 2056, 2054, 212}

$$\frac{2c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[a
]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
```

```
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx &= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{c\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx}{a}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{\left(c\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^n}}\,dx}{a\sqrt{cx}}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{\left(2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a(1-n)\sqrt{cx}}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{2c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 104, normalized size = 1.22

$$\frac{2\sqrt{cx}\left(\sqrt{a}\,\sqrt{x}-\sqrt{b}\,x^{n/2}\sqrt{1+\frac{ax^{1-n}}{b}}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-1+n)\sqrt{x}\,\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[a*
x + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax+b\,x^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax + bx^n)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)
```

```
[Out] Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^n+a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2),x)
[Out] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2), x)
```

3.402 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/c/n+2/a/c/n/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 272, 53, 65, 214}

$$\frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx &= \frac{\int \frac{1}{x(a+bx^n)^{3/2}}\,dx}{c} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^n\right)}{cn} \\
&= \frac{2}{acn\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{acn} \\
&= \frac{2}{acn\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{abcn} \\
&= \frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.96

$$\frac{\frac{2}{an\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] (2/(a*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*n)
)/c
```

Maple [A]
time = 0.42, size = 42, normalized size = 0.78

method	result	size
derivativedivides	$\frac{-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\,x^{n}}}}{cn}$	42
default	$\frac{-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\,x^{n}}}}{cn}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(a+b*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/n*(-2/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))+2/a/(a+b*x^n)^(1/2))
```

Maxima [A]
time = 0.50, size = 61, normalized size = 1.13

$$\frac{\frac{\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n}+\frac{2}{\sqrt{bx^{n}+a}\,an}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n) +
 2/(sqrt(b*x^n + a)*a*n))/c
```

Fricas [A]
time = 1.62, size = 148, normalized size = 2.74

$$\left[\frac{\left(\sqrt{a}\,bx^{n}+a^{\frac{3}{2}}\right)\log\left(\frac{bx^{n}-2\sqrt{bx^{n}+a}\,\sqrt{a}+2a}{x^{n}}\right)+2\sqrt{bx^{n}+a}\,a}{a^{2}bcnx^{n}+a^{3}cn}, \frac{2\left(\left(\sqrt{-a}\,bx^{n}+\sqrt{-a}\,a\right)\arctan\left(\frac{\sqrt{bx^{n}+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx^{n}+a}\,a\right)}{a^{2}bcnx^{n}+a^{3}cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x
^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*c*n*x^n + a^3*c*n), 2*((sqrt(-a)*b*x^n +
```

```
sqrt(-a)*a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a)*a)/(a^2*b*
c*n*x^n + a^3*c*n)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(42) = 84$.
time = 1.14, size = 185, normalized size = 3.43

$$\frac{\frac{2a^3\sqrt{1+\frac{bx^n}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^3\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^2bx^n\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}-\frac{2a^2bx^n\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x**n)**(3/2),x)
```

```
[Out] (2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x*
*n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) + 1)
/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*n +
 a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/2)*n
 + a**(7/2)*b*n*x**n))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{c\,x\,{\left(a+b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x^n)^(3/2)),x)
```

```
[Out] int(1/(c*x*(a + b*x^n)^(3/2)), x)
```

3.403 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2}{ac^2(1+n)\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^2/(1+n)/(c*
x)^(1/2)+2/a/c^2/(1+n)/(c*x)^(1/2)/(a/x+b*x^n)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2055, 2056, 2054, 212}

$$\frac{2}{ac^2(n+1)\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/
(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac^2\sqrt{cx}}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{ac^2(1+n)\sqrt{cx}}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}$$

Mathematica [A]

time = 0.08, size = 78, normalized size = 0.87

$$\frac{2\left(\sqrt{a}-\sqrt{a+bx^{1+n}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^{2}(1+n)\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^{n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])
)/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\left(\frac{a}{x}+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\left(\frac{a}{x}+bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)
[Out] Integral(1/((c*x)**(5/2)*(a/x + b*x**n)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,{\left(b\,x^n+\frac{a}{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)),x)
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)), x)
```

3.404 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/a^(3/2)/c^4/(2+n)+2/a/c^4/(2+n)/x
/(a/x^2+b*x^n)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2055, 2054, 212}

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2
+ b*x^n])])/(a^(3/2)*c^4*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx}{c^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} + \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{ac^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{ac^4(2+n)}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

Mathematica [A]

time = 0.08, size = 74, normalized size = 1.03

$$\frac{2\left(\sqrt{a}-\sqrt{a+bx^{2+n}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])
)/(a^(3/2)*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

```
[Out] int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^2\sqrt{\frac{a}{x^2}+bx^n}+bx^4x^n\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**n
)), x)/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{c^4\,x^4\,{\left(b\,x^n+\frac{a}{x^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)),x)
```

```
[Out] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)), x)
```

3.405 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^5/(3+n)/(
c*x)^(1/2)+2/a/c^4/(3+n)/(c*x)^(3/2)/(a/x^3+b*x^n)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2055, 2056, 2054, 212}

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt
[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} + \frac{\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^3}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} + \frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^5\sqrt{cx}}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} - \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+b}}\right.}{ac^5(3+n)\sqrt{cx}}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} - \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}$$

Mathematica [A]

time = 0.09, size = 78, normalized size = 0.87

$$\frac{2\left(\sqrt{a}-\sqrt{a+bx^{3+n}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])
)/(a^(3/2)*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}}\left(\frac{a}{x^3}+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

```
[Out] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{11/2}\,\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)),x)
[Out] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)), x)
```

3.406 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}$$

```
[Out] -2*arctanh(a^(1/2)/x^2/(a/x^4+b*x^n)^(1/2))/a^(3/2)/c^7/(4+n)+2/a/c^7/(4+n)
/x^2/(a/x^4+b*x^n)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2055, 2054, 212}

$$\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/
x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{c^7 x^7 \left(\frac{a}{x^4}+b x^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{x^7\left(\frac{a}{x^4}+b x^n\right)^{3/2}}\,dx}{c^7}$$

$$= \frac{2}{a c^7 (4+n) x^2 \sqrt{\frac{a}{x^4}+b x^n}} + \frac{\int \frac{1}{x^3\sqrt{\frac{a}{x^4}+b x^n}}\,dx}{a c^7}$$

$$= \frac{2}{a c^7 (4+n) x^2 \sqrt{\frac{a}{x^4}+b x^n}} - \frac{2\text{Subst}\left(\int \frac{1}{1-a x^2}\,dx, x, \frac{1}{x^2\sqrt{\frac{a}{x^4}+b x^n}}\right)}{a c^7 (4+n)}$$

$$= \frac{2}{a c^7 (4+n) x^2 \sqrt{\frac{a}{x^4}+b x^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+b x^n}}\right)}{a^{3/2} c^7 (4+n)}$$

Mathematica [A]

time = 0.08, size = 74, normalized size = 1.03

$$\frac{2\left(\sqrt{a}-\sqrt{a+bx^{4+n}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(4 + n)]*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]])
)/(a^(3/2)*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n])
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

```
[Out] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*x^7), x)/c^7
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^3\sqrt{\frac{a}{x^4}+bx^n}+bx^7x^n\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**n
)), x)/c**7
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{c^7\,x^7\,{\left(b\,x^n+\frac{a}{x^4}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)),x)
```

```
[Out] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)), x)
```

3.407 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

[Out] 2/3*arctanh(x*b^(1/2)/(a/x+b*x^2)^(1/2))/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2004, 2033, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x^3)/x],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x}+bx^2}}\,dx \\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 1.97

$$
\frac{2\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{3\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a+bx^3}{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/(Sqrt[b]*x^(3/2))])/(3*Sqrt[b]*S
qrt[x]*Sqrt[(a + b*x^3)/x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.41, size = 477, normalized size = 14.91

method	result
default	$-\frac{4(b\,x^3+a)\left(-1+i\sqrt{3}\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)xb}{\left(-1+i\sqrt{3}\right)\left(-bx+(-a\,b^2)^{\frac{1}{3}}\right)}}\,\left(-bx+(-a\,b^2)^{\frac{1}{3}}\right)^2\sqrt{\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}+2bx+(-a\,b^2)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+(-a\,b^2)^{\frac{1}{3}}\right)}}\,\sqrt{\cdots}}{b^2\sqrt{\frac{b\,x^3+a}{x}}\,\sqrt{x\,(b\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(b*x^3+a)*(-1+I*3^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*(-b*x+(-a*b^2)^(1/3))^2*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
/b^2*(EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-Elli
pticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+
I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2)))/((b*x^3+a)/x)^(1/2)/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b
^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^3 + a)/x), x)
```

Fricas [A]
time = 1.71, size = 102, normalized size = 3.19

$$\left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\,(2\,bx^5+ax^2)\sqrt{b}\,\sqrt{\frac{bx^3+a}{x}}\right)}{6\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{-b}\,x^2\sqrt{\frac{bx^3+a}{x}}}{2\,bx^3+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((b
*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(-b)*x^2*sqrt((b*x^3 + a)
/x)/(2*b*x^3 + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x**3)/x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^3+a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)/x)^(1/2),x)
```

```
[Out] int(1/((a + b*x^3)/x)^(1/2), x)
```

3.408 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*arctanh(x*b^(1/2)/(a/x^2+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2004, 2033, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^2}+bx^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 59, normalized size = 1.84

$$\frac{\sqrt{a+bx^4}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{b}\,x^2}\right)}{2\sqrt{b}\,x\sqrt{\frac{a+bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] (Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*x^4]/(Sqrt[b]*x^2)])/(2*Sqrt[b]*x*Sqrt[
(a + b*x^4)/x^2])
```

Maple [**A**]
time = 0.06, size = 49, normalized size = 1.53

method	result	size
default	$\frac{\sqrt{b\,x^4+a}\,\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2\sqrt{\frac{b\,x^4+a}{x^2}}\,x\sqrt{b}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((b*x^4+a)/x^2)^(1/2)/x*(b*x^4+a)^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
[Out] b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)
```

Fricas [A]
time = 1.68, size = 80, normalized size = 2.50

$$\left[\frac{\log\left(-2\,bx^4-2\,\sqrt{b}\,x^3\sqrt{\frac{bx^4+a}{x^2}}-a\right)}{4\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^3\sqrt{\frac{bx^4+a}{x^2}}}{bx^4+a}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2*
sqrt(-b)*arctan(sqrt(-b)*x^3*sqrt((b*x^4 + a)/x^2)/(b*x^4 + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a)/x**2)**(1/2),x)
[Out] Integral(1/sqrt((a + b*x**4)/x**2), x)
```

Giac [A]
time = 0.54, size = 40, normalized size = 1.25

$$\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{4\,\sqrt{b}}-\frac{\log\left(\left|-\sqrt{b}\,x^2+\sqrt{bx^4+a}\right|\right)}{2\,\sqrt{b}\,\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)
))/(sqrt(b)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^4+a}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)/x^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^4)/x^2)^(1/2), x)
```

3.409 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}} \, dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

[Out] 2/5*arctanh(x*b^(1/2)/(a/x^3+b*x^2)^(1/2))/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2004, 2033, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x^5)/x^3],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}+bx^2}}\,dx\\ &= \frac{2}{5}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}\end{aligned}$$

Mathematica [**A**]
time = 0.19, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^5}\tanh^{-1}\left(\frac{\sqrt{a+bx^5}}{\sqrt{b}\,x^{5/2}}\right)}{5\sqrt{b}\,x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^5)/x^3],x]
[Out] (2*Sqrt[a + b*x^5]*ArcTanh[Sqrt[a + b*x^5]/(Sqrt[b]*x^(5/2))])/(5*Sqrt[b]*x
^(3/2)*Sqrt[(a + b*x^5)/x^3])
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^5+a)/x^3)^(1/2),x)
[Out] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((b*x^5 + a)/x^3), x)
```

Fricas [A]
time = 3.92, size = 102, normalized size = 3.19

$$\left[\frac{\log\left(-8\,b^2x^{10}-8\,abx^5-a^2-4\,(2\,bx^9+ax^4)\sqrt{b}\,\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{-b}\,x^4\sqrt{\frac{bx^5+a}{x^3}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
[Out] [1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt(
(b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^5
+ a)/x^3)/(2*b*x^5 + a))/b]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**5+a)/x**3)**(1/2),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^5)/x^3)^(1/2),x)
```

```
[Out] int(1/((a + b*x^5)/x^3)^(1/2), x)
```

3.410 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2004, 2033, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx = \int \frac{1}{\sqrt{bx^2+ax^{2-n}}}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^{2-n}}}\right)}{n}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.
time = 0.05, size = 76, normalized size = 2.05

$$\frac{2\sqrt{a}\,x^{1-\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,n\sqrt{x^{2-n}(a+bx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}(a+b\,x^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

```
[Out] int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

Fricas [A]
time = 2.16, size = 102, normalized size = 2.76

$$\left[\frac{\log\left(\frac{2\,bxx^n+ax+2\,\sqrt{b}\,x^n\sqrt{\frac{bx^2x^n+ax^2}{x^n}}}{x}\right)}{\sqrt{b}\,n}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{bx^2x^n+ax^2}{x^n}}}{bx}\right)}{bn} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(sq
rt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^2*x^n + a*x^2)/x^n)/(b*x))/
(b*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\,(a+bx^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**(2 - n)*(a + b*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^{2-n}\,(a+b\,x^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2 - n)*(a + b*x^n))^(1/2),x)
[Out] int(1/(x^(2 - n)*(a + b*x^n))^(1/2), x)
```

3.411 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}} dx$

Optimal. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctan(x*b^(1/2)/(a/x-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2004, 2033, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a}{x}-bx^2}}\,dx$$
$$= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}-bx^2}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

Mathematica [A]
time = 0.14, size = 66, normalized size = 2.00

$$-\frac{2\sqrt{a-bx^3}\,\tan^{-1}\left(\frac{\sqrt{a-bx^3}}{\sqrt{b}\,x^{3/2}}\right)}{3\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a-bx^3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (-2*Sqrt[a - b*x^3]*ArcTan[Sqrt[a - b*x^3]/(Sqrt[b]*x^(3/2))])/(3*Sqrt[b]*S
qrt[x]*Sqrt[(a - b*x^3)/x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.05, size = 466, normalized size = 14.12

method	result
default	$-\frac{4\left(-b\,x^{3}+a\right)\left(1+i\sqrt{3}\right)\sqrt{-\frac{\left(i\sqrt{3}+3\right)xb}{\left(1+i\sqrt{3}\right)\left(-bx+\left(a\,b^{2}\right)^{\frac{1}{3}}\right)}}\,\left(-bx+\left(a\,b^{2}\right)^{\frac{1}{3}}\right)^{2}\sqrt{\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(a\,b^{2}\right)^{\frac{1}{3}}}{\left(-1+i\sqrt{3}\right)\left(-bx+\left(a\,b^{2}\right)^{\frac{1}{3}}\right)}}\,\sqrt{\frac{i\sqrt{3}\,(\cdots}{(1+i\sqrt{\cdots}}}\cdots}{b^{2}\sqrt{\frac{-b\,x^{3}+a}{x}}\,\sqrt{x\,(-b\,x^{3}\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(-b*x^3+a)*(1+I*3^(1/2))*(-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)
^(1/3)))^(1/2)*(-b*x+(a*b^2)^(1/3))^2*((I*3^(1/2)*(a*b^2)^(1/3)-2*b*x-(a*b^
2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(a*b^2)^(1
/3)+2*b*x+(a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2)/b^2*(Ell
ipticF((-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)-3)*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-EllipticPi((-(I
*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2),(1+I*3^(1/2))/(I*
3^(1/2)+3),((I*3^(1/2)-3)*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
))/((-b*x^3+a)/x)^(1/2)/(x*(-b*x^3+a))^(1/2)/(I*3^(1/2)+3)/(-1/b^2*x*(-b*x+
(a*b^2)^(1/3))*(I*3^(1/2)*(a*b^2)^(1/3)-2*b*x-(a*b^2)^(1/3))*(I*3^(1/2)*(a*
b^2)^(1/3)+2*b*x+(a*b^2)^(1/3)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

Fricas [A]
time = 3.10, size = 111, normalized size = 3.36

$$\left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^6+8\,abx^3-a^2+4\,(2\,bx^5-ax^2)\sqrt{-b}\,\sqrt{-\frac{bx^3-a}{x}}\right)}{6\,b},\,-\frac{\arctan\left(\frac{2\,\sqrt{b}\,x^2\sqrt{-\frac{bx^3-a}{x}}}{2\,bx^3-a}\right)}{3\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt(
-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(2*sqrt(b)*x^2*sqrt(-(b*x^3 - a)/x)
/(2*b*x^3 - a))/sqrt(b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**3+a)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a - b*x**3)/x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^3)/x)^(1/2),x)
```

```
[Out] int(1/((a - b*x^3)/x)^(1/2), x)
```

3.412 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*arctan(x*b^(1/2)/(a/x^2-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2004, 2033, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a}{x^2}-bx^2}}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 1.88

$$-\frac{\sqrt{a-bx^4}\tan^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{b}\,x^2}\right)}{2\sqrt{b}\,x\sqrt{\frac{a-bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] -1/2*(Sqrt[a - b*x^4]*ArcTan[Sqrt[a - b*x^4]/(Sqrt[b]*x^2)])/(Sqrt[b]*x*Sqr
t[(a - b*x^4)/x^2])
```

Maple [A]
time = 0.07, size = 51, normalized size = 1.55

method	result	size
default	$\frac{\sqrt{-b\,x^4+a}\;\arctan\left(\frac{x^2\sqrt{b}}{\sqrt{-b\,x^4+a}}\right)}{2\sqrt{\frac{-b\,x^4+a}{x^2}}\;x\sqrt{b}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((-b*x^4+a)/x^2)^(1/2)/x*(-b*x^4+a)^(1/2)*arctan(x^2*b^(1/2)/(-b*x^4+a)
^(1/2))/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
[Out] b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 +
a)
```

Fricas [A]
time = 2.90, size = 88, normalized size = 2.67

$$\left[-\frac{\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-b}\,x^3\sqrt{-\frac{bx^4-a}{x^2}}-a\right)}{4\,b}, -\frac{\arctan\left(\frac{\sqrt{b}\,x^3\sqrt{-\frac{bx^4-a}{x^2}}}{bx^4-a}\right)}{2\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
[Out] [-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b,
-1/2*arctan(sqrt(b)*x^3*sqrt(-(b*x^4 - a)/x^2)/(b*x^4 - a))/sqrt(b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
[Out] Integral(1/sqrt((a - b*x**4)/x**2), x)
```

Giac [A]
time = 0.55, size = 47, normalized size = 1.42

$$\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{4\,\sqrt{-b}}-\frac{\log\left(\left|-\sqrt{-b}\,x^2+\sqrt{-bx^4+a}\right|\right)}{2\,\sqrt{-b}\,\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(-b) - 1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 +
 a)))/(sqrt(-b)*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)/x^2)^(1/2),x)
```

```
[Out] int(1/((a - b*x^4)/x^2)^(1/2), x)
```

3.413 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx$

Optimal. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

```
[Out] 2/5*arctan(x*b^(1/2)/(a/x^3-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2004, 2033, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}-bx^2}}\,dx\\ &= \frac{2}{5}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}\end{aligned}$$

Mathematica [**A**]
time = 0.20, size = 66, normalized size = 2.00

$$-\frac{2\sqrt{a-bx^5}\,\tan^{-1}\left(\frac{\sqrt{a-bx^5}}{\sqrt{b}\,x^{5/2}}\right)}{5\sqrt{b}\,x^{3/2}\sqrt{\frac{a-bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (-2*Sqrt[a - b*x^5]*ArcTan[Sqrt[a - b*x^5]/(Sqrt[b]*x^(5/2))])/(5*Sqrt[b]*x
^(3/2)*Sqrt[(a - b*x^5)/x^3])
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{-b\,x^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

```
[Out] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(b*x^5 - a)/x^3), x)
```

Fricas [**A**]
time = 3.75, size = 111, normalized size = 3.36

$$\left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^{10}+8\,abx^5-a^2+4\,(2\,bx^9-ax^4)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10\,b}, -\frac{\arctan\left(\frac{2\,\sqrt{b}\,x^4\sqrt{-\frac{bx^5-a}{x^3}}}{2\,bx^5-a}\right)}{5\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
[Out] [-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sqr
t(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 - a
)/x^3)/(2*b*x^5 - a))/sqrt(b)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**5+a)/x**3)**(1/2),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^5}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^5)/x^3)^(1/2),x)
```

```
[Out] int(1/((a - b*x^5)/x^3)^(1/2), x)
```

3.414 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2004, 2033, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \int \frac{1}{\sqrt{-bx^2+ax^{2-n}}}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{n}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(38) = 76.
time = 0.06, size = 78, normalized size = 2.05

$$\frac{2\sqrt{a}\,x^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,n\sqrt{x^{2-n}(a-bx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a]
])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}(a-b\,x^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
[Out] int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

Fricas [A]
time = 1.92, size = 106, normalized size = 2.79

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{2\,bxx^{n}-ax-2\,\sqrt{-b}\,x^{n}\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{x}\right)}{bn}, -\frac{2\arctan\left(\frac{\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{\sqrt{b}\,x}\right)}{\sqrt{b}\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2)
/x^n))/x)/(b*n), -2*arctan(sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(sqrt(b)*x))/(sqr
t(b)*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)
[Out] Integral(1/sqrt(x**(2 - n)*(a - b*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^{2-n}\,\left(a-b\,x^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2 - n)*(a - b*x^n))^(1/2),x)
[Out] int(1/(x^(2 - n)*(a - b*x^n))^(1/2), x)
```

3.415 $\int \frac{1}{\sqrt{x^n \left(a+b x^{2-n}\right)}} \, dx$

Optimal. Leaf size=37

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{b x^2+a x^n}}\right)}{\sqrt{b}\, (2-n)}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2004, 2033, 212}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a x^n+b x^2}}\right)}{\sqrt{b}\, (2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\int \frac{1}{\sqrt{bx^2+ax^n}}\,dx$$
$$=\frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}$$
$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.
time = 0.07, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(-2+n)\sqrt{bx^2+ax^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^n\left(a+b\,x^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
[Out] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{n}\left(a+bx^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a + b*x**(2 - n))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

Mupad [**B**]
time = 5.21, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\;x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\;x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\;\,1\mathrm{i}}{\sqrt{b}\;\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a + b*x^(2 - n)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.416 $\int \frac{1}{\sqrt{x^2 (b+ax^{-2+n})}} dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2004, 2033, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2\,(b+ax^{-2+n})}}\,dx=\int \frac{1}{\sqrt{bx^2+ax^n}}\,dx$$
$$=\frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}$$
$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.
time = 0.02, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(-2+n)\sqrt{bx^2+ax^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\,(b+a\,x^{-2+n})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
[Out] int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

Fricas [A]
time = 1.39, size = 109, normalized size = 2.95

$$\left[\frac{\sqrt{b}\log\left(\frac{axx^{n-2}+2\,bx-2\,\sqrt{ax^2x^{n-2}+bx^2}\,\sqrt{b}}{xx^{n-2}}\right)}{bn-2\,b}, \frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{ax^2x^{n-2}+bx^2}\,\sqrt{-b}}{bx}\right)}{bn-2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt(
b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) + b*
x^2)*sqrt(-b)/(b*x))/(b*n - 2*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\,(ax^{n-2}+b)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) + b)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

Mupad [B]
time = 5.32, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\;x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\;x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\;1\mathrm{i}}{\sqrt{b}\;\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b + a*x^(n - 2)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.417 $\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2004, 2033, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx = \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx$$
$$= \frac{2\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.
time = 0.02, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(-2+n)\sqrt{bx^2+ax^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(bx+a\,x^{-1+n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
[Out] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)
[Out] Integral(1/sqrt(x*(a*x**(n - 1) + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

Mupad [B]
time = 5.13, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x + a*x^(n - 1)))^(1/2),x)
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.418 $\int \frac{1}{\sqrt{x^n (a - bx^{2-n})}} \, dx$

Optimal. Leaf size=38

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{-bx^2 + ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2004, 2033, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\, x}{\sqrt{ax^n - bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^n (a-bx^{2-n})}}\,dx = \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.
time = 0.07, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(-2+n)\sqrt{-bx^2+ax^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^n (a-b\,x^{2-n})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

```
[Out] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a - b*x**(2 - n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

Mupad [B]
time = 5.17, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a - b*x^(2 - n)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.419 $\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2004, 2033, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx = \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 80 vs. $2(38) = 76$.
time = 0.02, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(-2+n)\sqrt{-bx^2+ax^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(-b+a\,x^{-2+n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
[Out] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

Fricas [A]
time = 2.16, size = 109, normalized size = 2.87

$$\left[-\frac{\sqrt{-b}\log\left(\frac{axx^{n-2}-2\,bx-2\,\sqrt{ax^2x^{n-2}-bx^2}\,\sqrt{-b}}{xx^{n-2}}\right)}{bn-2\,b}, \frac{2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{ax^2x^{n-2}-bx^2}}{\sqrt{b}\,x}\right)}{bn-2\,b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sqr
t(-b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2) -
b*x^2)/(sqrt(b)*x))/(b*n - 2*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(ax^{n-2}-b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) - b)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

Mupad [B]
time = 5.10, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2*(b - a*x^(n - 2)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.420 $\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2004, 2033, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx$$
$$=\frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}$$
$$=\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.
time = 0.02, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(-2+n)\sqrt{-bx^2+ax^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(-bx+a\,x^{-1+n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

```
[Out] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}-bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a*x**(n - 1) - b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

Mupad [B]
time = 5.12, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(b*x - a*x^(n - 1)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.421 $\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx$

Optimal. Leaf size=107

$$\frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\ {}_2F_1\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n};1+\frac{1+m+\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*b*x^(1+n)*(c*x)^m*hypergeom([-3/2, (1+m+3/2*n)/(j-n)],[1+(1+m+3/2*n)/(j-n
)],-a*x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 372, 371}

$$\frac{2bx^{n+1}(cx)^m\sqrt{ax^j+bx^n}\ {}_2F_1\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n};\frac{m+\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + m +
 (3*n)/2)/(j - n), 1 + (1 + m + (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/((2
+ 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(ax^j+bx^n\right)^{3/2}\,dx &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(b+ax^{j-n}\right)^{3/2}\,dx}{\sqrt{b+ax^{j-n}}}\\ &= \frac{\left(bx^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\ &= \frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n};1+\frac{1+m+\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(107) = 214.
time = 0.27, size = 218, normalized size = 2.04

$$\frac{2(cx)^m\left((2+4j+2m-n)x^{-m}(ax^j+bx^n)\left(a(2-j+2m+4n)x^{1+j+m}+b(2+2j+2m+n)x^{1+m+n}\right)+3a^2(j-n)^2x^{1+2j}\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{2+4j+2m-n}{2j-2n};\frac{2+6j+2m-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{(2+4j+2m-n)(2+2j+2m+n)(2+2m+3n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*(c*x)^m*(((2 + 4*j + 2*m - n)*(a*x^j + b*x^n)*(a*(2 - j + 2*m + 4*n)*x^(
1 + j + m) + b*(2 + 2*j + 2*m + n)*x^(1 + m + n)))/x^m + 3*a^2*(j - n)^2*x^
(1 + 2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 4*j + 2*m -
 n)/(2*j - 2*n), (2 + 6*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(
(2 + 4*j + 2*m - n)*(2 + 2*j + 2*m + n)*(2 + 2*m + 3*n)*Sqrt[a*x^j + b*x^n]
)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (cx)^m \left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(ax^j + bx^n\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(3/2),x)
[Out] Integral((c*x)**m*(a*x**j + b*x**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^m\,\left(a\,x^j+b\,x^n\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j + b*x^n)^(3/2),x)
[Out] int((c*x)^m*(a*x^j + b*x^n)^(3/2), x)
```

3.422 $\int (cx)^m \sqrt{ax^j + bx^n}\, dx$

Optimal. Leaf size=100

$$\frac{2x(cx)^m\sqrt{ax^j+bx^n}\ {}_2F_1\left(-\frac{1}{2},\frac{1+m+\frac{n}{2}}{j-n};1+\frac{2+2m+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*x*(c*x)^m*hypergeom([-1/2, (1+m+1/2*n)/(j-n)],[1+(2+2*m+n)/(2*j-2*n)],-a*
x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2057, 372, 371}

$$\frac{2x(cx)^m\sqrt{ax^j+bx^n}\ {}_2F_1\left(-\frac{1}{2},\frac{m+\frac{n}{2}+1}{j-n};\frac{2m+n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (1 + m + n/2)/(j -
 n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*Sqr
t[1 + (a*x^(j - n))/b])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^m\sqrt{ax^j+bx^n}\,dx &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{n}{2}}\sqrt{b+ax^{j-n}}\,dx}{\sqrt{b+ax^{j-n}}}\\
&= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\
&= \frac{2x(cx)^m\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{1+m+\frac{n}{2}}{j-n};1+\frac{2+2m+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1+\frac{ax^{j-n}}{b}}}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 156, normalized size = 1.56

$$\frac{2x(cx)^m\left((2+2j+2m-n)\left(ax^j+bx^n\right)-a(j-n)x^j\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{2+2j+2m-n}{2j-2n};\frac{2+4j+2m-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{(2+2j+2m-n)(2+2m+n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
[Out] (2*x*(c*x)^m*((2 + 2*j + 2*m - n)*(a*x^j + b*x^n) - a*(j - n)*x^j*Sqrt[1 +
(a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*j + 2*m - n)/(2*j - 2*n), (2
 + 4*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + 2*j + 2*m - n)
*(2 + 2*m + n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (cx)^m\sqrt{a\,x^j+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
[Out] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**m*sqrt(a*x**j + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^m\,\sqrt{a\,x^j + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^m*(a*x^j + b*x^n)^(1/2), x)
```

3.423 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$

Optimal. Leaf size=102

$$\frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n};1+\frac{1+m-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x*(c*x)^m*hypergeom([1/2, (1+m-1/2*n)/(j-n)],[1+(1+m-1/2*n)/(j-n)],-a*x^(
j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/(2+2*m-n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2057, 372, 371}

$$\frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n};\frac{m-\frac{n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (1 + m - n/2)
/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sq
rt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n};1+\frac{1+m-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

Mathematica [A]
time = 0.09, size = 106, normalized size = 1.04

$$\frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{2+2m-n}{2j-2n};1+\frac{2+2m-n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*m - n)
/(2*j - 2*n), 1 + (2 + 2*m - n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m
 - n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**m/sqrt(a*x**j + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^m}{\sqrt{a\,x^j + b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(1/2), x)
```

3.424 $\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\ {}_2F_1\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n};1+\frac{1+m-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-n)*(c*x)^m*hypergeom([3/2, (1+m-3/2*n)/(j-n)],[1+(1+m-3/2*n)/(j-n)],
-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2+2*m-3*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2057, 372, 371}

$$\frac{2x^{1-n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\ {}_2F_1\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n};\frac{m-\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 +
m - (3*n)/2)/(j - n), 1 + (1 + m - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(
b*(2 + 2*m - 3*n)*Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{3n}{2}}}{(b+ax^{j-n})^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{3n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}$$
$$= \frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n};1+\frac{1+m-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}$$

Mathematica [A]
time = 0.12, size = 116, normalized size = 1.05

$$\frac{2x^{1-j}(cx)^m\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{2-2j+2m-n}{2j-2n};\frac{2+2m-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
[Out] (2*x^(1 - j)*(c*x)^m*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2,
 (2 - 2*j + 2*m - n)/(2*j - 2*n), (2 + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j -
n))/b)]))/(a*(j - n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(a\,x^j+b\,x^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
[Out] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(3/2),x)
[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^m}{(a\,x^j + b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(3/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

3.425 $\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\ {}_2F_1\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n};1+\frac{1+m-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-2*n)*(c*x)^m*hypergeom([5/2, (1+m-5/2*n)/(j-n)],[1+(1+m-5/2*n)/(j-n)
],-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2+2*m-5*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2057, 372, 371}

$$\frac{2x^{1-2n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\ {}_2F_1\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n};\frac{m-\frac{5n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1
+ m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])
/(b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{5n}{2}}}{(b+ax^{j-n})^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{5n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}$$
$$= \frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n};1+\frac{1+m-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}$$

Mathematica [A]
time = 0.36, size = 166, normalized size = 1.50

$$\frac{2x^{1-2j}(cx)^m\left(-2+2j-2m+3n-\frac{a(j-n)x^j}{ax^j+bx^n}-(-2+2j-2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{2-4j+2m-n}{2j-2n};\frac{2-2j+2m-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{3a^2(j-n)^2\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(c*x)^m*(-2 + 2*j - 2*m + 3*n - (a*(j - n)*x^j)/(a*x^j + b*x
^n) - (-2 + 2*j - 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/
2, (2 - 4*j + 2*m - n)/(2*j - 2*n), (2 - 2*j + 2*m - 3*n)/(2*j - 2*n), -((a
*x^(j - n))/b)]))/(3*a^2*(j - n)^2*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(a\,x^j+b\,x^n)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(5/2),x)
[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c\,x)}^m}{{\left(a\,x^j+b\,x^n\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(5/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

3.426 $\int \left(ax^j+bx^n\right)^{3/2}\,dx$

Optimal. Leaf size=97

$$\frac{2bx^{1+n}\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n};\frac{2+2j+n}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*b*x^(1+n)*hypergeom([-3/2, (1+3/2*n)/(j-n)],[1/2*(2+2*j+n)/(j-n)],-a*x^(j
-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2036, 372, 371}

$$\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n};\frac{2j+n+2}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n)/2)/(j
 - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)])/((2 + 3*n)*Sqrt[1 +
(a*x^(j - n))/b])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(b+ax^{j-n}\right)^{3/2}\,dx}{\sqrt{b+ax^{j-n}}}$$
$$=\frac{\left(bx^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}$$
$$=\frac{2bx^{1+n}\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n};\frac{2+2j+n}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

Mathematica [A]
time = 0.16, size = 177, normalized size = 1.82

$$\frac{2x\left((2+4j-n)\left(ax^j+bx^n\right)\left(a(2-j+4n)x^j+b(2+2j+n)x^n\right)+3a^2(j-n)^2x^{2j}\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{2+4j-n}{2j-2n};\frac{2+6j-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{(2+4j-n)(2+2j+n)(2+3n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x*((2 + 4*j - n)*(a*x^j + b*x^n)*(a*(2 - j + 4*n)*x^j + b*(2 + 2*j + n)*
x^n) + 3*a^2*(j - n)^2*x^(2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[
1/2, (2 + 4*j - n)/(2*j - 2*n), (2 + 6*j - 3*n)/(2*j - 2*n), -((a*x^(j - n)
)/b)]))/((2 + 4*j - n)*(2 + 2*j + n)*(2 + 3*n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((a*x^j+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^j + bx^n)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(3/2),x)
[Out] Integral((a*x**j + b*x**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

Mupad [B]
time = 5.25, size = 82, normalized size = 0.85

$$\frac{x\,(a\,x^j + b\,x^n)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\,\frac{\frac{3\,n}{2}+1}{j-n};\;\frac{\frac{3\,n}{2}+1}{j-n}+1;\;-\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{3\,n}{2}+1\right)\,\left(\frac{a\,x^{j-n}}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(3/2),x)
```

```
[Out] (x*(a*x^j + b*x^n)^(3/2)*hypergeom([-3/2, ((3*n)/2 + 1)/(j - n)], ((3*n)/2
+ 1)/(j - n) + 1, -(a*x^(j - n))/b))/(((3*n)/2 + 1)*((a*x^(j - n))/b + 1)^(
3/2))
```

3.427 $\int \sqrt{ax^j + bx^n}\, dx$

Optimal. Leaf size=87

$$\frac{2x\sqrt{ax^j+bx^n}\ {}_2F_1\left(-\frac{1}{2},\frac{2+n}{2(j-n)};1+\frac{2+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*x*hypergeom([-1/2, 1/2*(2+n)/(j-n)],[1+(2+n)/(2*j-2*n)],-a*x^(j-n)/b)*(a*
x^j+b*x^n)^(1/2)/(2+n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2036, 372, 371}

$$\frac{2x\sqrt{ax^j+bx^n}\ {}_2F_1\left(-\frac{1}{2},\frac{n+2}{2(j-n)};\frac{n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 + n)/(2*(j - n)), 1 + (
2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + n)*Sqrt[1 + (a*x^(j - n))/b]
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{ax^j+bx^n}\,dx = \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{b+ax^{j-n}}\,dx}{\sqrt{b+ax^{j-n}}}$$
$$= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}$$
$$= \frac{2x\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{2+n}{2(j-n)};1+\frac{2+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

Mathematica [A]
time = 0.12, size = 134, normalized size = 1.54

$$\frac{2x\left(-\left((2+2j-n)\left(ax^j+bx^n\right)\right)+a(j-n)x^j\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{2+2j-n}{2j-2n};\frac{2+4j-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{(2+n)(-2-2j+n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^j + b*x^n],x]
[Out] (2*x*(-((2 + 2*j - n)*(a*x^j + b*x^n)) + a*(j - n)*x^j*Sqrt[1 + (a*x^(j - n
))/b]*Hypergeometric2F1[1/2, (2 + 2*j - n)/(2*j - 2*n), (2 + 4*j - 3*n)/(2*
j - 2*n), -((a*x^(j - n))/b)]))/((2 + n)*(-2 - 2*j + n)*Sqrt[a*x^j + b*x^n]
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^j+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j+b*x^n)^(1/2),x)
[Out] int((a*x^j+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(1/2),x)
[Out] Integral(sqrt(a*x**j + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

Mupad [B]
time = 5.23, size = 82, normalized size = 0.94

$$\frac{x\,\sqrt{a\,x^j+b\,x^n}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{\frac{n}{2}+1}{j-n};\ \frac{\frac{n}{2}+1}{j-n}+1;\ -\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{n}{2}+1\right)\,\sqrt{\frac{a\,x^{j-n}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2),x)
```

```
[Out] (x*(a*x^j + b*x^n)^(1/2)*hypergeom([-1/2, (n/2 + 1)/(j - n)], (n/2 + 1)/(j
- n) + 1, -(a*x^(j - n))/b))/((n/2 + 1)*((a*x^(j - n))/b + 1)^(1/2))
```

3.428 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$

Optimal. Leaf size=93

$$\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};1+\frac{1-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x*hypergeom([1/2, 1/2*(2-n)/(j-n)],[1+1/2*(2-n)/(j-n)],-a*x^(j-n)/b)*(1+a
*x^(j-n)/b)^(1/2)/(2-n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2036, 372, 371}

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};\frac{1-\frac{n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)),
1 + (1 - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 - n)*Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx &= \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-n/2}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}\\ &= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-n/2}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}\\ &= \frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\ {}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};1+\frac{1-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 88, normalized size = 0.95

$$-\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\ {}_2F_1\left(\frac{1}{2},\frac{-2+n}{2(-j+n)};1+\frac{-2+n}{2(-j+n)};-\frac{ax^{j-n}}{b}\right)}{(-2+n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^j + b*x^n],x]
[Out] (-2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (-2 + n)/(2*(-j + n)
), 1 + (-2 + n)/(2*(-j + n)), -((a*x^(j - n))/b)])/((-2 + n)*Sqrt[a*x^j + b
*x^n])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,x^{j}+b\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(1/2),x)
[Out] int(1/(a*x^j+b*x^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(1/2),x)
[Out] Integral(1/sqrt(a*x**j + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

Mupad [B]
time = 5.27, size = 83, normalized size = 0.89

$$-\frac{x\,\sqrt{\frac{b\,x^{n-j}}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{\frac{j}{2}-1}{j-n};\ \frac{\frac{j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{j}{2}-1\right)\,\sqrt{a\,x^j+b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j + b*x^n)^(1/2),x)
[Out] -(x*((b*x^(n - j))/a + 1)^(1/2)*hypergeom([1/2, (j/2 - 1)/(j - n)], (j/2 -
1)/(j - n) + 1, -(b*x^(n - j))/a))/((j/2 - 1)*(a*x^j + b*x^n)^(1/2))
```

3.429 $\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};1+\frac{1-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-n)*hypergeom([3/2, (1-3/2*n)/(j-n)],[1+(2-3*n)/(2*j-2*n)],-a*x^(j-n)
/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2-3*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2036, 372, 371}

$$\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};\frac{1-\frac{3n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n)/2)
/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b*(2 - 3*n)*Sqrt
[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx = \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-3n/2}}{(b+ax^{j-n})^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-3n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}$$

$$= \frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};1+\frac{1-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

Mathematica [A]
time = 0.08, size = 104, normalized size = 1.03

$$\frac{2x^{1-j}\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{1}{2},-\frac{-2+2j+n}{2(j-n)};\frac{2-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - j)*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, -1/2*(-
2 + 2*j + n)/(j - n), (2 - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(a*(j -
n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^{j} + bx^{n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

Mupad [B]
time = 5.48, size = 83, normalized size = 0.82

$$-\frac{x\,{\left(\frac{b\,x^{n-j}}{a}+1\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(\frac{3}{2},\frac{\frac{3\,j}{2}-1}{j-n};\ \frac{\frac{3\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{3\,j}{2}-1\right)\,{\left(a\,x^j+b\,x^n\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j + b*x^n)^(3/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(3/2)*hypergeom([3/2, ((3*j)/2 - 1)/(j - n)], ((3
*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((3*j)/2 - 1)*(a*x^j + b*x^n)^(
3/2))
```

3.430 $\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

Optimal. Leaf size=101

$$\frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};1+\frac{1-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-2*n)*hypergeom([5/2, (1-5/2*n)/(j-n)],[1+(2-5*n)/(2*j-2*n)],-a*x^(j-
n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2-5*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2036, 372, 371}

$$\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};\frac{1-\frac{5n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)/
2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n)*
Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx &= \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-5n/2}}{(b+ax^{j-n})^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}\\ &= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-5n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}\\ &= \frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};1+\frac{1-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 185, normalized size = 1.83

$$\frac{2x^{1-2j}\left(-((-2+4j+n)\left(a(-2+j+4n)x^j+b(-2+2j+3n)x^n\right))+(4+8j^2-8n+3n^2+2j(-6+7n))\sqrt{1+\frac{ax^{j-n}}{b}}\,(ax^j+bx^n)\;{}_2F_1\left(\frac{1}{2},-\frac{-2+4j+n}{2(j-n)};\frac{2-2j-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{3a^2(2-4j-n)(j-n)^2\left(ax^j+bx^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(-((-2 + 4*j + n)*(a*(-2 + j + 4*n)*x^j + b*(-2 + 2*j + 3*n)
*x^n)) + (4 + 8*j^2 - 8*n + 3*n^2 + 2*j*(-6 + 7*n))*Sqrt[1 + (a*x^(j - n))/
b]*(a*x^j + b*x^n)*Hypergeometric2F1[1/2, -1/2*(-2 + 4*j + n)/(j - n), (2 -
 2*j - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(3*a^2*(2 - 4*j - n)*(j - n)
^2*(a*x^j + b*x^n)^(3/2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(5/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^j + bx^n)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(5/2),x)
[Out] Integral((a*x**j + b*x**n)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

Mupad [B]
time = 5.60, size = 83, normalized size = 0.82

$$-\frac{x\left(\frac{b\,x^{n-j}}{a}+1\right)^{5/2}{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{\frac{5\,j}{2}-1}{j-n};\ \frac{\frac{5\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{5\,j}{2}-1\right)\,\left(a\,x^j+b\,x^n\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j + b*x^n)^(5/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(5/2)*hypergeom([5/2, ((5*j)/2 - 1)/(j - n)], ((5
*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((5*j)/2 - 1)*(a*x^j + b*x^n)^(
5/2))
```

3.431 $\int \sqrt{\frac{1+x}{x^5}}\, dx$

Optimal. Leaf size=18

$$-\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6$$

```
[Out] -2/3*(1/x^5+1/x^4)^(3/2)*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2004, 2025}

$$-\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x^5],x]
```

```
[Out] (-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{x^5}}\, dx &= \int \sqrt{\frac{1}{x^5}+\frac{1}{x^4}}\, dx\\ &= -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6\end{aligned}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.06

$$-\frac{2}{3}x(1+x)\sqrt{\frac{1+x}{x^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x^5],x]
[Out] (-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3
```

Maple [A]
time = 0.06, size = 26, normalized size = 1.44

method	result	size
gosper	$-\frac{2x(x+1)\sqrt{\frac{x+1}{x^5}}}{3}$	16
trager	$-\frac{2x(x+1)\sqrt{-\frac{-1-x}{x^5}}}{3}$	19
default	$-\frac{2\sqrt{\frac{x+1}{x^5}}\,(x^2+x)^{\frac{3}{2}}}{3\sqrt{x\,(x+1)}}$	26
risch	$-\frac{2\sqrt{\frac{x+1}{x^5}}\,x(x^2+2x+1)}{3(x+1)}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/x^5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*((x+1)/x^5)^(1/2)/(x*(x+1))^(1/2)*(x^2+x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x + 1)/x^5), x)
```

Fricas [A]
time = 1.35, size = 16, normalized size = 0.89

$$-\frac{2}{3}\left(x^2+x\right)\sqrt{\frac{x+1}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")
[Out] -2/3*(x^2 + x)*sqrt((x + 1)/x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x+1}{x^5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x**5)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x**5), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(14) = 28.
time = 1.31, size = 50, normalized size = 2.78

$$\frac{2\left(3\left(x-\sqrt{x^2+x}\right)^2\operatorname{sgn}(x)+3\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+\operatorname{sgn}(x)\right)}{3\left(x-\sqrt{x^2+x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x)
)/(x - sqrt(x^2 + x))^3
```

Mupad [B]
time = 5.26, size = 15, normalized size = 0.83

$$-\frac{2\,x\,\sqrt{\frac{x+1}{x^5}}\,\left(x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x^5)^(1/2),x)
```

```
[Out] -(2*x*((x + 1)/x^5)^(1/2)*(x + 1))/3
```

3.432 $\int \sqrt{x+x^{5/2}}\, dx$

Optimal. Leaf size=20

$$\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

```
[Out] 4/9*(x+x^(5/2))^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2025}

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(5/2)],x]
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{x+x^{5/2}}\, dx = \frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(5/2)],x]
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Maple [A]
time = 0.37, size = 18, normalized size = 0.90

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(1+x^{\frac{3}{2}}\right)}{9\sqrt{x}}$	18
default	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(1+x^{\frac{3}{2}}\right)}{9\sqrt{x}}$	18
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(2+2x^{\frac{3}{2}}\right)\sqrt{1+x^{\frac{3}{2}}}}{3}}{3\sqrt{\pi}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(5/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(1+x^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^(5/2) + x), x)
```

Fricas [A]
time = 1.57, size = 19, normalized size = 0.95

$$\frac{4\sqrt{x^{\frac{5}{2}}+x}\,\left(x^2+\sqrt{x}\right)}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")
[Out] 4/9*sqrt(x^(5/2) + x)*(x^2 + sqrt(x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{\frac{5}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(5/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(5/2) + x), x)
```

Giac [A]
time = 1.51, size = 11, normalized size = 0.55

$$\frac{4}{9}\left(x^{\frac{3}{2}}+1\right)^{\frac{3}{2}}-\frac{4}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/9*(x^(3/2) + 1)^(3/2) - 4/9
```

Mupad [B]
time = 5.32, size = 27, normalized size = 1.35

$$\frac{2\,x\,\sqrt{x+x^{5/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},1;\ 2;\ -x^{3/2}\right)}{3\,\sqrt{x^{3/2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(5/2))^(1/2),x)
```

```
[Out] (2*x*(x + x^(5/2))^(1/2)*hypergeom([-1/2, 1], 2, -x^(3/2)))/(3*(x^(3/2) + 1
)^(1/2))
```

3.433 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arctan(x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 65, 209}

$$2\text{ArcTan}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + x^(3/2))^(-1),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,(1+x)}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\tan^{-1}\left(\sqrt{x}\right)$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + x^(3/2))^(-1),x]
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A]
time = 0.44, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$2\arctan\left(\sqrt{x}\right)$	7
default	$2\arctan\left(\sqrt{x}\right)$	7
meijerg	$2\arctan\left(\sqrt{x}\right)$	7
trager	$\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\text{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{x+1}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*arctan(x^(1/2))
```

Maxima [A]
time = 0.51, size = 6, normalized size = 0.75

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

Fricas [A]
time = 1.67, size = 6, normalized size = 0.75

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

Sympy [A]
time = 0.07, size = 7, normalized size = 0.88

$$2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(3/2)+x**(1/2)),x)
[Out] 2*atan(sqrt(x))
```

Giac [A]
time = 1.12, size = 6, normalized size = 0.75

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

Mupad [B]
time = 5.24, size = 6, normalized size = 0.75

$$2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(3/2)),x)
[Out] 2*atan(x^(1/2))
```

3.434 $\int x\sqrt{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=25

$$\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

```
[Out] 2/9*(x^2*(b*x^3+a))^(3/2)/b/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1602}

$$\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{x^2(a+bx^3)}\,dx = \frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Maple [A]
time = 0.38, size = 29, normalized size = 1.16

method	result	size
gosper	$\frac{2(b\,x^3+a)\,\sqrt{x^2\,(b\,x^3+a)}}{9bx}$	29
default	$\frac{2(b\,x^3+a)\,\sqrt{x^2\,(b\,x^3+a)}}{9bx}$	29
trager	$\frac{2(b\,x^3+a)\,\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
risch	$\frac{2(b\,x^3+a)\,\sqrt{x^2\,(b\,x^3+a)}}{9bx}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.56

$$\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Fricas [A]
time = 1.40, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{bx^5+ax^2}\,(bx^3+a)}{9\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\,(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2*(b*x**3+a))**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

Giac [A]
time = 1.05, size = 27, normalized size = 1.08

$$\frac{2\,(bx^3+a)^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

Mupad [B]
time = 5.23, size = 22, normalized size = 0.88

$$\frac{2\,{\left(b\,x^3+a\right)}^{3/2}\,\sqrt{x^2}}{9\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2*(a + b*x^3))^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(3/2)*(x^2)^(1/2))/(9*b*x)
```

3.435 $\int x\sqrt{ax^2+bx^5}\,dx$

Optimal. Leaf size=25

$$\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

```
[Out] 2/9*(b*x^5+a*x^2)^(3/2)/b/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1602\}$

$$\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Maple [A]
time = 0.38, size = 29, normalized size = 1.16

method	result	size
gosper	$\frac{2(b\,x^{3}+a)\,\sqrt{b\,x^{5}+a\,x^{2}}}{9bx}$	29
default	$\frac{2(b\,x^{3}+a)\,\sqrt{b\,x^{5}+a\,x^{2}}}{9bx}$	29
trager	$\frac{2(b\,x^{3}+a)\,\sqrt{b\,x^{5}+a\,x^{2}}}{9bx}$	29
risch	$\frac{2(b\,x^{3}+a)\,\sqrt{x^{2}\,(b\,x^{3}+a)}}{9bx}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(b*x^3+a)/b/x*(b*x^5+a*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.56

$$\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Fricas [A]
time = 1.61, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{bx^5+ax^2}\,(bx^3+a)}{9\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\,(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

Giac [A]
time = 0.92, size = 27, normalized size = 1.08

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

Mupad [B]
time = 5.26, size = 29, normalized size = 1.16

$$\frac{\left(\frac{2\,a}{9\,b}+\frac{2\,x^3}{9}\right)\,\sqrt{b\,x^5+a\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] (((2*a)/(9*b) + (2*x^3)/9)*(a*x^2 + b*x^5)^(1/2))/x
```

3.436 $\int \sqrt{x^4 \left(a+bx^3\right)}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^4+bx^7\right)^{3/2}}{9bx^6}$$

```
[Out] 2/9*(b*x^7+a*x^4)^(3/2)/b/x^6
```

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2004, 2025}

$$\frac{2\left(ax^4+bx^7\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x^4 \left(a+bx^3\right)}\,dx &= \int \sqrt{ax^4+bx^7}\,dx \\ &= \frac{2\left(ax^4+bx^7\right)^{3/2}}{9bx^6} \end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\left(x^4\left(a+bx^3\right)\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)
```

Maple [A]
time = 0.38, size = 29, normalized size = 1.16

method	result	size
gosper	$\frac{2(b\,x^3+a)\,\sqrt{x^4\,(b\,x^3+a)}}{9b\,x^2}$	29
default	$\frac{2(b\,x^3+a)\,\sqrt{x^4\,(b\,x^3+a)}}{9b\,x^2}$	29
trager	$\frac{2(b\,x^3+a)\,\sqrt{b\,x^7+a\,x^4}}{9b\,x^2}$	29
risch	$\frac{2(b\,x^3+a)\,\sqrt{x^4\,(b\,x^3+a)}}{9b\,x^2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.56

$$\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Fricas [A]
time = 1.52, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{bx^7+ax^4}\,(bx^3+a)}{9\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4 (a + bx^3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4*(b*x**3+a))**(1/2),x)
```

```
[Out] Integral(sqrt(x**4*(a + b*x**3)), x)
```

Giac [**A**]
time = 2.26, size = 14, normalized size = 0.56

$$\frac{2 (bx^3 + a)^{\frac{3}{2}}}{9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Mupad [**B**]
time = 5.21, size = 22, normalized size = 0.88

$$\frac{2 (b x^3 + a)^{3/2} \sqrt{x^4}}{9 b x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3))^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(3/2)*(x^4)^(1/2))/(9*b*x^2)
```

3.437 $\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx$

Optimal. Leaf size=988

$$-\frac{45a^2\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}}{14\sqrt[3]{2}\,b^3\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}-\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}+\frac{9\left(a+b\sqrt[3]{x}\right.}{7b\sqrt[3]{a\sqrt[3]{x}+}}$$

```
[Out] -45/28*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(1/3)+9/7*(a+b*x^(
1/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(1/3)-45/28*a^2*(a+2*b*x^(1/3))*(-b*(
a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*2^(2/3)/b^3/(a*x^(1/3)+b*x^(2/3))^(1/3)/(1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))+15/14*3^(3/4)*a^4*(1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2
)^(1/3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/
(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1
+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^(1/3))*x
^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))
^2)^(1/2)*2^(1/6)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(1/3)/((-1+2^(2
/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1
/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)-45/56*3^(1/4)*a^4*(1-2^(2/3)*(-b*(a+b*x^(1
/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*EllipticE((1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-b*(a+b*x
^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-b*(a+b*x^(
1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(2/3))/(1
-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2
)+1/2*2^(1/2))*2^(2/3)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(1/3)/((-1
+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))
*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 1.56, antiderivative size = 988, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2036, 348, 52, 63, 636, 633, 241, 310, 225, 1893}

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{-\frac{b\left(\sqrt[3]{x}a+bx^{2/3}\right)}{a^2}}E\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)a^4}{28\sqrt[3]{2}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}+\frac{15\,3^{3/4}\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{-\frac{b\left(\sqrt[3]{x}a+bx^{2/3}\right)}{a^2}}F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)a^4}{7\,2^{5/6}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}-\frac{45\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(\sqrt[3]{x}a+bx^{2/3}\right)}{a^2}}a^2}{14\sqrt[3]{2}b^3\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}-\frac{45\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}a}{28b^2\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

```
[Out] (-45*a^2*(a + 2*b*x^(1/3))*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3))/(14*
2^(1/3)*b^3*(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/
3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) - (45*a*(a + b*x^(1/3))*x^(1/3))/(28*b^2
*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (9*(a + b*x^(1/3))*x^(2/3))/(7*b*(a*x^(1/
3) + b*x^(2/3))^(1/3)) - (45*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticE[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(28*2^(1/
3)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b
*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (15*3^(3/4)*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticF[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(7*2^(5/6
)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*
x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/(a*c + (b*c + a*d)*x + b*d*x^2)^m), Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = \frac{\left(\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\int \frac{1}{\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}}\,dx}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{\left(3\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\text{Subst}\left(\int \frac{x^{5/3}}{\sqrt[3]{a+bx}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\text{Subst}\left(\int \frac{x^{2/3}}{\sqrt[3]{a+bx}}\,dx, x, \sqrt[3]{x}\right)}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\text{Subst}\Big(\ldots}{14b^2\sqrt[3]{a\sqrt[3]{x}\ldots}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{(15a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx, x\ldots\right.}{14b^2}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Su}\ldots}{14b^2\sqrt[3]{a\ldots}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a^4\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{S}\ldots}{14\sqrt[3]{2}\,b\ldots}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\,\sqrt[3]{-\frac{b\,(\ldots}{\ldots}}\ldots\right.}{2\ldots}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\,\sqrt[3]{-\frac{b\,(\ldots}{\ldots}}\ldots\right.}{2\ldots}$$

$$\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 61, normalized size = 0.06

$$\frac{9\sqrt[3]{1+\frac{b\sqrt[3]{x}}{a}}\,x\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
[Out] (9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1/
3))/a)])/(8*((a + b*x^(1/3))*x^(1/3))^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(1/3),x)
```

```
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

Mupad [B]
time = 5.41, size = 42, normalized size = 0.04

$$\frac{9\,x\,\left(\frac{b\,x^{1/3}}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{8\,\left(a\,x^{1/3}+b\,x^{2/3}\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3) + b*x^(2/3))^(1/3),x)
```

```
[Out] (9*x*((b*x^(1/3))/a + 1)^(1/3)*hypergeom([1/3, 8/3], 11/3, -(b*x^(1/3))/a))
/(8*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

3.438 $\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$

Optimal. Leaf size=487

$$-\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{6\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)}{}$$

```
[Out] -18/5*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(2/3)+9/5*(a+b*x^(1
/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(2/3)+6/5*2^(1/3)*3^(3/4)*a^4*(1-2^(2/
3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^(2/
3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(1-2^
(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(
1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3
)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3
)/a^2)^(1/3)-3^(1/2))^2)^(1/2)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(2
/3)/((-1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*
x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2036, 348, 52, 63, 636, 633, 242, 225}

$$\frac{6\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}F\left(\text{ArcSin}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}-\frac{18a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9x^{2/3}\left(a+b\sqrt[3]{x}\right)}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (-18*a*(a + b*x^(1/3))*x^(1/3))/(5*b^2*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (9*
(a + b*x^(1/3))*x^(2/3))/(5*b*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (6*2^(1/3)*3
```

```
^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^
2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3) + 2
*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2]*(-((b*(a*x^(1/3) + b*x^(2/3
)))/a^2))^(2/3)*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3
))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/
3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b
*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3)
)*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(2/3)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/(a*c + (b*c + a*d)*x + b*d*x^2)^m), Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
```

```
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = \frac{\left(\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}}\,dx}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= \frac{\left(3\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\text{Subst}\left(\int \frac{x^{4/3}}{(a+bx)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} - \frac{\left(12a\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\text{Subst}\left(\int \frac{\sqrt[3]{x}}{(a+bx)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{\left(6a^2\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\text{Subs}}{5b^2\left(a\sqrt[3]{x}\right.}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{(6a^2)\,\text{Subst}\left(\int \frac{1}{(ax+bx^2)^{2/3}}\,dx, x\right.}{5b^2}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{\left(6a^2\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right)}{5b^2\left(a\sqrt[3]{\right.}}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} - \frac{\left(6\sqrt[3]{2}\,a^4\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)\right.}{5b}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} - \frac{\left(9\sqrt[3]{2}\,a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right.}{}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{6\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\left(1-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 61, normalized size = 0.13

$$\frac{9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2/3}x\,{}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1/
3))/a)])/(7*((a + b*x^(1/3))*x^(1/3))^(2/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

```
[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sqrt[3]{x} + bx^{\frac{2}{3}}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(2/3),x)
```

```
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

Mupad [B]
time = 5.25, size = 42, normalized size = 0.09

$$\frac{9\,x\,\left(\frac{b\,x^{1/3}}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{7\,\left(a\,x^{1/3}+b\,x^{2/3}\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3) + b*x^(2/3))^(2/3),x)
```

```
[Out] (9*x*((b*x^(1/3))/a + 1)^(2/3)*hypergeom([2/3, 7/3], 10/3, -(b*x^(1/3))/a))
/(7*(a*x^(1/3) + b*x^(2/3))^(2/3))
```

3.439 $\int x^m \left(ax^j+bx^n\right)^p \, dx$

Optimal. Leaf size=89

$$\frac{x^{1+m}(ax^j+bx^n)^p\,(a+bx^{-j+n})\;{}_2F_1\left(1,1+p+\frac{1+m+jp}{-j+n};1+\frac{1+m+jp}{-j+n};-\frac{bx^{-j+n}}{a}\right)}{a(1+m+jp)}$$

```
[Out] x^(1+m)*(a*x^j+b*x^n)^p*(a+b*x^(-j+n))*hypergeom([1, 1+p+(j*p+m+1)/(-j+n)],
[1+(j*p+m+1)/(-j+n)],-b*x^(-j+n)/a)/a/(j*p+m+1)
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2057, 372, 371}

$$\frac{x^{m+1}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}(ax^j+bx^n)^p\;{}_2F_1\left(-p,\frac{m+np+1}{j-n};\frac{m+np+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x^j + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^m\left(ax^j+bx^n\right)^p\,dx=\left(x^{-np}\left(b+ax^{j-n}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(b+ax^{j-n}\right)^p\,dx$$
$$=\left(x^{-np}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(1+\frac{ax^{j-n}}{b}\right)^p\,dx$$
$$=\frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\,{}_2F_1\left(-p,\frac{1+m+np}{j-n};1+\frac{1+m+np}{j-n};-\frac{ax^{j-n}}{b}\right)}{1+m+np}$$

Mathematica [**A**]
time = 0.10, size = 92, normalized size = 1.03

$$\frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\,{}_2F_1\left(-p,\frac{1+m+np}{j-n};1+\frac{1+m+np}{j-n};-\frac{ax^{j-n}}{b}\right)}{1+m+np}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x^j + b*x^n)^p,x]
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

Maple [**F**]
time = 0.20, size = 0, normalized size = 0.00

$$\int x^m\left(a\,x^j+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x^j+b*x^n)^p,x)
[Out] int(x^m*(a*x^j+b*x^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x^j + b*x^n)^p*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(ax^j + bx^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a*x**j+b*x**n)**p,x)
```

```
[Out] Integral(x**m*(a*x**j + b*x**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(a\,x^j+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x^j + b*x^n)^p,x)
```

```
[Out] int(x^m*(a*x^j + b*x^n)^p, x)
```

3.440 $\int x^{-1-pq}(bx^n+ax^q)^p\,dx$

Optimal. Leaf size=69

$$-\frac{x^{-pq}(a+bx^{n-q})\,(bx^n+ax^q)^p\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)}$$

```
[Out] -(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, 1+p],[2+p],1+b*x^(n-q)/a)/a/(1
+p)/(n-q)/(x^(p*q))
```

Rubi [A]

time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {2057, 272, 67}

$$-\frac{x^{-pq}(a+bx^{n-q})\,(ax^q+bx^n)^p\ {}_2F_1\left(1,p+1;p+2;\frac{bx^{n-q}}{a}+1\right)}{a(p+1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1-pq}(bx^n+ax^q)^p\,dx &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}(bx^n+ax^q)^p\right)\int \frac{(a+bx^{n-q})^p}{x}\,dx\\
&= \frac{\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}(bx^n+ax^q)^p\right)\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^{n-q}\right)}{n-q}\\
&= -\frac{x^{-pq}\left(a+bx^{n-q}\right)(bx^n+ax^q)^p\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 73, normalized size = 1.06

$$
\frac{x^{-pq}(bx^n+ax^q)^p\left(1+\frac{ax^{-n+q}}{b}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{ax^{-n+q}}{b}\right)}{p(n-q)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
[Out] ((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((a*x^(-n + q))/b)])/(
p*(n - q)*x^(p*q)*(1 + (a*x^(-n + q))/b)^p)
```

Maple [**F**]
time = 0.20, size = 0, normalized size = 0.00

$$
\int x^{-pq-1}(b\,x^n+a\,x^q)^p\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
[Out] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-pq-1}(ax^q + bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-p*q-1)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Integral(x**(-p*q - 1)*(a*x**q + b*x**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^n + a\,x^q)^p}{x^{p\,q+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(p*q + 1),x)
```

```
[Out] int((b*x^n + a*x^q)^p/x^(p*q + 1), x)
```

3.441 $\int x^{-1-np}(bx^n+ax^q)^p\,dx$

Optimal. Leaf size=66

$$-\frac{x^{-np}(a+bx^{n-q})\left(bx^n+ax^q\right)^p\ {}_2F_1\left(1,1;1-p;-\frac{bx^{n-q}}{a}\right)}{ap(n-q)}$$

```
[Out] -(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, 1],[1-p],-b*x^(n-q)/a)/a/p/(n-
q)/(x^(n*p))
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2057, 372, 371}

$$-\frac{x^{-np}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}\left(ax^q+bx^n\right)^p\ {}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1-np}(bx^n+ax^q)^p\,dx &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}(bx^n+ax^q)^p\right)\int x^{-1-np+pq}\left(a+bx^{n-q}\right)^p\,dx\\
&= \left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\right)\int x^{-1-np+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p\,dx\\
&= -\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 74, normalized size = 1.12

$$
-\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$
\int x^{-np-1}(b\,x^n+a\,x^q)^p\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
[Out] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-np-1}(ax^q + bx^n)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p-1)*(b*x**n+a*x**q)**p,x)
[Out] Integral(x**(-n*p - 1)*(a*x**q + b*x**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^n + a\,x^q)^p}{x^{n\,p+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(n*p + 1),x)
[Out] int((b*x^n + a*x^q)^p/x^(n*p + 1), x)
```

3.442 $\int x^{-1-n-(-1+p)q}(bx^n + ax^q)^p \, dx$

Optimal. Leaf size=69

$$\frac{bx^{-pq}(a + bx^{n-q})\,(bx^n + ax^q)^p \;{}_2F_1\left(2, 1 + p; 2 + p; 1 + \frac{bx^{n-q}}{a}\right)}{a^2(1 + p)(n - q)}$$

```
[Out] b*(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([2, 1+p],[2+p],1+b*x^(n-q)/a)/a^2
/(1+p)/(n-q)/(x^(p*q))
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2057, 272, 67}

$$\frac{bx^{-pq}(a + bx^{n-q})\,(ax^q + bx^n)^p \;{}_2F_1\left(2, p + 1; p + 2; \frac{bx^{n-q}}{a} + 1\right)}{a^2(p + 1)(n - q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx = \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}(bx^n+ax^q)^p\right)\int x^{-1-n-(-1+p)q+pq}\left(a+bx^{n-q}\right)^p\,dx$$

$$= \frac{\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}(bx^n+ax^q)^p\right)\operatorname{Subst}\left(\int\frac{(a+bx)^p}{x^2}\,dx,x,x^{n-q}\right)}{n-q}$$

$$= \frac{bx^{-pq}\left(a+bx^{n-q}\right)(bx^n+ax^q)^p\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a^2(1+p)(n-q)}$$

Mathematica [A]
time = 0.14, size = 82, normalized size = 1.19

$$\frac{x^{-n+q-pq}(bx^n+ax^q)^p\left(1+\frac{ax^{-n+q}}{b}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;-\frac{ax^{-n+q}}{b}\right)}{(-1+p)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(-n + q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((
a*x^(-n + q))/b)])/((-1 + p)*(n - q)*(1 + (a*x^(-n + q))/b)^p)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int x^{-1-n-(-1+p)q}(b\,x^n+a\,x^q)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n-(-1+p)*q)*(b*x**n+a*x**q)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^n+a\,x^q)^p}{x^{n+q\,(p-1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1),x)
[Out] int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1), x)
```

3.443 $\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx$

Optimal. Leaf size=84

$$\frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

```
[Out] x^(-n*p+n-q)*(b*x^n+a*x^q)^p*hypergeom([-p, 1-p],[2-p],-b*x^(n-q)/a)/(1-p)/
(n-q)/((1+b*x^(n-q)/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2057, 372, 371}

$$\frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}(ax^q+bx^n)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b
*x^(n - q))/a)])/((1 - p)*(n - q)*(1 + (b*x^(n - q))/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx = \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}(bx^n+ax^q)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(a+bx^{n-q}\right)^p\,dx$$
$$= \left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p$$
$$= \frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

Mathematica [A]
time = 0.12, size = 83, normalized size = 0.99

$$-\frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(-1+p)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
[Out] -((x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(
(b*x^(n - q))/a)])/((-1 + p)*(n - q)*(1 + (b*x^(n - q))/a)^p))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int x^{-1-n(-1+p)-q}(b\,x^n+a\,x^q)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x)
[Out] int(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p + n - q - 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(-1+p)-q)*(b*x**n+a*x**q)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^n+a\,x^q)^p}{x^{q+n\,(p-1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1),x)
[Out] int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1), x)
```

3.444 $\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx$

Optimal. Leaf size=44

$$\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+1))^(1+p)/b/(1+p)/(m*p+1)/(x^(m*(1+p)))
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2025}

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + m + m*p))^p,x]
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 0.98

$$\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+mp}\right)\right)^{1+p}}{b(1+p)(1+mp)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + m + m*p))^p,x]
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a\,x^{m}+b\,x^{m p +m +1}\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(m*p+m+1))^p,x)
[Out] int((a*x^m+b*x^(m*p+m+1))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

Fricas [A]
time = 2.30, size = 64, normalized size = 1.45

$$\frac{(bxx^{mp+m+1}+axx^{m})(bx^{mp+m+1}+ax^{m})^{p}}{(bmp^{2}+(bm+b)p+b)x^{mp+m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")
[Out] (b*x*x^(m*p + m + 1) + a*x*x^m)*(b*x^(m*p + m + 1) + a*x^m)^p/((b*m*p^2 + (
b*m + b)*p + b)*x^(m*p + m + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{m}+bx^{mp+m+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(m*p+m+1))**p,x)
[Out] Integral((a*x**m + b*x**(m*p + m + 1))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

Mupad [B]
time = 5.30, size = 76, normalized size = 1.73

$$\frac{a\,{\left(a\,x^m+b\,x^{m+m\,p+1}\right)}^p\,\left(\frac{b\,x^{m\,p+1}}{a}-\frac{1}{{\left(\frac{b\,x^{m\,p+1}}{a}+1\right)}^p}+1\right)}{b\,x^{m\,p}\,\left(m\,p+1\right)\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(m + m*p + 1))^p,x)
```

```
[Out] (a*(a*x^m + b*x^(m + m*p + 1))^p*((b*x^(m*p + 1))/a - 1/((b*x^(m*p + 1))/a
+ 1)^p + 1))/(b*x^(m*p)*(m*p + 1)*(p + 1))
```

3.445 $\int (x^m(a+bx^{1+mp}))^p \, dx$

Optimal. Leaf size=44

$$\frac{x^{-m(1+p)}(ax^m+bx^{1+m+mp})^{1+p}}{b(1+p)(1+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+1))^(1+p)/b/(1+p)/(m*p+1)/(x^(m*(1+p)))
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2004, 2025}

$$\frac{x^{-m(p+1)}(ax^m+bx^{mp+m+1})^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (x^m(a+bx^{1+mp}))^p \, dx &= \int (ax^m+bx^{1+m+mp})^p \, dx\\ &= \frac{x^{-m(1+p)}(ax^m+bx^{1+m+mp})^{1+p}}{b(1+p)(1+mp)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 0.98

$$\frac{x^{-m(1+p)}(x^m(a+bx^{1+mp}))^{1+p}}{b(1+p)(1+mp)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(x^{m} \left(a +b \,x^{m p +1}\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

```
[Out] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

Fricas [A]
time = 2.21, size = 61, normalized size = 1.39

$$\frac{(bxx^{mp+1} + ax)(bx^{mp+1}x^m + ax^m)^p}{(bmp^2 + (bm + b)p + b)x^{mp+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b)
*p + b)*x^(m*p + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{m} \left(a + bx^{mp+1}\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**m*(a+b*x**(m*p+1)))**p,x)
```

```
[Out] Integral((x**m*(a + b*x**(m*p + 1)))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x^m\,\left(a+b\,x^{m\,p+1}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*x^(m*p + 1)))^p,x)
[Out] int((x^m*(a + b*x^(m*p + 1)))^p, x)
```

3.446 $\int x^n(x^m(a+bx^{1+n+mp}))^p\,dx$

Optimal. Leaf size=46

$$\frac{x^{-m(1+p)}(ax^m+bx^{1+m+n+mp})^{1+p}}{b(1+p)(1+n+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+n+1))^(1+p)/b/(1+p)/(m*p+n+1)/(x^(m*(1+p)))
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2005, 2039}

$$\frac{x^{-m(p+1)}(ax^m+bx^{mp+m+n+1})^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))
```

Rule 2005

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx &= \int x^n\left(ax^m+bx^{1+m+n+mp}\right)^p\,dx\\ &= \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.98

$$\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+n+mp}\right)\right)^{1+p}}{b(1+p)(1+n+mp)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int x^{n}\left(x^{m}\left(a+b\,x^{m p+n+1}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
[Out] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

Fricas [A]
time = 2.08, size = 76, normalized size = 1.65

$$\frac{\left(bxx^{mp+n+1}x^{n}+axx^{n}\right)\left(bx^{mp+n+1}x^{m}+ax^{m}\right)^{p}}{\left(bmp^{2}+bn+(bm+bn+b)p+b\right)x^{mp+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")
[Out] (b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*m
*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^n\,\left(x^m\,\left(a+b\,x^{n+m\,p+1}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p,x)
```

```
[Out] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p, x)
```

3.447 $\int x^n(ax^m + bx^{1+m+n+mp})^p \, dx$

Optimal. Leaf size=46

$$\frac{x^{-m(1+p)}(ax^m + bx^{1+m+n+mp})^{1+p}}{b(1+p)(1+n+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+n+1))^(1+p)/b/(1+p)/(m*p+n+1)/(x^(m*(1+p)))
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2039}

$$\frac{x^{-m(p+1)}(ax^m + bx^{mp+m+n+1})^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \frac{x^{-m(1+p)}\left(ax^m + bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 0.98

$$\frac{x^{-m(1+p)}(x^m(a + bx^{1+n+mp}))^{1+p}}{b(1+p)(1+n+mp)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int x^n \left(a\,x^m + b\,x^{mp+m+n+1}\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

```
[Out] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

Fricas [A]
time = 2.45, size = 79, normalized size = 1.72

$$\frac{\left(bxx^{mp+m+n+1}x^n + axx^m x^n\right)\left(bx^{mp+m+n+1} + ax^m\right)^p}{(bmp^2 + bn + (bm + bn + b)p + b)x^{mp+m+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + n + 1)*x^n + a*x*x^m*x^n)*(b*x^(m*p + m + n + 1) + a*x^m)
^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + m + n + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(a*x**m+b*x**(m*p+m+n+1))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="giac")
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^n\,\left(a\,x^m+b\,x^{m+n+m\,p+1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p,x)
[Out] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p, x)
```

3.448 $\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx$

Optimal. Leaf size=44

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}$$

```
[Out] 2/3*x^(3-3*n)*(a/(x^(2-2*n))+b*x^(-2+3*n))^(3/2)/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2004, 2025}

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx &= \int \sqrt{ax^{2(-1+n)}+bx^{2(-1+n)+n}}\,dx\\ &= \frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 0.82

$$\frac{2x^{3-3n}\left(x^{-2+2n}(a+bx^n)\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
[Out] (2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)
```

Maple [A]
time = 0.44, size = 40, normalized size = 0.91

method	result	size
risch	$\frac{2\sqrt{\frac{x^{2n}(a+b\,x^{n})}{x^{2}}}\,(a+b\,x^{n})x^{-n}x}{3bn}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(-2+2*n)*(a+b*x^n))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1/x^2*(x^n)^2*(a+b*x^n))^(1/2)*(a+b*x^n)/(x^n)*x/b/n
```

Maxima [A]
time = 0.30, size = 17, normalized size = 0.39

$$\frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)
```

Fricas [A]
time = 2.38, size = 44, normalized size = 1.00

$$\frac{2\,(bxx^n+ax)\sqrt{\frac{bx^{3\,n}+ax^{2\,n}}{x^2}}}{3\,bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x^{2\,n-2}\,\left(a+b\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 2)*(a + b*x^n))^(1/2),x)
```

```
[Out] int((x^(2*n - 2)*(a + b*x^n))^(1/2), x)
```

3.449 $\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}$$

```
[Out] 3/4*x^(4-4*n)*(a/(x^(3-3*n))+b*x^(-3+4*n))^(4/3)/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2004, 2025}

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

```
[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx &= \int \sqrt[3]{ax^{3(-1+n)}+bx^{3(-1+n)+n}}\,dx\\ &= \frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.82

$$\frac{3x^{4-4n}\left(x^{-3+3n}(a+bx^n)\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
[Out] (3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)
```

Maple [A]
time = 0.43, size = 40, normalized size = 0.91

method	result	size
risch	$\frac{3\left(\frac{x^{3n}\left(a+b\,x^{n}\right)}{x^{3}}\right)^{\frac{1}{3}} x\,x^{-n}\left(a+b\,x^{n}\right)}{4bn}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(-3+3*n)*(a+b*x^n))^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/4*(1/x^3*(x^n)^3*(a+b*x^n))^(1/3)*x/(x^n)*(a+b*x^n)/b/n
```

Maxima [A]
time = 0.29, size = 17, normalized size = 0.39

$$\frac{3\left(bx^{n}+a\right)^{\frac{4}{3}}}{4\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="maxima")
[Out] 3/4*(b*x^n + a)^(4/3)/(b*n)
```

Fricas [A]
time = 1.98, size = 44, normalized size = 1.00

$$\frac{3\left(bxx^{n}+ax\right)\left(\frac{bx^{4\,n}+ax^{3\,n}}{x^{3}}\right)^{\frac{1}{3}}}{4\,bnx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")
[Out] 3/4*(b*x*x^n + a*x)*((b*x^(4*n) + a*x^(3*n))/x^3)^(1/3)/(b*n*x^n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(x^{3\,n-3}\,\left(a+b\,x^n\right)\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 3)*(a + b*x^n))^(1/3),x)
[Out] int((x^(3*n - 3)*(a + b*x^n))^(1/3), x)
```

3.450 $\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}$$

```
[Out] 4/5*x^(5-5*n)*(a/(x^(4-4*n))+b*x^(-4+5*n))^(5/4)/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2004, 2025}

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx &= \int \sqrt[4]{ax^{4(-1+n)}+bx^{4(-1+n)+n}}\,dx\\ &= \frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 0.82

$$\frac{4x^{5-5n}\left(x^{-4+4n}(a+bx^n)\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
[Out] (4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)
```

Maple [A]
time = 0.43, size = 40, normalized size = 0.91

method	result	size
risch	$\frac{4\left(\frac{x^{4n}(a+b\,x^{n})}{x^{4}}\right)^{\frac{1}{4}} x\,x^{-n}(a+b\,x^{n})}{5bn}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(-4+4*n)*(a+b*x^n))^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/5*(1/x^4*(x^n)^4*(a+b*x^n))^(1/4)*x/(x^n)*(a+b*x^n)/b/n
```

Maxima [A]
time = 0.30, size = 17, normalized size = 0.39

$$\frac{4\,(bx^n+a)^{\frac{5}{4}}}{5\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="maxima")
[Out] 4/5*(b*x^n + a)^(5/4)/(b*n)
```

Fricas [A]
time = 3.01, size = 44, normalized size = 1.00

$$\frac{4\,(bxx^n+ax)\left(\frac{bx^{5\,n}+ax^{4\,n}}{x^4}\right)^{\frac{1}{4}}}{5\,bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="fricas")
[Out] 4/5*(b*x*x^n + a*x)*((b*x^(5*n) + a*x^(4*n))/x^4)^(1/4)/(b*n*x^n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-4+4*n)*(a+b*x**n))**(1/4),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x^{4\,n-4}\,\left(a+b\,x^n\right)\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(4*n - 4)*(a + b*x^n))^(1/4),x)
```

```
[Out] int((x^(4*n - 4)*(a + b*x^n))^(1/4), x)
```

3.451 $\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx$

Optimal. Leaf size=57

$$\frac{px^{(1-n)(1+p)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}$$

```
[Out] p*x^((1-n)*(1+p))*(a/(x^((1-n)*p))+b*x^(n-(1-n)*p))^(1+1/p)/b/n/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2004, 2025}

$$\frac{px^{(1-n)(p+1)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1))
)/(b*n*(1 + p))
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx &= \int \left(ax^{(-1+n)p}+bx^{n+(-1+n)p}\right)^{\frac{1}{p}}\,dx\\ &= \frac{px^{(1-n)(1+p)}\left(ax^{-(1-n)p}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.82

$$\frac{x^{1-n}(a+bx^n)\left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}}{bn\left(1+\frac{1}{p}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-1
)))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(x^{(-1+n)p}(a+b\,x^n)\right)^{\frac{1}{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
[Out] int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

Fricas [A]
time = 2.85, size = 47, normalized size = 0.82

$$\frac{(bpxx^n+apx)\left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}}{(bnp+bn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")
[Out] (b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{p(n-1)}(a+bx^{n})\right)^{\frac{1}{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)
```

```
[Out] Integral((x**(p*(n - 1))*(a + b*x**n))**(1/p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(x^{p\,(n-1)}\,\left(a+b\,x^n\right)\right)}^{1/p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(p*(n - 1))*(a + b*x^n))^(1/p),x)
```

```
[Out] int((x^(p*(n - 1))*(a + b*x^n))^(1/p), x)
```

3.452 $\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p \, dx$

Optimal. Leaf size=61

$$\frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}$$

```
[Out] x^((1-n)*(1+p)/p)*(b*x^(n+(-1+n)/p)+a/(x^((1-n)/p)))^(1+p)/b/n/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2004, 2025}

$$\frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(
b*n*(1 + p))
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p \, dx &= \int \left(bx^{n+\frac{-1+n}{p}}+ax^{\frac{-1+n}{p}}\right)^p \, dx\\ &= \frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.74

$$\frac{x^{1-n}(a+bx^n)\left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p}{bn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(x^{\frac{-1+n}{p}}(a+b\,x^n)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((-1+n)/p)*(a+b*x^n))^p,x)
[Out] int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

Fricas [A]
time = 2.40, size = 54, normalized size = 0.89

$$\frac{(bxx^n+ax)\left(bx^nx^{\frac{n-1}{p}}+ax^{\frac{n-1}{p}}\right)^p}{(bnp+bn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")
[Out] (b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x^
n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{\frac{n-1}{p}}\left(a+bx^n\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)
```

```
[Out] Integral((x**((n - 1)/p)*(a + b*x**n))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(x^{\frac{n-1}{p}}\,\left(a+b\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((n - 1)/p)*(a + b*x^n))^p,x)
```

```
[Out] int((x^((n - 1)/p)*(a + b*x^n))^p, x)
```

3.453 $\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx$

Optimal. Leaf size=39

$$\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(n-p)(1+q)}$$

```
[Out] (a*x^n+b*x^p)^(1+q)/a/(n-p)/(1+q)/(x^(p*(1+q)))
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.040$, Rules used = {2039}

$$\frac{x^{-p(q+1)}(ax^n+bx^p)^{q+1}}{a(q+1)(n-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(n-p)(1+q)}$$

Mathematica [A]
time = 0.09, size = 40, normalized size = 1.03

$$-\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(-n+p)(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
[Out] -((a*x^n + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^(p*(1 + q))))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int x^{-1+n-p(q+1)}(a\,x^{n}+b\,x^{p})^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

```
[Out] int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="maxima")
```

```
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

Fricas [A]
time = 2.38, size = 76, normalized size = 1.95

$$\frac{(axx^{-pq+n-p-1}x^{n}+bxx^{-pq+n-p-1}x^{p})(ax^{n}+bx^{p})^{q}}{(an-ap+(an-ap)q)x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="fricas")
```

```
[Out] (a*x*x^(-p*q + n - p - 1)*x^n + b*x*x^(-p*q + n - p - 1)*x^p)*(a*x^n + b*x^
p)^q/((a*n - a*p + (a*n - a*p)*q)*x^n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^{n-p\,(q+1)-1}\,{\left(a\,x^n+b\,x^p\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q,x)
[Out] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q, x)
```

3.454 $\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx$

Optimal. Leaf size=40

$$-\frac{x^{-((n+p)(1+q))}(ax^n+bx^{n+p})^{1+q}}{ap(1+q)}$$

```
[Out] -(a*x^n+b*x^(n+p))^(1+q)/a/p/(1+q)/(x^((n+p)*(1+q)))
```

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2005, 2039}

$$-\frac{x^{-((q+1)(n+p))}(ax^n+bx^{n+p})^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Rule 2005

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx &= \int x^{-1-nq-p(1+q)}\left(ax^n+bx^{n+p}\right)^q\,dx\\ &= -\frac{x^{-(n+p)(1+q)}\left(ax^n+bx^{n+p}\right)^{1+q}}{ap(1+q)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 38, normalized size = 0.95

$$-\frac{x^{-((n+p)(1+q))}(x^n(a+bx^p))^{1+q}}{ap(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
[Out] -((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int x^{-1-qn-p(q+1)}(x^n(a+b\,x^p))^q\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-q*n-p*(q+1))*(x^n*(a+b*x^p))^q,x)
[Out] int(x^(-1-q*n-p*(q+1))*(x^n*(a+b*x^p))^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

Fricas [A]
time = 3.47, size = 64, normalized size = 1.60

$$-\frac{\left(bxx^{-(n+p)q-p-1}x^p+axx^{-(n+p)q-p-1}\right)(bx^nx^p+ax^n)^q}{apq+ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")
[Out] -(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p +
 a*x^n)^q/(a*p*q + a*p)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*q-p*(1+q))*(x**n*(a+b*x**p))**q,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^n\,\left(a+b\,x^p\right)\right)^q}{x^{n\,q+p\,\left(q+1\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1),x)
[Out] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```